Inflation Basics

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SSI 2004

Original Motivations (c. 1980)

1. Why is Universe so flat?

Today, $\Omega_{tot} = 1 \pm 0.02$ from CMB

1980: $\Omega_{tot} \simeq 0.3$ (more or less)

Either way, at BBN,

$\Omega_{tot} = 1 \pm 10^{-16}$,

and fine tuning even worse earlier.

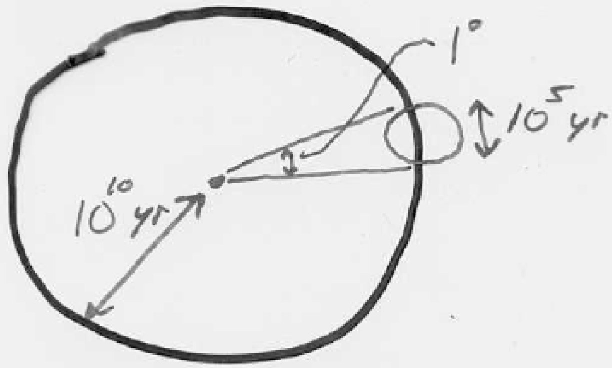
Why?

$$H^2 = \frac{8\pi G \rho_m}{3} + \frac{8\pi G \rho_{vac}}{3} - \frac{K}{a^2}$$

$\propto \frac{1}{a^3}$

Note: Inflation does <u>not</u> (generically) explain why $\Omega_m \sim \Omega_{vac}$ today.

2. Why is the Universe so smooth?



~40,000 causally-disconnected regions of early U have same T to ~10^{-5}.

Also, at BBN,

^4He, D, ^7Li
= nonlinear functions of ρ_b

⇒ Would get different abundances if ∃ small-scale fluctuations

Fluctuations must be small in early U on all scales.

⇒ perturbations ∃, but are small and nearly scale-invariant

3. Monopole (unwanted relic) problem:

GUTs $\Rightarrow$ 10^{16} GeV magnetic monopoles

"Kibble mechanism"

$\Rightarrow$ ~ 1 per horizon @ $T \sim 10^{16}$ GeV

$$H(t_{GUT}) \sim \frac{M_{GUT}^2}{m_{pl}} \qquad \rho_{GUT} \sim \frac{M_{GUT}}{(H_{GUT}^{-1})^3} \sim \frac{M_{GUT}^7}{m_{pl}^3}$$

$$\rho(\text{today}) \sim \frac{\rho_{GUT}}{(1+z_{GUT})^3}$$

$$\sim \frac{M_{GUT}^7}{m_{pl}^3}\left(\frac{T_{CMB}}{T_{GUT}}\right)^3 \sim \frac{M_{GUT}^4\, T_{CMB}^3}{M_{pl}^3}$$

$$\sim \frac{(10^{16})^4(10^{-12})^3}{(10^{19})^3} \sim 10^{-21}\ \mathrm{GeV}^4$$

$$\sim 10^{15}\ \mathrm{eV}^4 \gg \rho_{crit} \sim 10^{-11}\ \mathrm{eV}^4$$

Inflation: Basic Idea

As Universe expands, ~~the~~ Hubble length H^{-1} increases. During inflation,

$$\frac{d}{dt}\left[\frac{H^{-1}}{a}\right] < 0,$$

comoving Hubble length decreases with time, and objects/info/curvature exit horizon leaving (classically) smooth, empty Universe.

$$H = \frac{\dot{a}}{a} \Rightarrow \frac{d}{dt}\left[\frac{H^{-1}}{a}\right] = \frac{d}{dt}\left[\frac{1}{\dot{a}}\right] = \frac{-\ddot{a}}{\dot{a}^2}$$

$$\Rightarrow \ddot{a} > 0 \quad \text{for inflation}$$

2nd Friedmann eqn: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

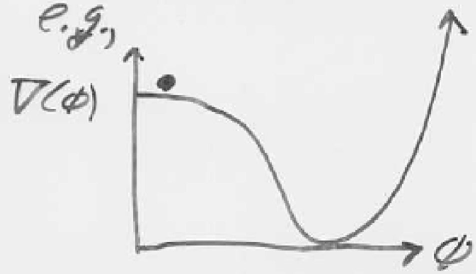
$\Rightarrow$ need $w = \frac{p}{\rho} < -\frac{1}{3}$

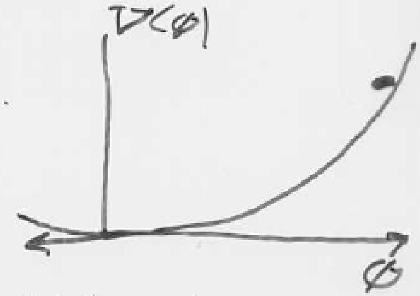
(~ dark energy)

Basic Mechanism

Postulate scalar field $\phi(\vec{x},t)$ with potential-energy density $V(\phi)$:

e.g.,





If field homogeneous (why?), then

energy density: $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$

pressure: $p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$,

so if $\dot{\phi}^2 < 2V(\phi) \Rightarrow$ inflation.

For flat $V(\phi)$, $\dot{\phi}^2 \propto a^{-6}$, $V(\phi) \propto a^0$;

so solution $\rightarrow$ inflation if $V(\phi)$ sufficiently flat.

Equations of motion:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_{pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} = \frac{-dV}{d\phi}$$

(i.e., $\Box\phi = \frac{-dV}{d\phi}$ in $ds^2 = dt^2 - a^2(t)\,d\vec{x}^2$)

Slow-roll approximation:

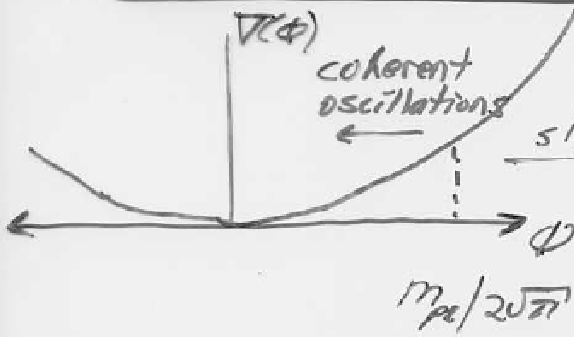
$$H^2 \simeq \frac{8\pi V}{3m_{pl}^2} \qquad 3H\dot{\phi} \simeq -V'(\phi)$$

if $\epsilon(\phi) \equiv \frac{m_{pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 \ll 1$

$$\eta(\phi) \equiv \left|\frac{m_{pl}^2}{8\pi}\frac{V''}{V}\right| \ll 1$$

(and require $\dot{\phi}$ sufficiently small at onset).

Simple Example: Chaotic Inflation



$V(\phi)=\frac{1}{2}m^2\phi^2 \qquad \frac{V'}{V}=\frac{2}{\phi} \qquad \frac{V''}{V}=\frac{1}{2\phi^2}$

slow-roll: $(\epsilon \lesssim 1)$ for $\phi \gtrsim \frac{1}{2\sqrt{\pi}} m_{pl}$

and $\eta \lesssim 1$ for $\phi \gtrsim \frac{m_{pl}}{4\sqrt{\pi}}$

For $\phi \lesssim \frac{m_{pl}}{2\sqrt{\pi}}$, field oscillates coherently:

$$\ddot{\phi} + 3\cancel{H\dot{\phi}} + m^2\phi = 0$$

when $H \ll m$, $\phi(t) \propto e^{\pm imt - \frac{3}{2}Ht}$,

so $\bar{\rho}(t) \propto \langle \dot{\phi}^2 \rangle \propto e^{-3Ht}$

i.e., gas of zero-momentum ϕ particles

Reheating: ϕ particles decay to SM particles which make up primordial plasma

More generally,

$$\ddot{a} = \dot{H} + H^2 \text{ must be } > 0.$$

Always true for $\dot{H} > 0$.

If $\dot{H} < 0$, require $\frac{-\dot{H}}{H^2} < 1$, but

$$-\frac{\dot{H}}{H^2} \simeq \frac{m_{pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 = \epsilon.$$

$\therefore$ Slow-roll ($\epsilon \ll 1$) guarantees inflation.

Some Exact Solutions:

Power-law inflation:

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{2}{p}}\frac{\phi}{m_{pl}}\right) \qquad a = a_0 t^p$$

$$\frac{\phi}{m_{pl}} = \sqrt{2p}\,\ln\left(\sqrt{\frac{V_0}{p(3p-1)}}\,\frac{t}{m_{pl}}\right)$$

$$p > 1 \Rightarrow \text{inflation} \qquad \epsilon = \frac{n}{2} = \frac{1}{p}$$

Intermediate inflation:

$$a(t) \propto \exp(At^f) \qquad 0 < f < 1 \quad A > 0$$

$$V(\phi) \propto \left(\frac{\phi}{m_{pl}}\right)^{-\beta}\left(1 - \frac{\beta^2}{6}\frac{m_{pl}^2}{\phi^2}\right)$$

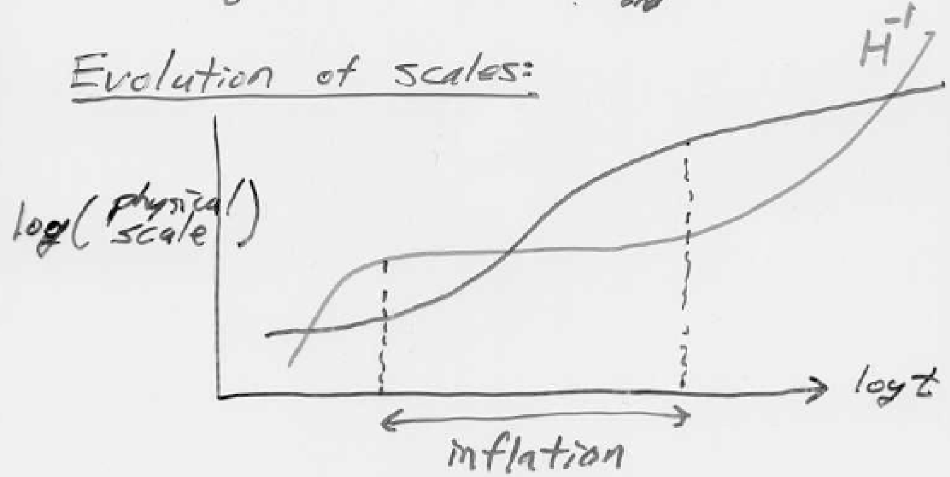
$$\beta = 4\left(\frac{1}{f} - 1\right)$$

Duration of Inflation:

$$N(t) \equiv \ln \frac{a(t_{end})}{a(t)} = \begin{pmatrix} \text{\# e-folding} \\ \text{between } t \text{ and} \\ \text{end of inflation} \end{pmatrix}$$

$$= \int_t^{t_{end}} H\,dt \simeq \frac{1}{m_{pl}^2} \int_{\phi_{end}}^{\phi} \frac{V}{V'}\,d\phi$$

Evolution of scales:



biggest co-moving scales exit horizon <u>first</u> during inflation, and <u>last</u> during matter/radiation domination

$$\frac{k_{phys}}{a_0 H_0} = \frac{a_k H_k}{a_0 H_0}$$

$$= \frac{a_k}{a_{end}} \frac{a_{end}}{a_{reh}} \frac{a_{reh}}{a_{eq}} \frac{a_{eq}}{a_0} \frac{H_k}{H_0}$$

so

$$N(k) = 62 - \ln \frac{k_{phys}}{a_0 H_0} - \ln \frac{10^{16}\,\mathrm{GeV}}{V_k^{1/4}}$$

$$+ \ln \frac{V_k^{1/4}}{V_{end}^{1/4}} - \frac{1}{3} \ln \frac{V_{end}^{1/4}}{\rho_{reh}^{1/4}}$$

$\Rightarrow$ require $\gtrsim 62$ e-foldings of inflation to explain current smooth horizon.

Hamilton-Jacobi Formalism:

replace $t \to \phi$

$$\Rightarrow \quad [H'(\phi)]^2 - \frac{12\pi}{m_{pl}^2} H^2(\phi) = \frac{-4\pi}{m_{pl}^4} V(\phi)$$

is <u>exact</u>

- Generates exact solutions
- Can make conditions for inflation (ϵ_H, η_H) precise
- Can show that if $H_0(\phi)$ is inflating solution, then another solution $H(\phi) = H_0(\phi) + \delta H(\phi)$ will have δH decay exponentially; i.e., perturbations decay during inflation.

Density Perturbations

Free scalar field: ← in Minkowski space

$$L = \frac{1}{2}\int d^3\vec{r}\,[\dot{\phi}^2 - (\vec{\nabla}\phi)^2 - m^2\phi^2]$$

$$= \frac{1}{2}\sum_{\vec{k}}[|\dot{\phi}_{\vec{k}}|^2 - E_{\vec{k}}^2|\phi_{\vec{k}}|^2]$$

with $E_{\vec{k}}^2 = \vec{k}^2 + m^2$

Quantization:

$$\phi_{\vec{k}} = w_{\vec{k}}(t)\,a_{\vec{k}} + w_{-\vec{k}}^*(t)\,a_{-\vec{k}}^+$$

$$w_{\vec{k}} = \sqrt{\frac{1}{2E_{\vec{k}}}}\,e^{-iE_{\vec{k}}t}$$

and $[a_{\vec{k}}, a_{\vec{k}'}^+] = \delta_{\vec{k}\vec{k}'}$ $\quad [a_{\vec{k}}, a_{\vec{k}'}] = 0$

i.e., each $\vec{k}$ mode is independent SHO.

Vacuum Fluctuations:

$$\langle 0|\,|\phi_{\vec{k}}|^2\,|0\rangle \equiv \langle |\phi_{\vec{k}}|^2\rangle = |w_{\vec{k}}|^2 = \frac{1}{2E_{\vec{k}}}$$

Power spectrum: $P_\phi(k) = \frac{k^3}{2\pi^2}|w_{\vec{k}}|^2$

$$= \frac{k^3}{4\pi^2 E_{\vec{k}}}$$

Amplitudes $\phi_{\vec{k}}$ have Gaussian wave fns in vacuum

Inflaton perturbations:

$$\ddot{\phi} + 3H\dot{\phi} + \nabla^2\phi + \frac{dV}{d\phi} = 0$$

Let $\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$. Then

$$(\ddot{\delta\phi})_{\vec{k}} + 3H(\dot{\delta\phi})_{\vec{k}} + \left(\frac{k}{a}\right)^2 \delta\phi_{\vec{k}} + \frac{1}{2}m^2\delta\phi_{\vec{k}} = 0$$

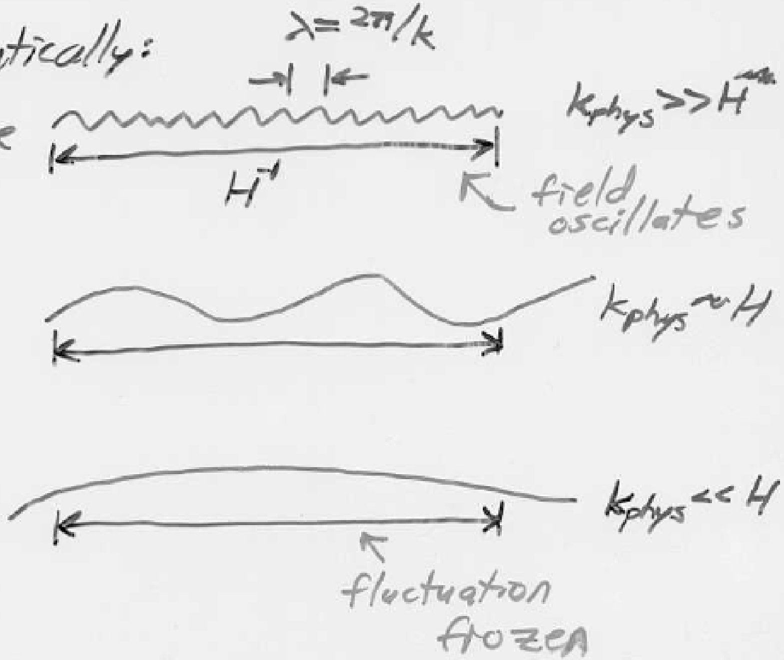
where $m^2 = V''$ and $(\delta\phi) \ll \phi$

I.e., to lowest order in $\delta\phi$, each $\vec{k}$ mode evolves independently

Schematically:

early time
$\frac{k}{a} \gg H$
$\lambda \ll H^{-1}$

late time
$\frac{k}{a} \ll H$
$\lambda \gg H^{-1}$



Slow-roll: $\eta \propto \frac{V''}{V} << 1 \Rightarrow m^2 << H^2$, so

$$(\delta \ddot{\phi}_k) + 3H(\delta \dot{\phi}_k) + (k/a)^2 \delta\phi_k = 0,$$

or, writing $\delta\phi_k = w_k(t)\, a_k + w_k^*(t)\, a_k^\dagger$,

$$\ddot{w}_k + 3H \dot{w}_k + (k/a)^2 w_k = 0$$

which, for H=const, has soln:

$$w_k(t) = L^{-3/2} \frac{H}{(2k)^{3/2}} \left(i + \frac{k}{aH}\right) e^{ik/aH}$$

$$\stackrel{\text{early}}{=} \left(\frac{1}{aL}\right)^{3/2} \sqrt{\frac{1}{2E_k}}\, e^{-iE_k t}$$ flat-space result

(with $E_k = k/a$),

which has, at early times, $\langle |\delta\phi_k|^2 \rangle = |w_k|^2$.

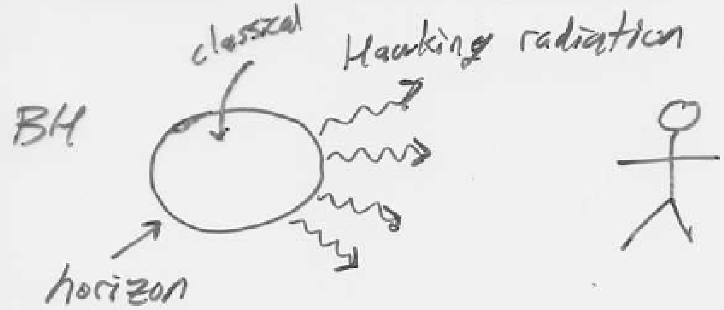
At time t_* shortly after horizon exit,

$$\langle |\delta\phi_k|^2 \rangle = \frac{H^2(t_*)}{2L^3 k^3},$$

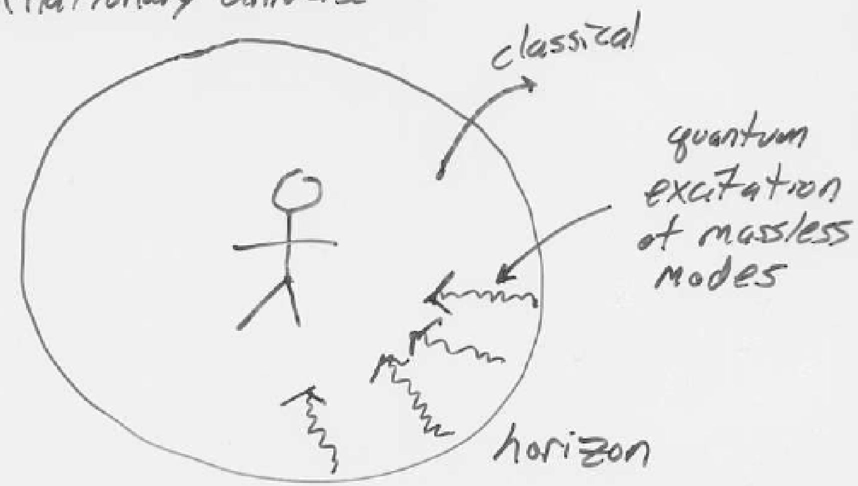
and classical spectrum of frozen ϕ fluctuations emerges with power spectrum,

and Gaussian! $$P_\phi(k, t_*) = \left(\frac{H}{2\pi}\right)^2 \Big|_{k=aH}$$

Black-hole analogy



Inflationary Universe



Evolution of perturbations from horizon exit to horizon re-entry to today is fully classical, but requires some nasty G.R. Result for power spectrum of curvature $\mathcal{R}^*$ (total density) is

$$P_{\mathcal{R}}(k) = \frac{8}{3\pi^2}\frac{V}{m_{pl}^2\,\epsilon} \propto \frac{V^3}{V'^2}$$

with V, ϵ evaluated at $k = aH$

COBE: $\left[P_{\mathcal{R}}(k)\right]^{1/2} \simeq 5\times 10^{-5}$ at $k = a_0 H_0$

$$\Rightarrow \frac{V^{3/2}\, 16\sqrt{2}\,\pi^{3/2}}{V'\, m_{pl}^3} = 5.2\times 10^{-4}$$

(assuming no GW contribution).

or $\dfrac{V^{1/4}}{\epsilon^{1/4}} = 6.6\times 10^{16}$ GeV

$$\Rightarrow V^{1/4} \lesssim 6\times 10^{16}\ \text{GeV}$$

* $\mathcal{R} \rightarrow \Phi$ = Newtonian potential on subhorizon scales

The (scalar or density-perturbation) spectral index.

Matter power spectrum $P(k) \propto k^{n_s}$
related to Φ through Poisson,

$$\nabla^2 \Phi = 4\pi G \bar{\rho}\, \delta \qquad \delta \equiv \frac{\delta\rho}{\rho}$$

So $P_{\mathcal{R}} \propto k^{n_s - \cancel{4}1}$

$$\Rightarrow \quad n_s - \cancel{4}1 \equiv \frac{d \ln P_{\mathcal{R}}}{d \ln k}$$

Using $d\ln k = \frac{dk}{k} = \frac{H\,da}{Ha} = \frac{da}{a} = \frac{\dot{a}}{a} dt = H dt$

and $dt = -(3H/V') \, d\phi$,

$$\frac{d}{d\ln k} = \frac{-m_{pl}^2}{8\pi} \frac{V'}{V} \frac{d}{d\phi}$$

$$\Rightarrow \boxed{n_s = 1 - 6\epsilon + 2\eta}$$

WMAP: $n_s = 0.99 \pm 0.04$

$$n_s \begin{cases} > 1 & \text{"blue" spectrum} \\ = 1 & \text{Peebles-Harrison-Zeldovich} \\ < 1 & \end{cases}$$

"Running" of the spectral index

$$\frac{dn}{d\ln k} = -16\epsilon\eta + 24\epsilon^2 + 2\xi^2 + HOT$$

$$\xi^2 \equiv \frac{m_{pl}^4}{64\pi^2}\frac{V'V'''}{V^2}$$

<u>Adiabatic vs. Isocurvature:</u>

If ϕ = inflaton, and $\delta\phi \to \delta\rho$, then decay of inflaton is same everywhere, so $\delta_x \equiv \delta\rho_x/\rho_x = \delta\rho/\rho$

for x = baryons, DM, ν's, γ's
i.e., no "entropy" perturbations
$\Longrightarrow$ <u>adiabatic</u>

<u>Isocurvature</u>: Suppose DM comes from decay of <u>non</u>-inflaton χ (a "spectator") that attains fluctuations $\delta\chi$ during inflation.
Then may have $\frac{\delta\rho_b}{\rho_b} \neq \frac{\delta\rho_\gamma}{\rho_\gamma} \neq \frac{\delta\rho_{dm}}{\rho_{dm}}$

Gravitational Waves:

Tensor perturbations h_{ij} to metric,

$$ds^2 = -dt^2 + a^2(t)\,d\vec{x}^2 + a^2 h_{ij}\,dx^i dx^j$$

satisfy KG eqn:

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + (k/a)^2 h_{ij} = 0$$

i.e., propagating massless modes = gravitons.
Get excited QM during inflation.
2 Polarization states $(+, \times)$ get power spectra,

$$P_+(k) = P_\times(k) = \tfrac{1}{2} P_{GW}(k) \propto \left(\frac{H}{2\pi}\right)^2.$$

Multiplicative coeff obtained by expanding Einstein-Hilbert action,

$$S = \frac{1}{16\pi G}\int \sqrt{-g}\, R\, d^4x$$

to quadratic order in h_+, $h_\times$. Result:

$$\Rightarrow P_{GW}(k) = \frac{m_{pl}^2}{4\pi}\left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH}$$

primordial!!

GW spectral index:

$$n_{grav} = \frac{d\ln P_{GW}(k)}{d\ln k} = -2\epsilon$$

Processed spectrum:

For $\epsilon = 0$, all h same as enter horizon. Then, h decays by

$$\rho_{GW} \propto \frac{1}{a^4} \propto \dot{h}^2 \propto (k/a)^2 h^2$$

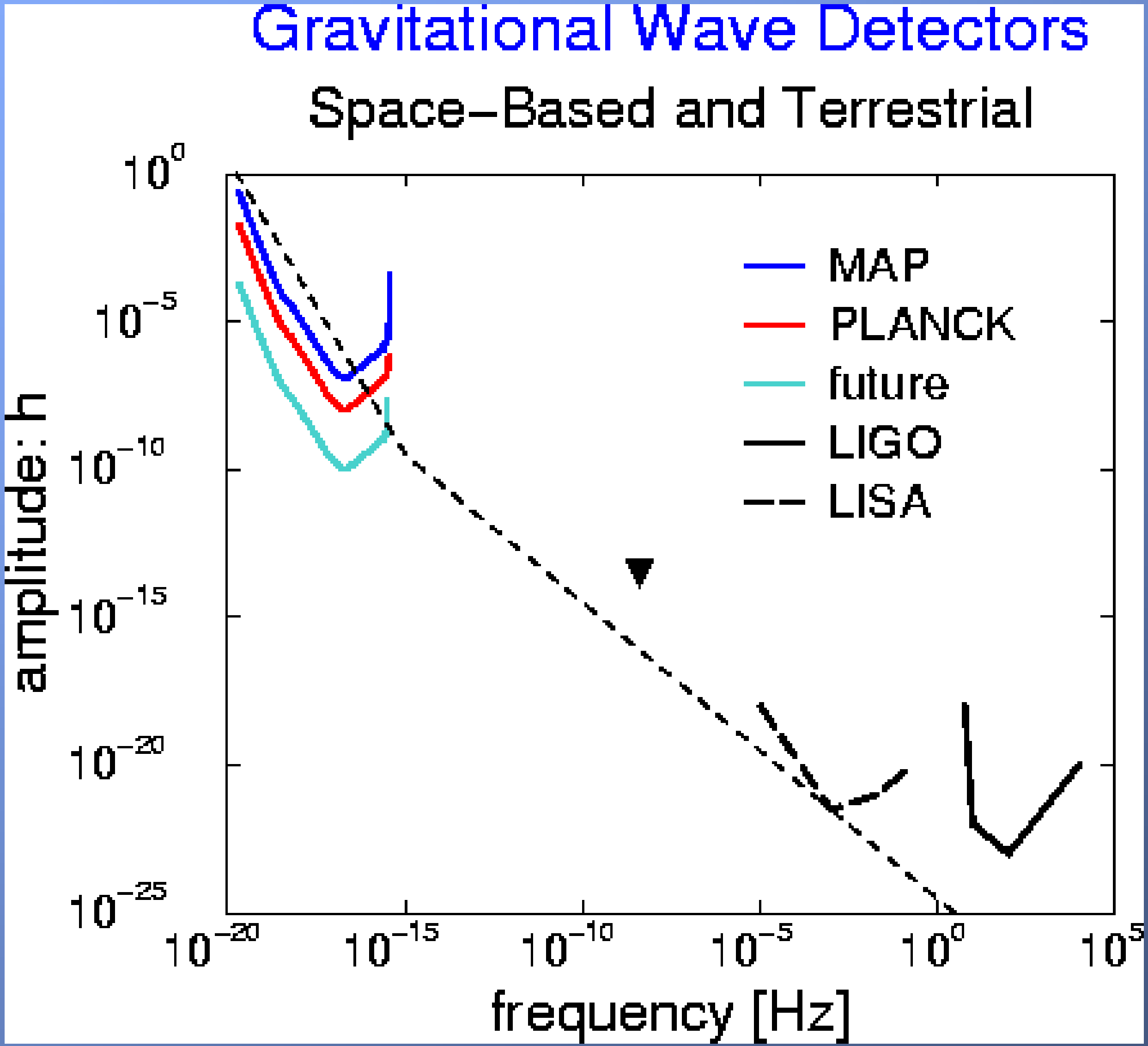
$\Rightarrow h \propto a^{-1}(t)$ after horizon crossing.

For modes $k > k_{eq}$ that enter horizon during RD, $aH \propto a^{-1}$, so $a \propto k^{-1}$ at horizon crossing, and so

$h_{\vec{k}} \propto k^{-1}$ today for $k > k_{eq}$.

(similar arguments for $k < k_{eq}$ leading to steeper low-k spectrum).

Important: $h_{\vec{k}} \propto \sqrt{V}$



Caldwell, MK, Wadley

Non-Gaussianity:

(e.g., Allen, Grinstein, Wise 1987)

Lowest order in $\delta\phi$: $\vec{k}$ modes uncorrelated and Gaussian distributed, and $(\delta\rho/\rho)$ and h uncorrelated.

Higher order: cubic terms in $V(\phi)$ and nonlinearities in Einstein action induce non-Gaussianity, to lowest order, in form of non-zero 3-pt function.

Roughly, the bispectrum amplitude is

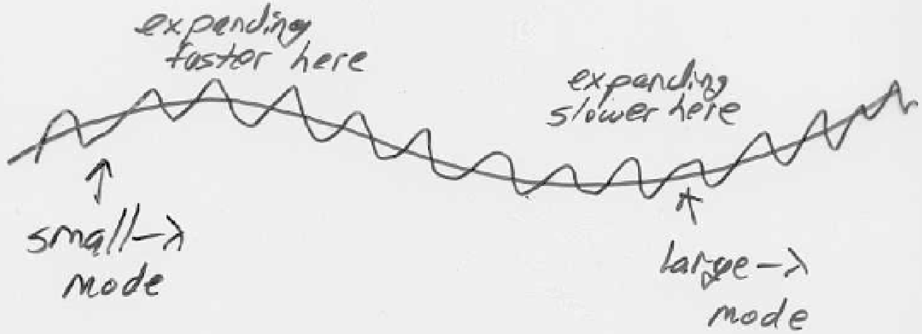
$$\frac{\langle \delta^3 \rangle}{\langle \delta^2 \rangle^2} \propto \epsilon.$$

(e.g., Wang, MK 2000)

Non-Gaussianity also in h, and will get $\langle \delta h h \rangle \neq 0$ $\langle \delta \delta h \rangle \neq 0$

(e.g., Maldacena 2002)

Why $\frac{\langle\delta^3\rangle}{\langle\delta^2\rangle^2} \propto \epsilon$:



If $\epsilon = 0$, fluctuations are scale-invariant, so small-λ modes in overdense regions have same amplitude as those in underdense regions.

Summary of Inflationary Observables

① Density-perturbation amplitude

$$\frac{\delta\rho}{\rho} \propto \frac{V^{3/2}}{V'}$$

② Spectral index for $\delta\rho/\rho$:

$$n_s = 1 - 6\epsilon + 2\eta$$

$$\epsilon \propto \left(\frac{V'}{V}\right)^2 \qquad \eta \propto \frac{V''}{V}$$

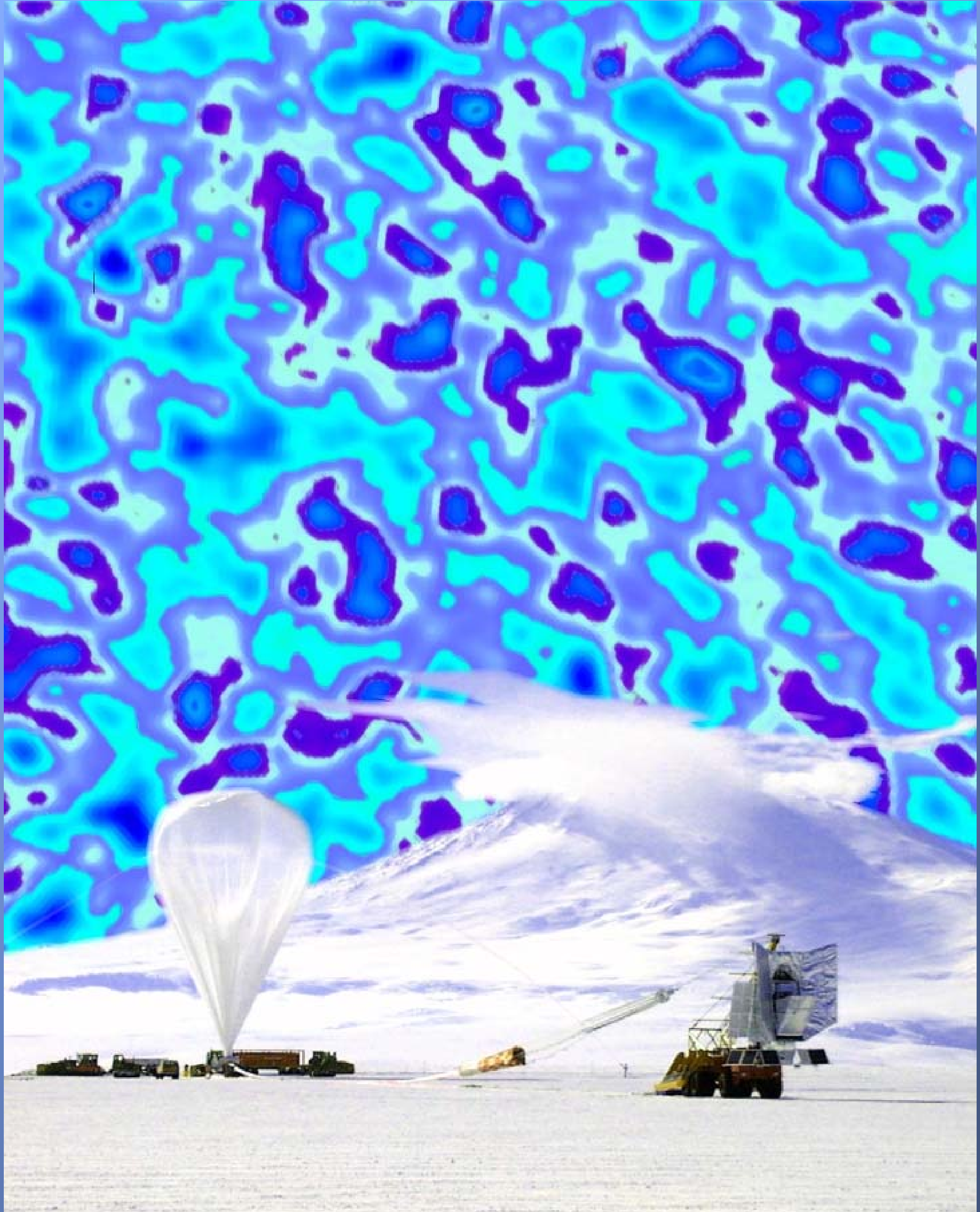
③ Running of spectral index
(higher order in $\frac{d}{d\phi}$ of $V(\phi)$)

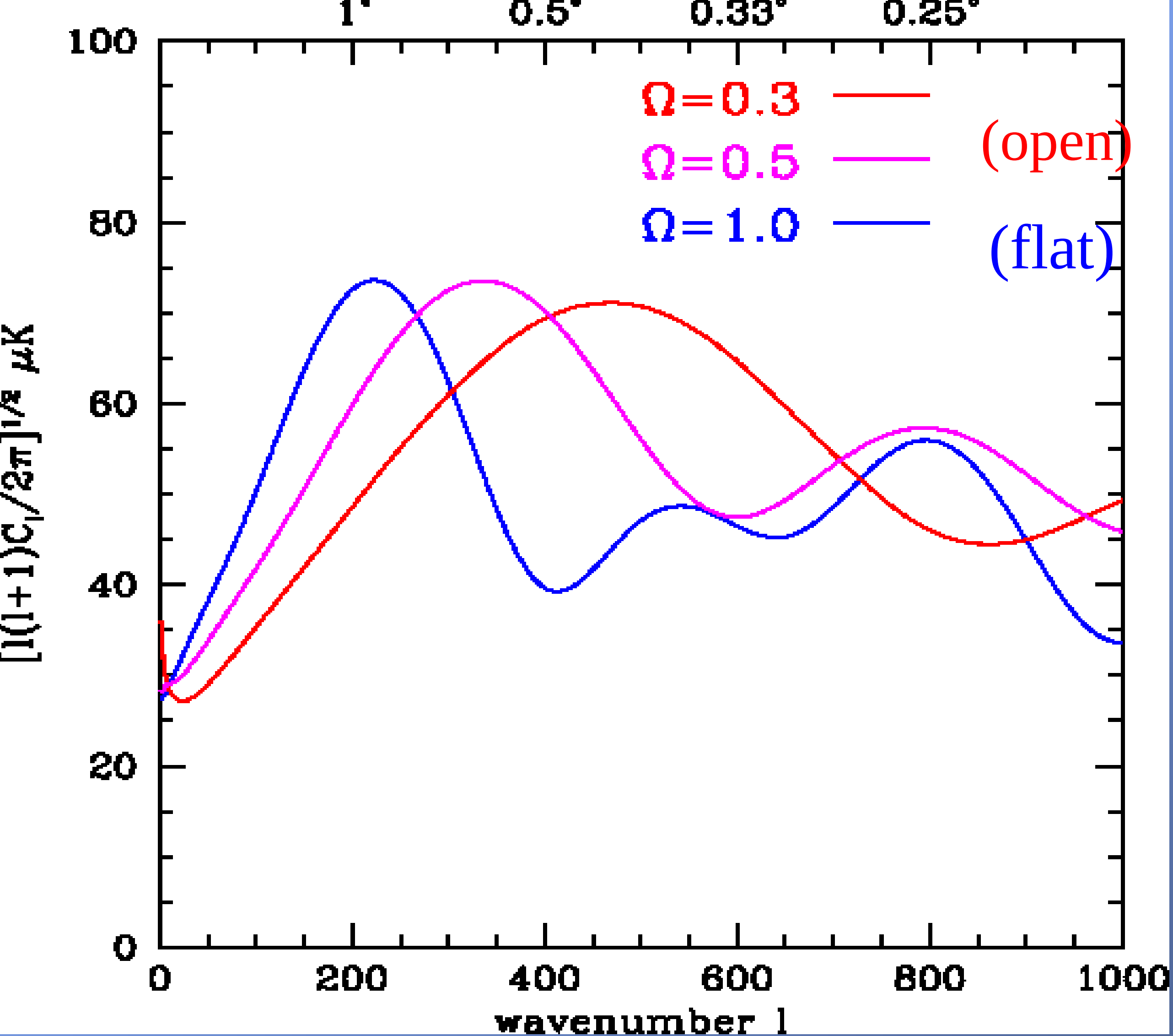
④ GW amplitude $\propto V$

⑤ GW spectral index $n_{grav} \propto \epsilon$

⑥ Non-gaussianity:

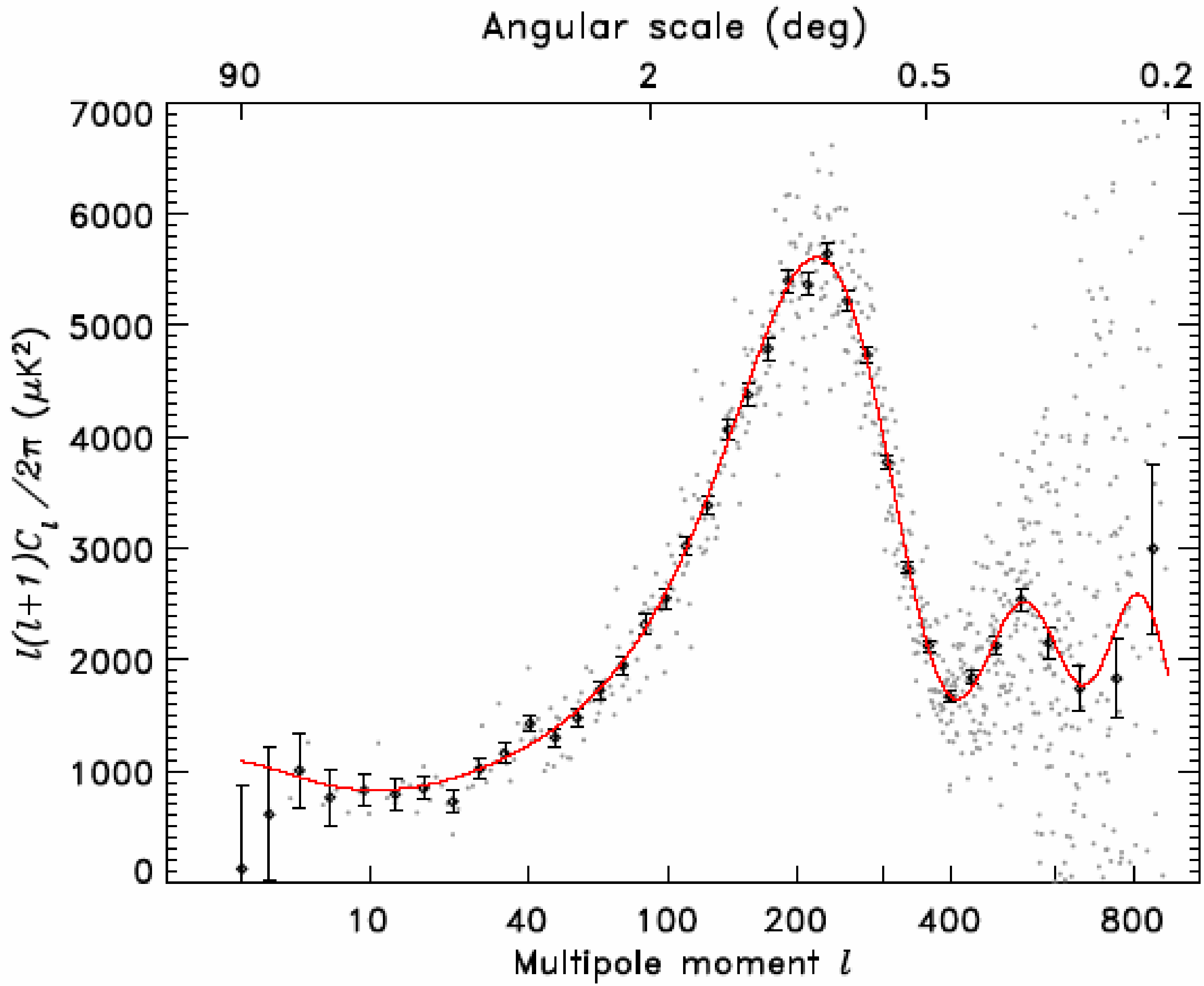
Bispectrum $\propto \epsilon$





CMB determination of the geometry

(MK, Spergel, and Sugiyama, 1994)


Angular scale (deg)
90
2
0.5
0.2
7000
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l/2\pi$ (μK^2)
10
40
100
200
400
800
Multipole moment l

Where did large scale structure (e.g., galaxies, clusters, larger-scale clustering) come from?

explosions

Late-time phase transitions

Superconducting cosmic strings

Cosmic strings

textures

Global monopoles

Soft phase transitions

Isocurvature CDM perturbations

Seed models

Primordial adiabatic perturbations

Isocurvature baryon perturbations

Rolling scalar fields

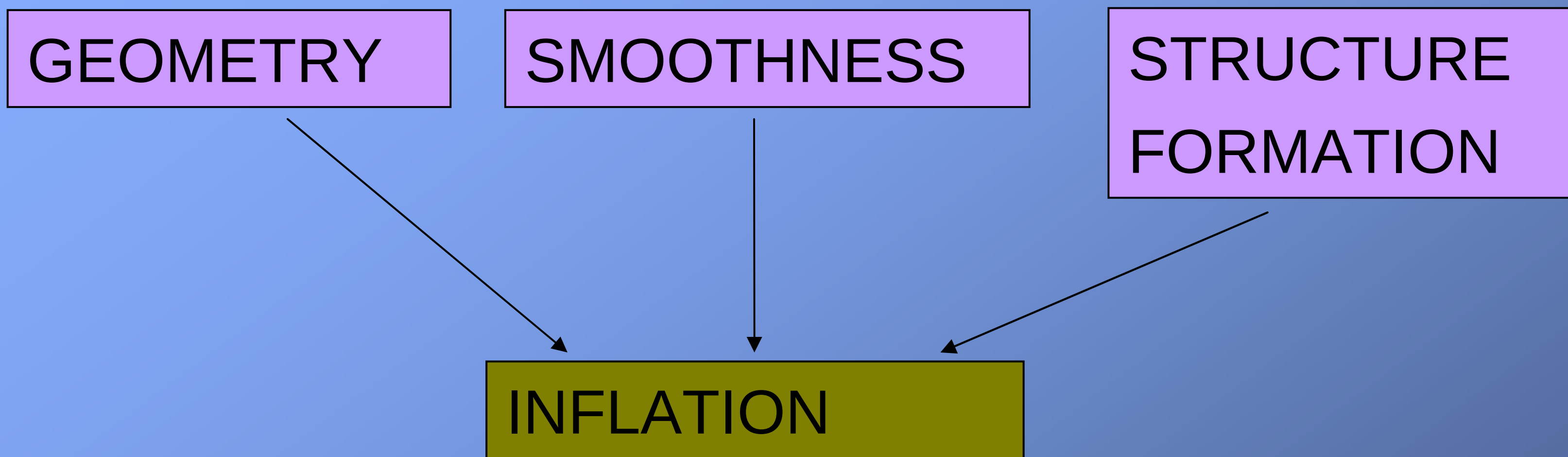
Loitering universe

Pre-CMB

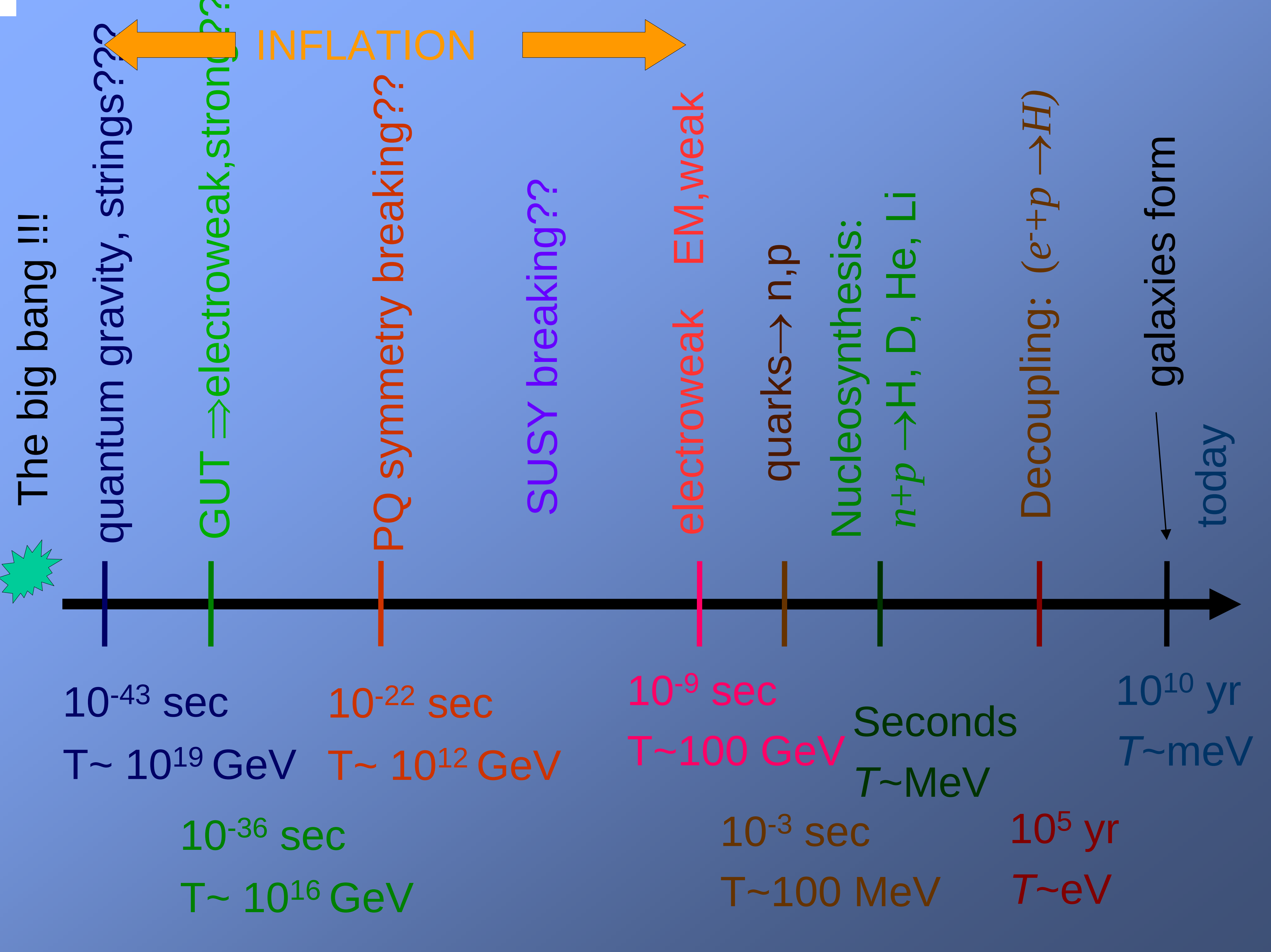
Where did large-scale structure (e.g., galaxies, clusters, larger-scale clustering) come from?

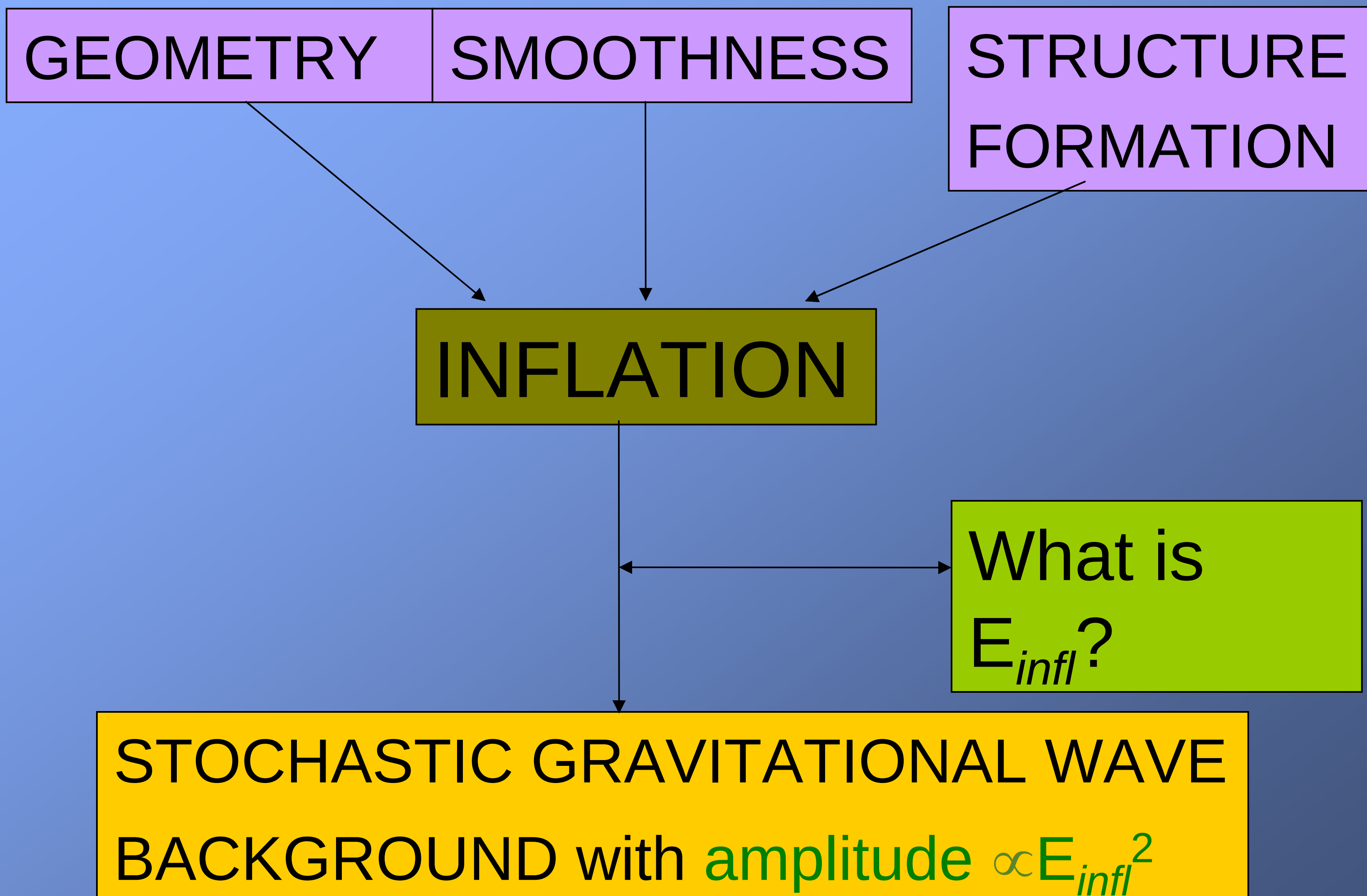
Post CMB:

gravitational infall from nearly scale-invariant spectrum of primordial adiabatic perturbations


GEOMETRY
SMOOTHNESS
STRUCTURE FORMATION
INFLATION

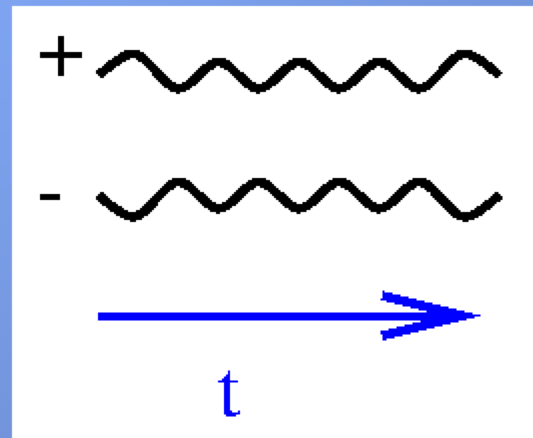
WHAT NEXT???


INFLATION
The big bang !!!
quantum gravity, strings???
GUT ⇒electroweak,strong??
PQ symmetry breaking??
SUSY breaking??
electroweak EM,weak
quarks→ n,p
Nucleosynthesis:
$n+p \rightarrow$H, D, He, Li
Decoupling: $(e^{-}+p \rightarrow H)$
galaxies form
today
10^{-43} sec
T~ 10^{19} GeV
10^{-36} sec
T~ 10^{16} GeV
10^{-22} sec
T~ 10^{12} GeV
10^{-9} sec
T~100 GeV
10^{-3} sec
T~100 MeV
Seconds
T~MeV
10^{5} yr
T~eV
10^{10} yr
T~meV


GEOMETRY
SMOOTHNESS
STRUCTURE FORMATION
INFLATION
What is E_{infl}?
STOCHASTIC GRAVITATIONAL WAVE BACKGROUND with amplitude $\propto E_{infl}^2$

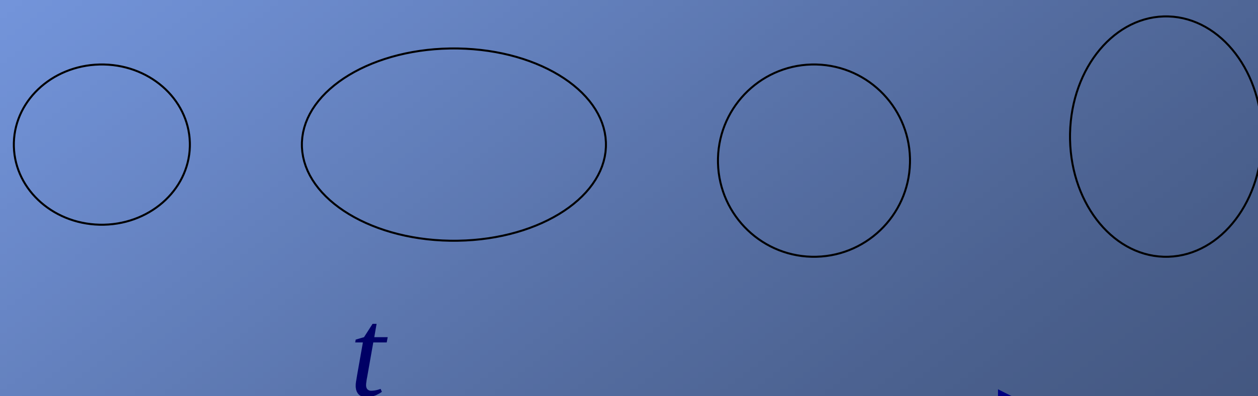
Detection of electromagnetic wave:

look for oscillations of test charges

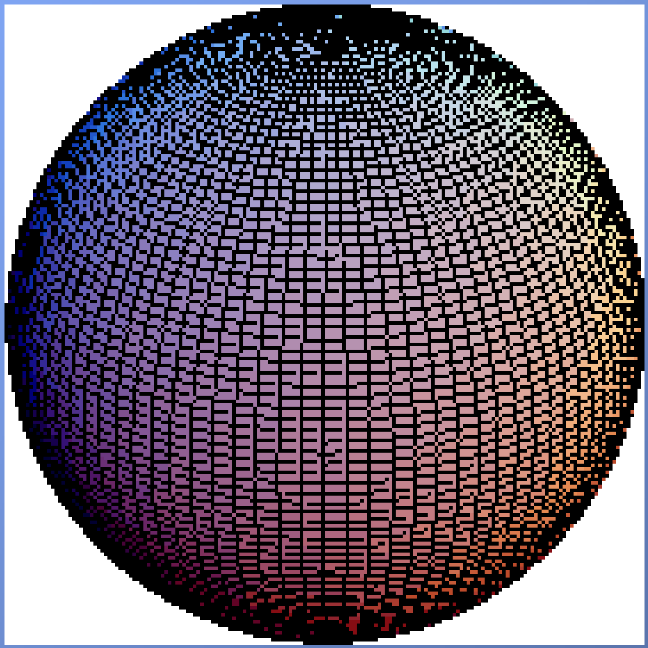


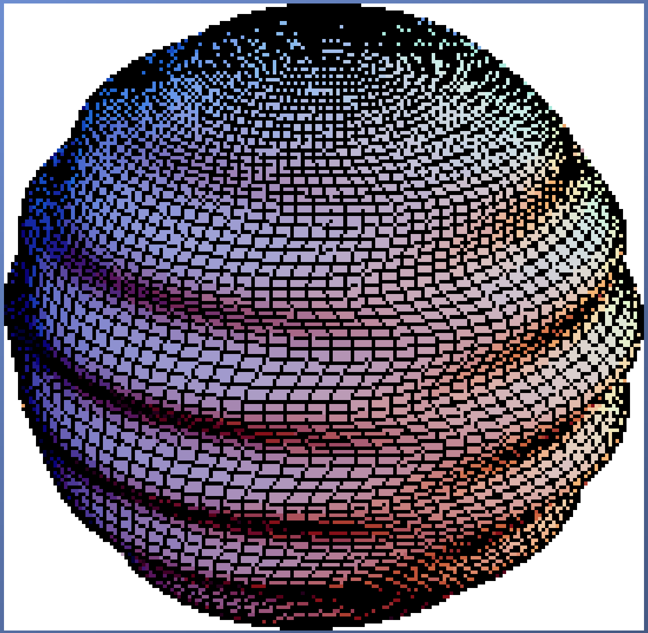
Detection of gravitational wave:

look for quadrupole oscillations of a ring of test masses

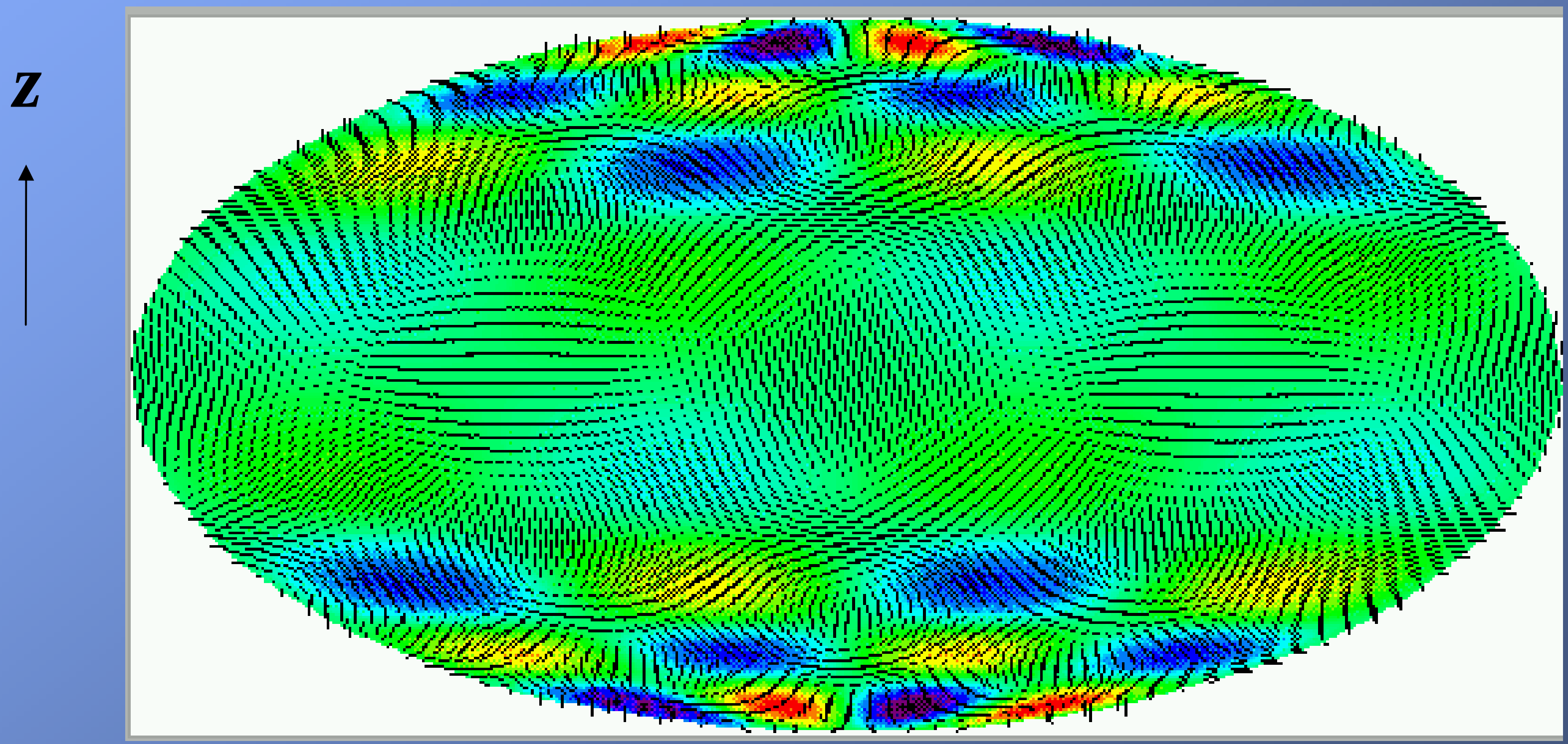


Detection of ultra-long-wavelength GWs from inflation: use plasma at CMB surface of last scatter as sphere of test masses.

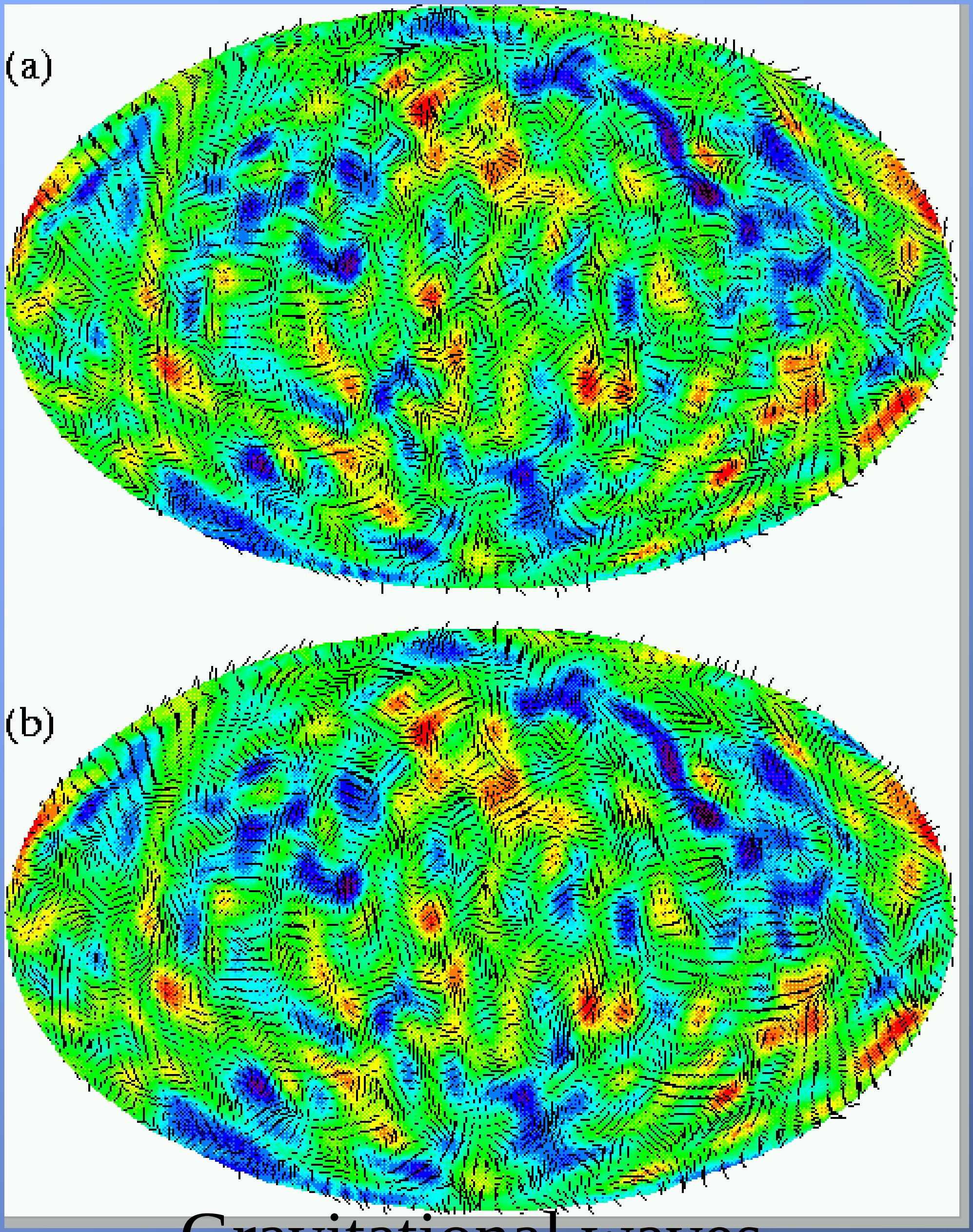




Temperature pattern produced by one gravitational wave oriented in ***z*** direction



Density perturbations



Gravitational waves

Inflation predicts *stochastic* background of GWs

Density perts and GWs can produce identical temperature patterns

Can be distinguished by the polarization!!

Detection of gravitational waves with CMB polarization

Temperature map: $T(\hat{n})$

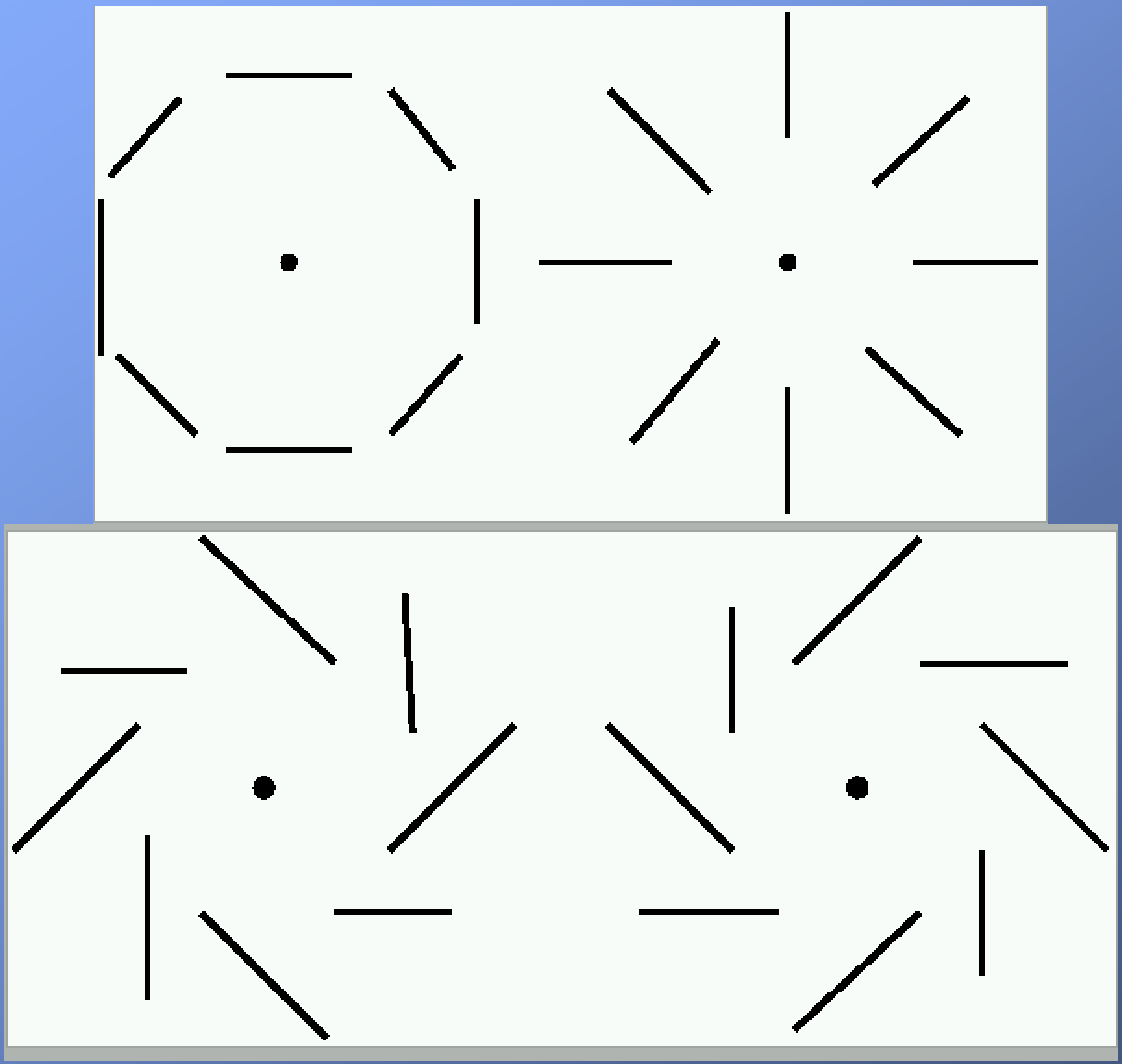
Polarization Map: $\vec{P}(\hat{n}) = \vec{\nabla} A + \vec{\nabla} \times \vec{B}$

Density perturbations have no handedness"
so they can*not* produce a polarization with a curl

Gravitational waves do have a handedness, so they can (and do) produce a curl

Model-independent probe of gravitational waves!

“Curl-free” polarization patterns



“curl” patterns

Recall, GW amplitude is $\propto E_{\mathrm{infl}}^{2}$

GWs $\Rightarrow$ ΔT

And from COBE, $E_{\mathrm{infl}} < 3\times10^{16}$ GeV

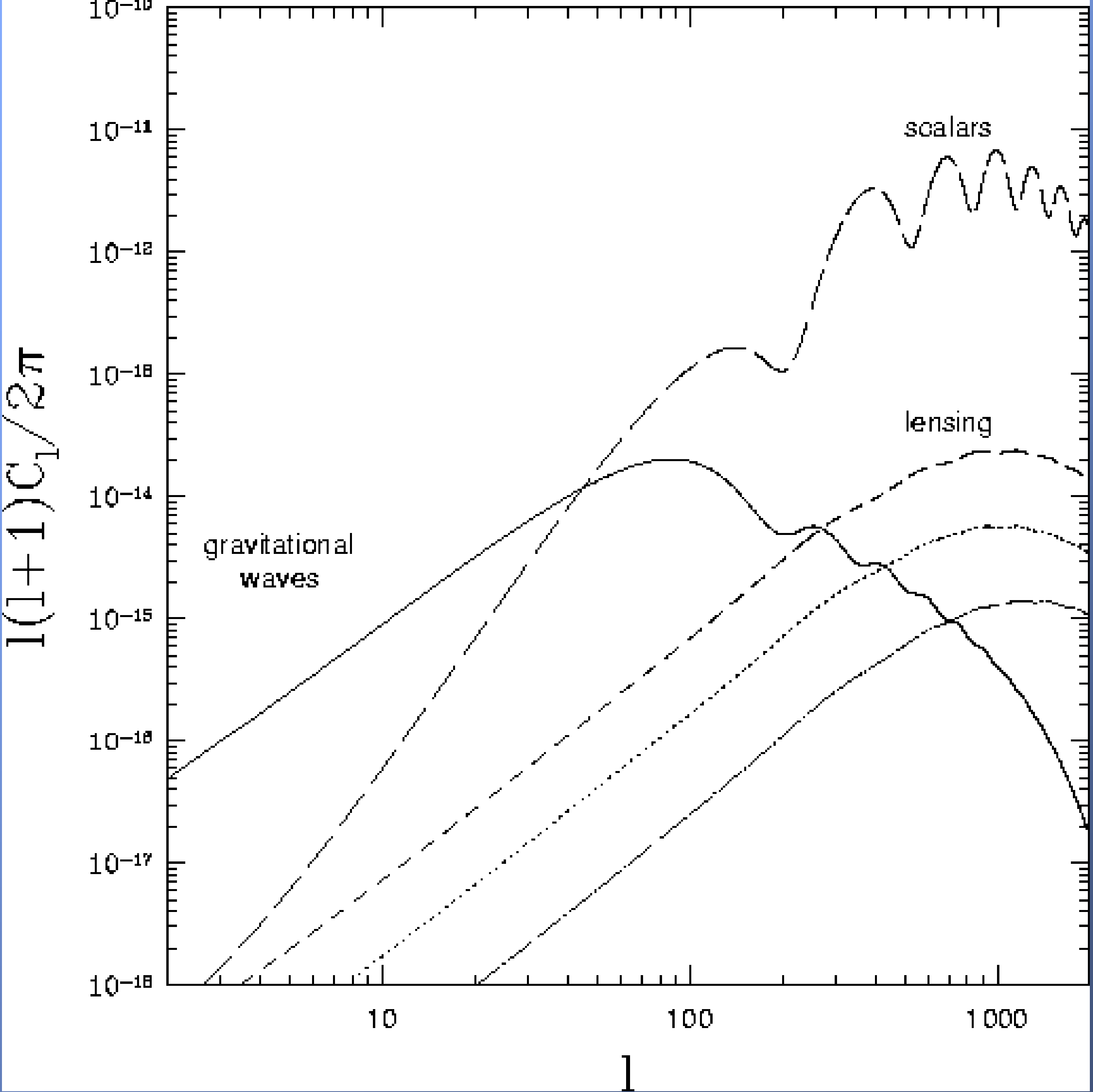
GWs $\Rightarrow$ unique polarization pattern. Is it detectable?

If $E << 10^{15}$ GeV (e.g., if inflation from PQSB), then polarization far too small to ever be detected.

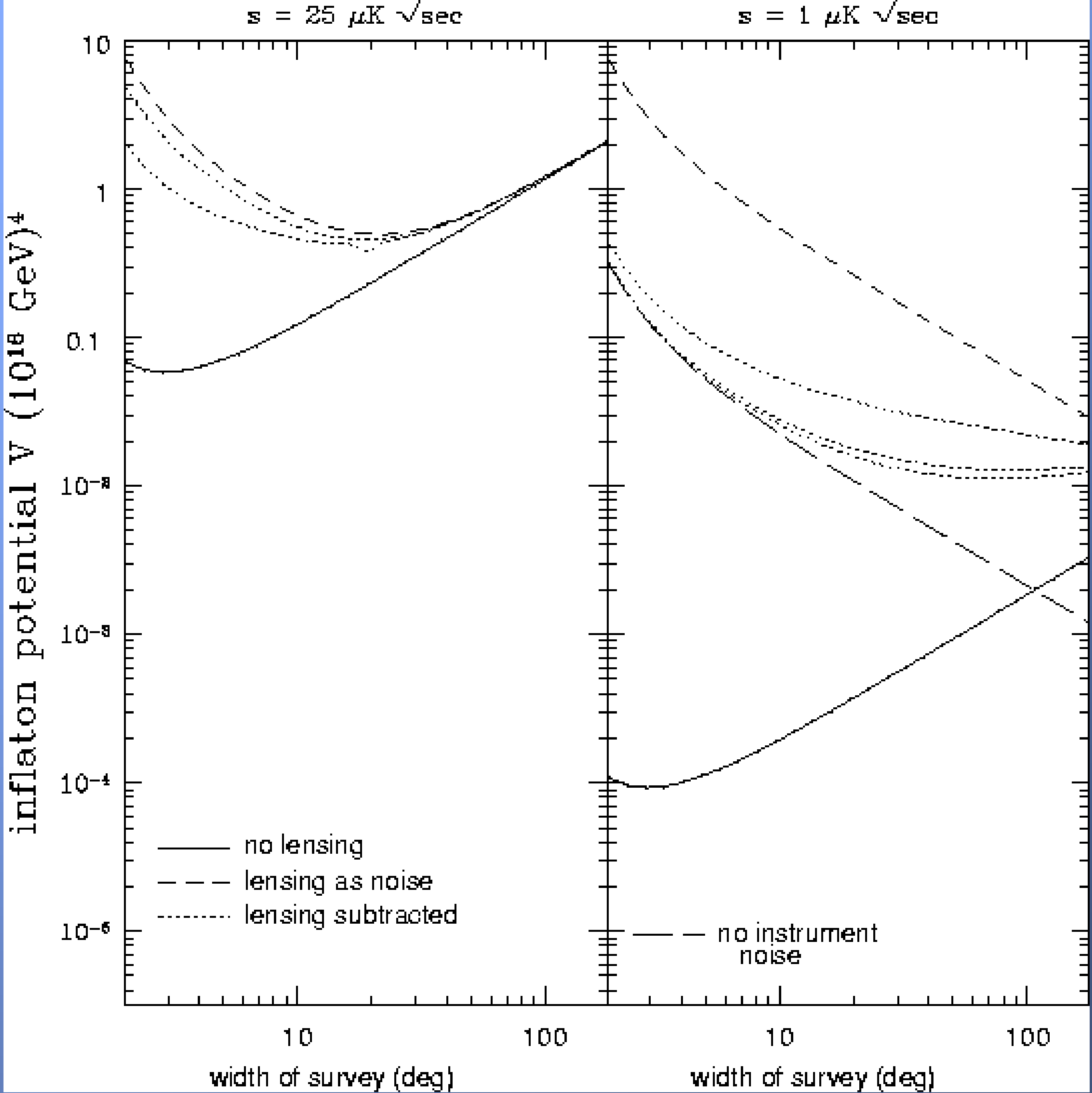
But, if $E \sim 10^{15\text{-}16}$ GeV (i.e., if inflation has something to do with GUTs), then polarization signal is conceivably detectable by Planck or realistic post-Planck experiment!!!

(Kesden, Cooray, MK 2002;
Knox, Song 2002)

To go beyond Planck, will require high resolution temperature and polarization maps to disentangle cosmic shear contribution to curl component from that due to inflationary gravitational waves.



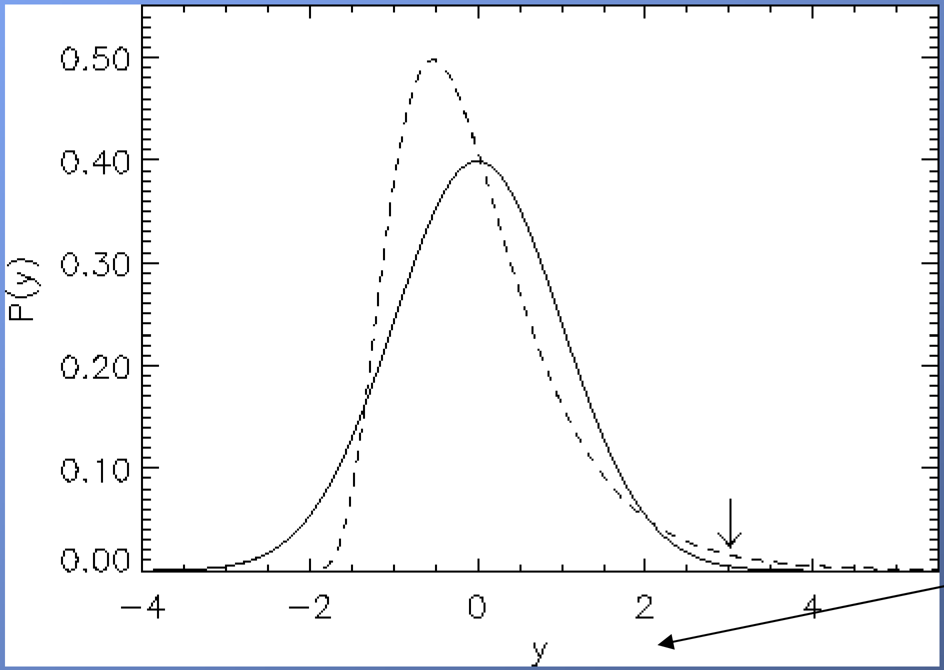
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10
100
1000
$l(l+1)C_l/2\pi$
l
scalars
lensing
gravitational waves



s = 25 μK √sec
s = 1 μK √sec
10
1
0.1
10⁻²
10⁻³
10⁻⁴
10⁻⁵
inflaton potential V (10¹⁶ GeV)⁴
no lensing
lensing as noise
lensing subtracted
no instrument noise
10
100
width of survey (deg)
width of survey (deg)

What Else??

Inflation predicts distribution of primordial density perturbations is Gaussian (e.g., Wang &MK, 2000).



But how do we tell if primordial perturbations were Gaussian??

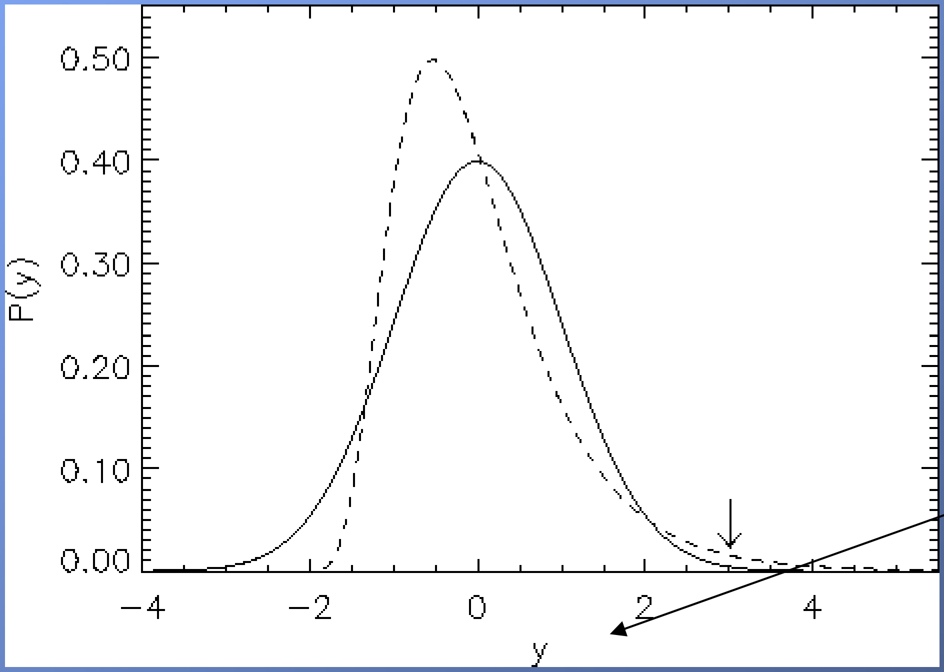
δ/σ

How do we tell if primordial perturbations were Gaussian??

(1) With the CMB:

Advantage: see primordial perturbation directly

Disadvantage: perturbations are small



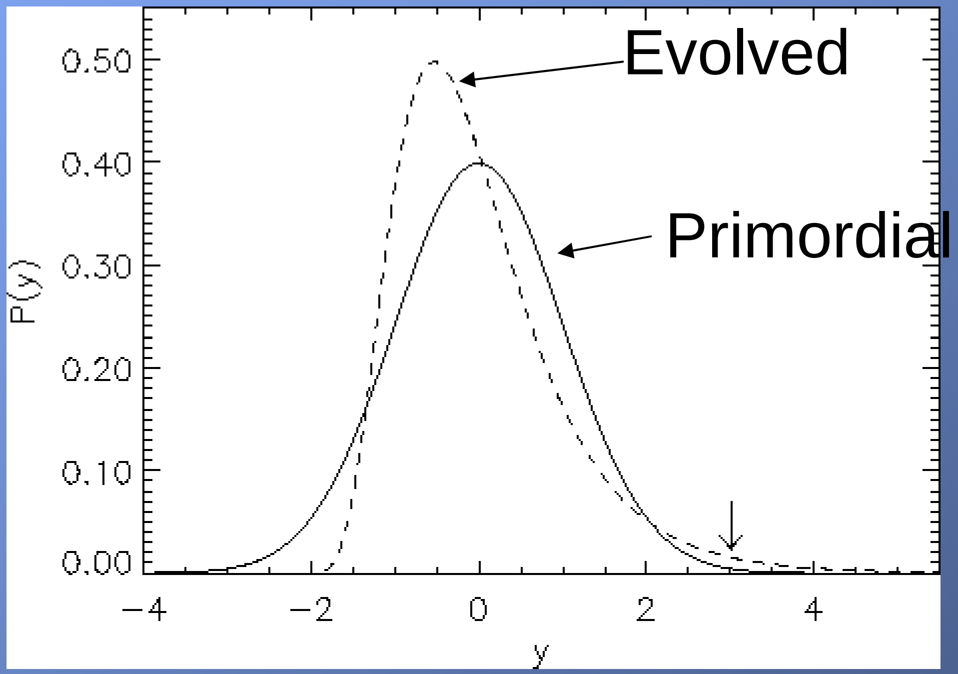
$\Delta T/T$

How do we tell if primordial perturbations were Gaussian??

(2) With galaxy surveys:

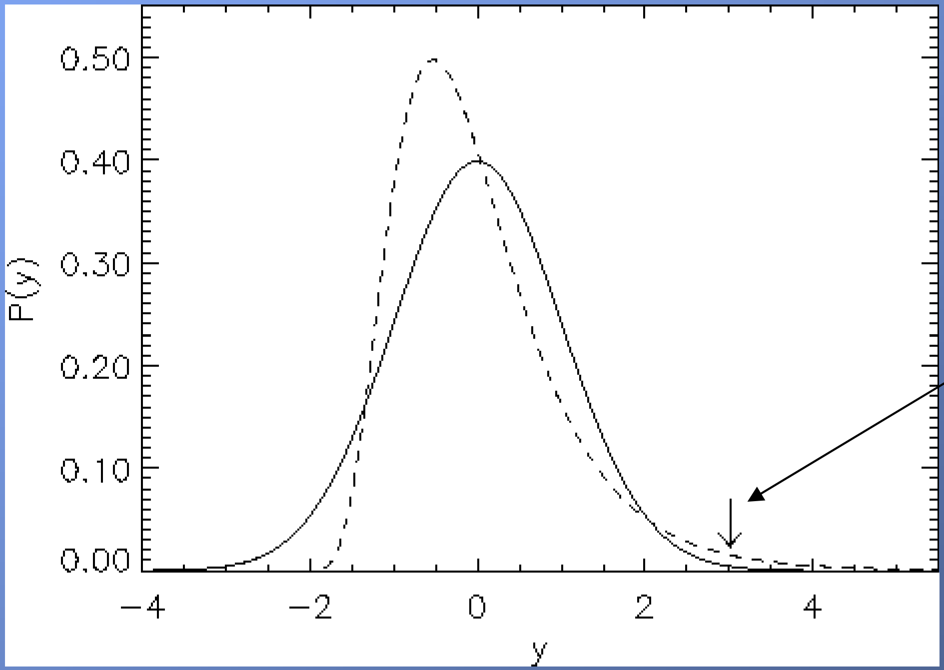
Advantage: perturbations are bigger

Disadvantage: gravitational infall induces non-Gaussianity (as may biasing)



How do we tell if primordial perturbations were Gaussian??

(3) With abundances of clusters (e.g., Robinson, Gawiser & Silk 2000) or high redshift galaxies (e.g., Matarrese, Verde & Jimenez 2000):

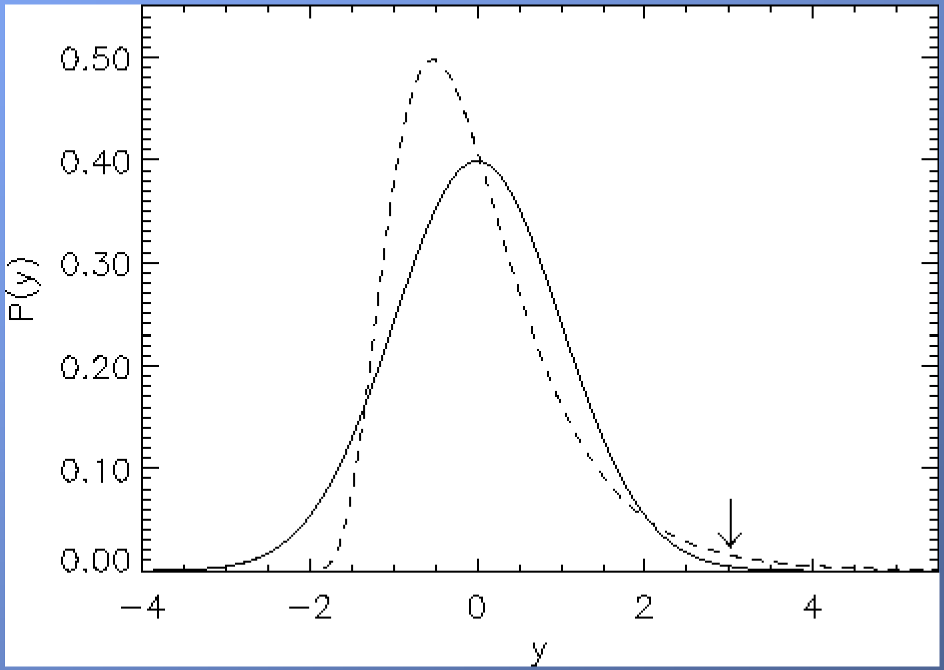


Rare objects form here

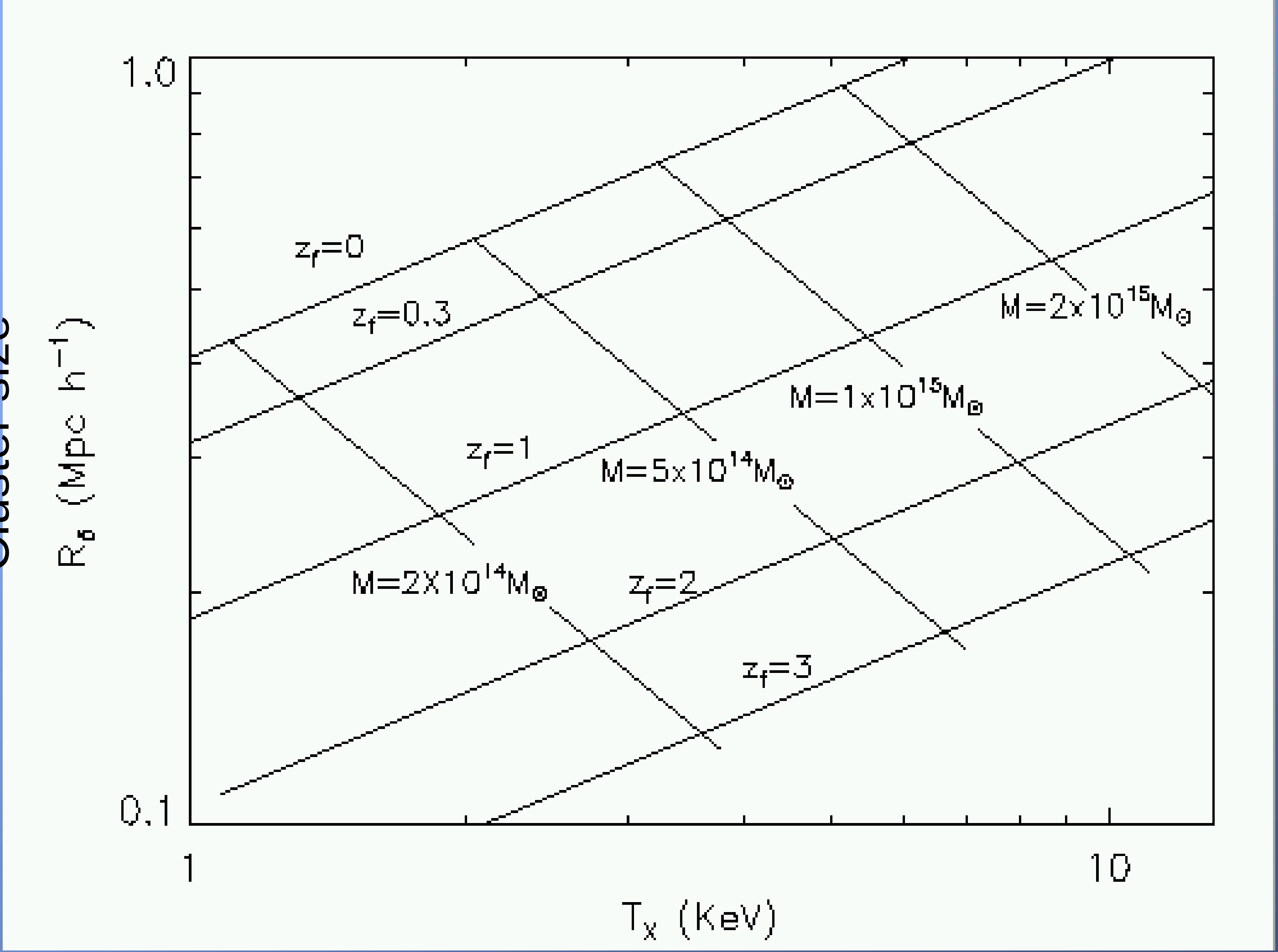
How do we tell if primordial perturbations were Gaussian??

(4) With distribution of cluster sizes

(Verde, MK, Mohr & Benson, MNRAS, 2001):

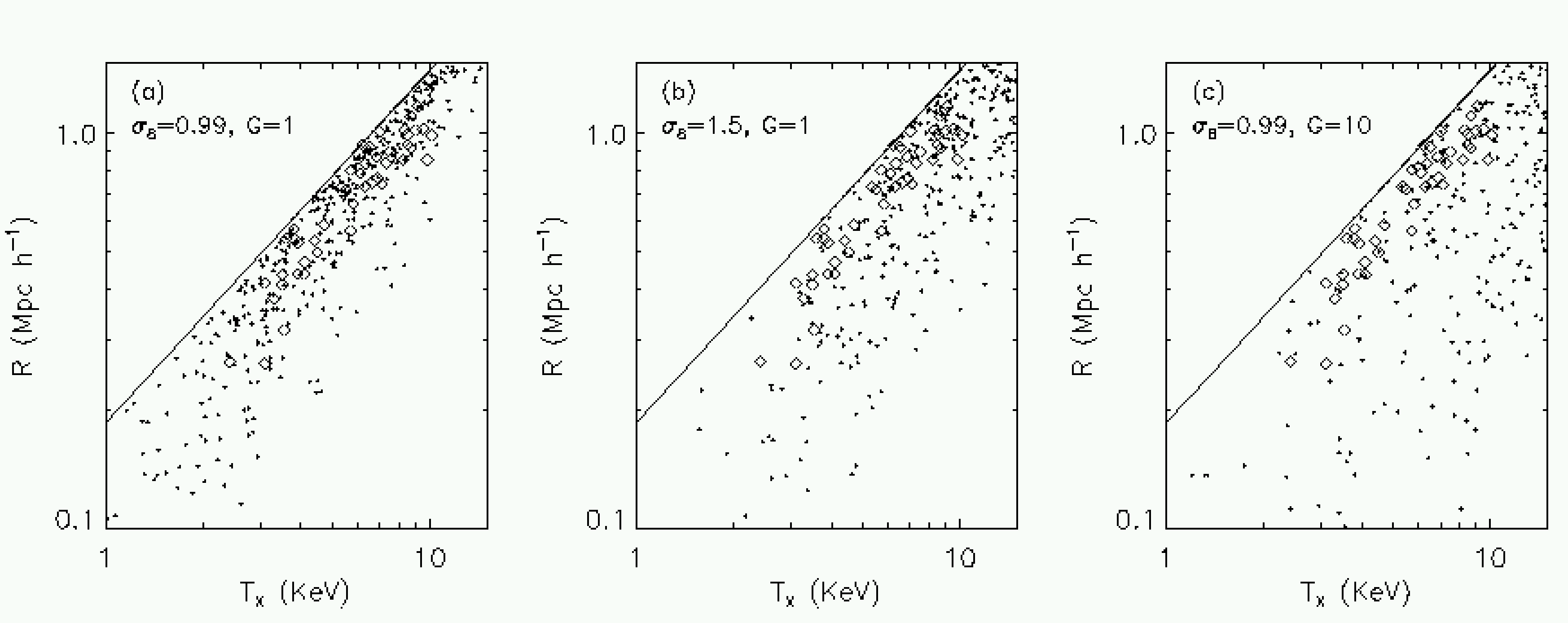


Broader distribution of >3σ peaks leads to broader formation redshift distribution and thus to broader size distribution

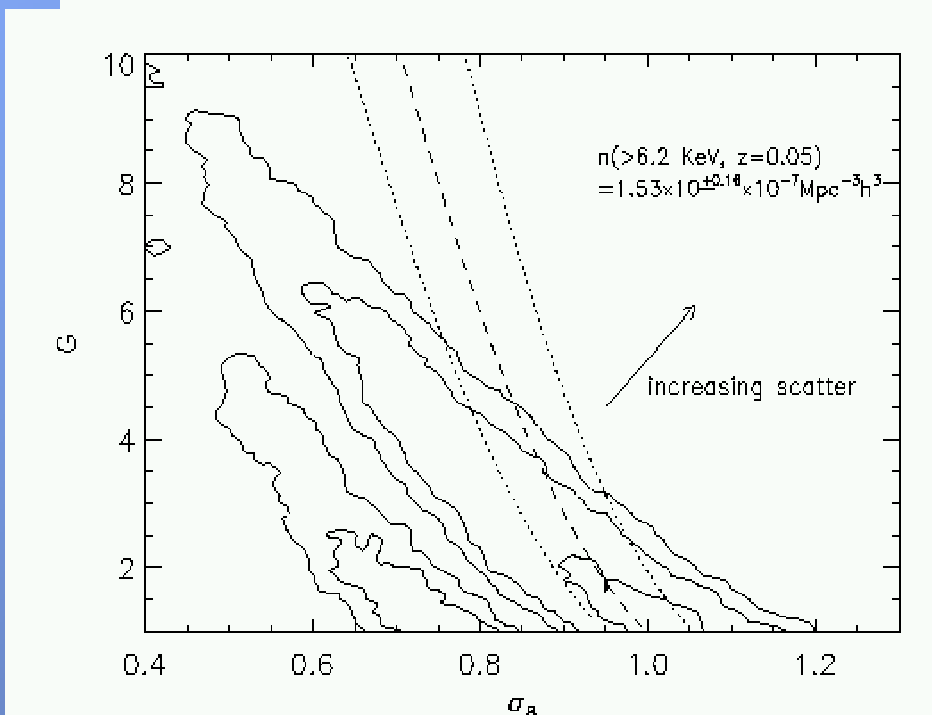


Cluster size

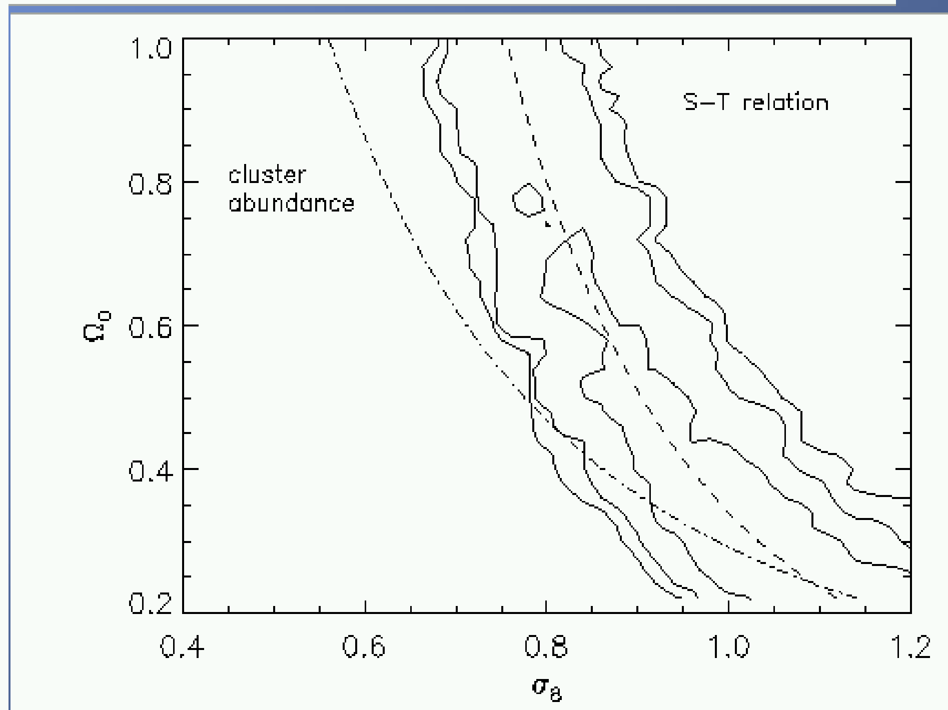
Cluster temperature



(a)
σ8=0.99, G=1
(b)
σ8=1.5, G=1
(c)
σ8=0.99, G=10
R (Mpc h⁻¹)
Tx (KeV)



n(>6.2 KeV, z=0.05)
=1.53×10^±0.16×10^-7 Mpc^-3 h^3
increasing scatter
G
σ8



S–T relation
cluster
abundance
Ωo
σ8

How do we tell if primordial perturbations were Gaussian??

How do these different avenues compare

For just about any nonGaussianity with long range correlations (e.g., from topological defects or funny inflation), CMB> LSS (Verde, Wang, Heavens & MK, 2000).

Cluster and high *z* abundances do better at probing nonGaussianity from topological defects than CMB/LSS, while CMB remains best probe of that from funny inflation (Verde, Jimenez, Matarrese & MK, MNRAS 2001).

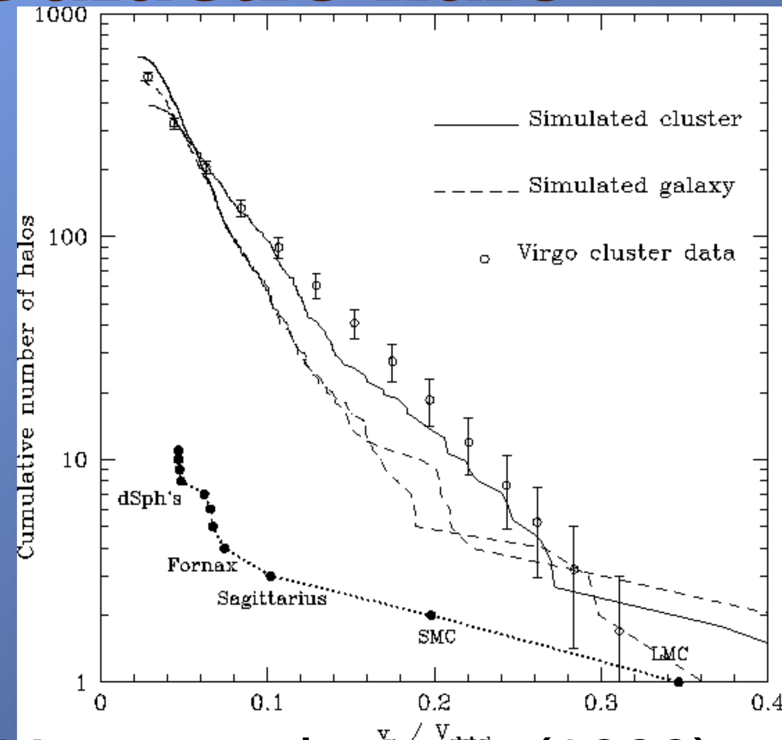
Inflation: What Else?

Clumpiness in the Galactic halo

cluster

galactic halo

300 kpc



Moore et al., ApJL (1999)

also Klypin et al, 2000

Kaufmann, White, Guiderdoni, 1993

Why are there so many more dwarfs predicted than observed?

- Are there but gas has been blown out? probably not
- Even if gas has been blown out and baryons remain dark, is difficult to see how disk could have formed in strongly fluctuating potential of such a clumpy halo.

Possible resolutions:

- Self-interacting dark matter (Spergel&Steinhardt 1999)

  ….requires very unusual particles (elastic-scattering cross sections more than 10^{10} annihilation cross sections!)
- Very low-mass scalars or generalized dark matter

  (e.g., Sahni&Wang 1999)
- Power suppression on small scales from hot dark matter

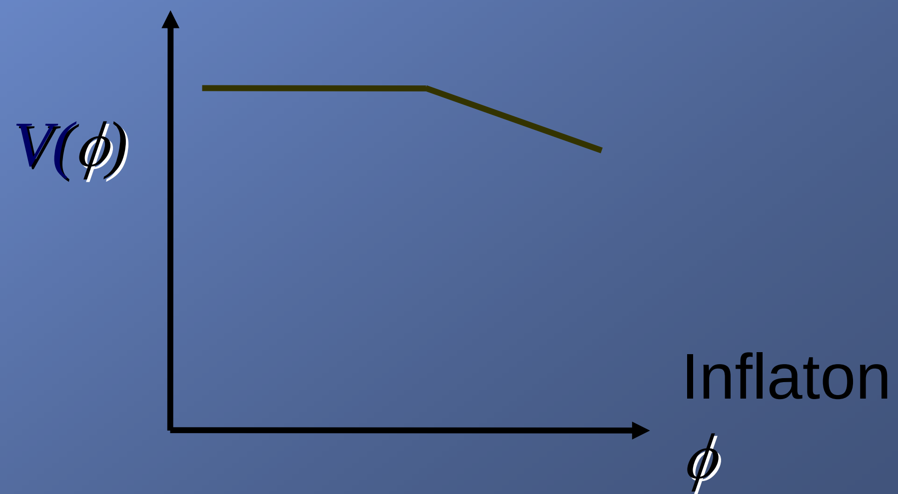
  …probably suppresses power on slightly larger scales too much for Lyman-alpha forest

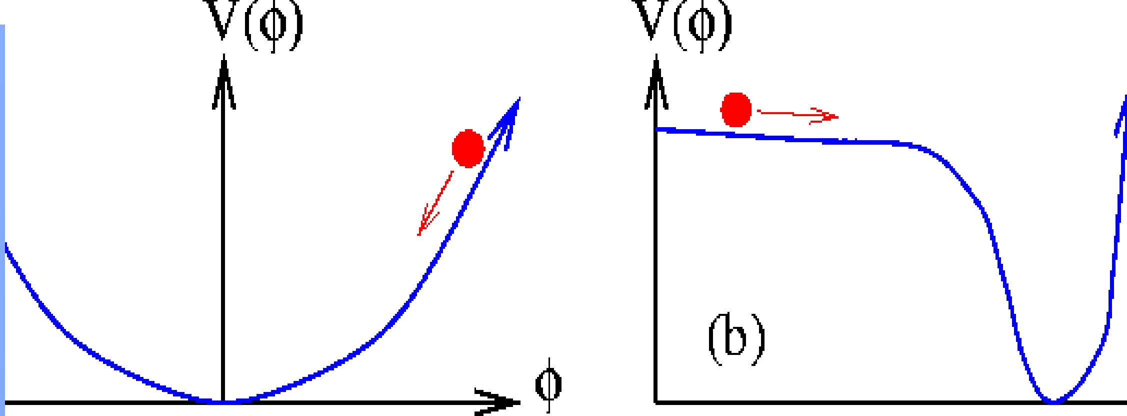
Another possible resolution:

Power suppression on small scales from inflation with broken scale invariance

MK&Liddle, PRL 84, 4525 (2000)

In inflation, amplitude of density perturbation on some comoving scale is proportional to $V^{3/2}/V'$, where V is the inflaton-potential height, and V' its derivative with respect to the inflaton. Thus, a break in the slope of the inflaton potential could yield a suppression of power on small scales





Inflaton potential

Power spectrum

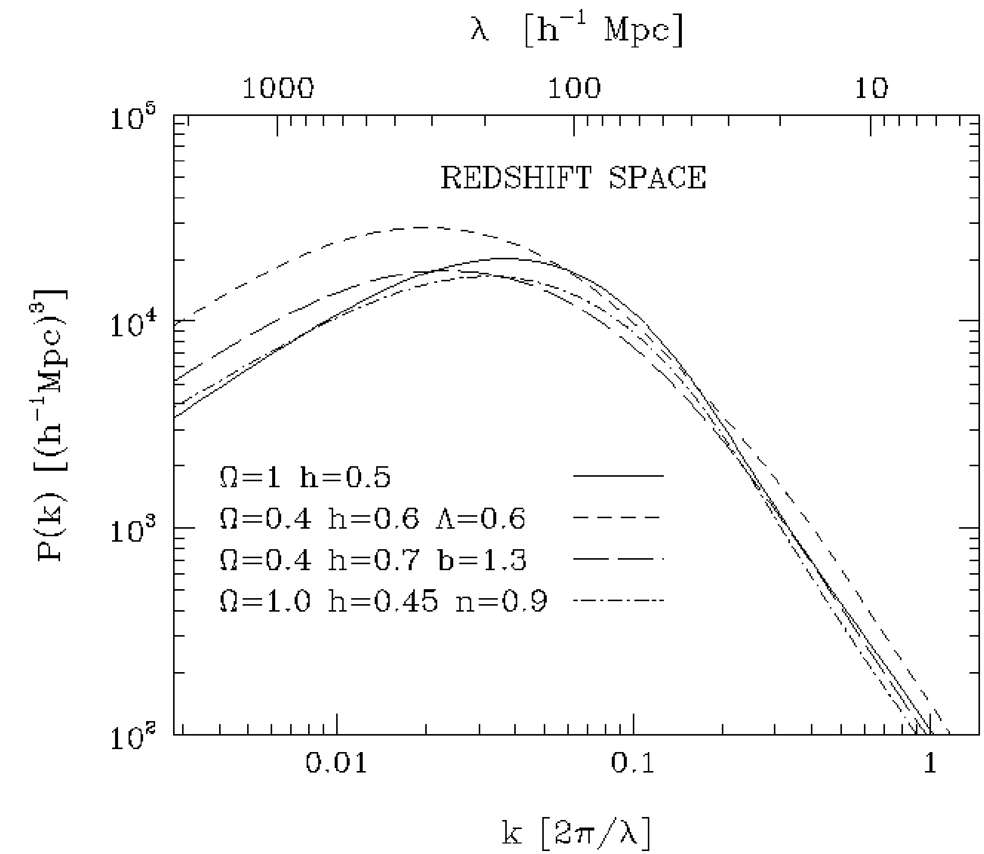
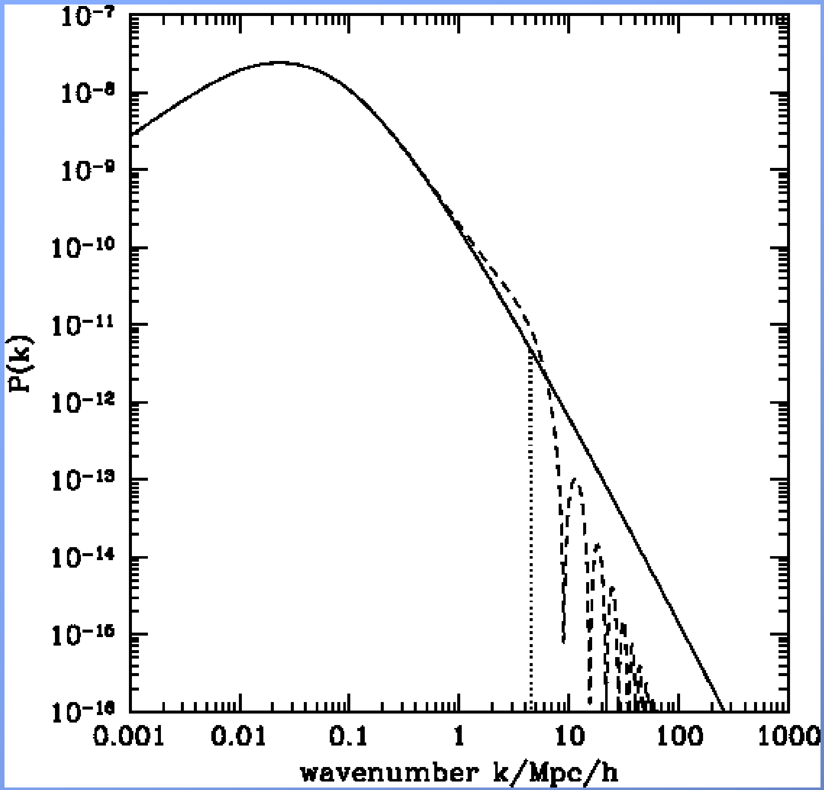
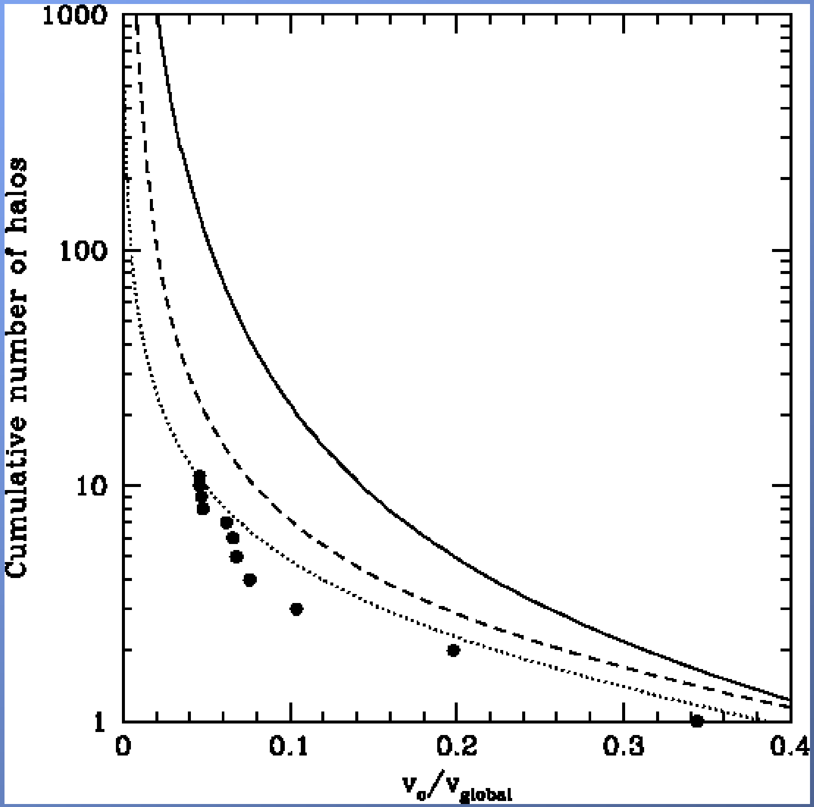


Fig. 2a.— Non-linear redshift-space power spectra for a variety of COBE-normalized CDM models.



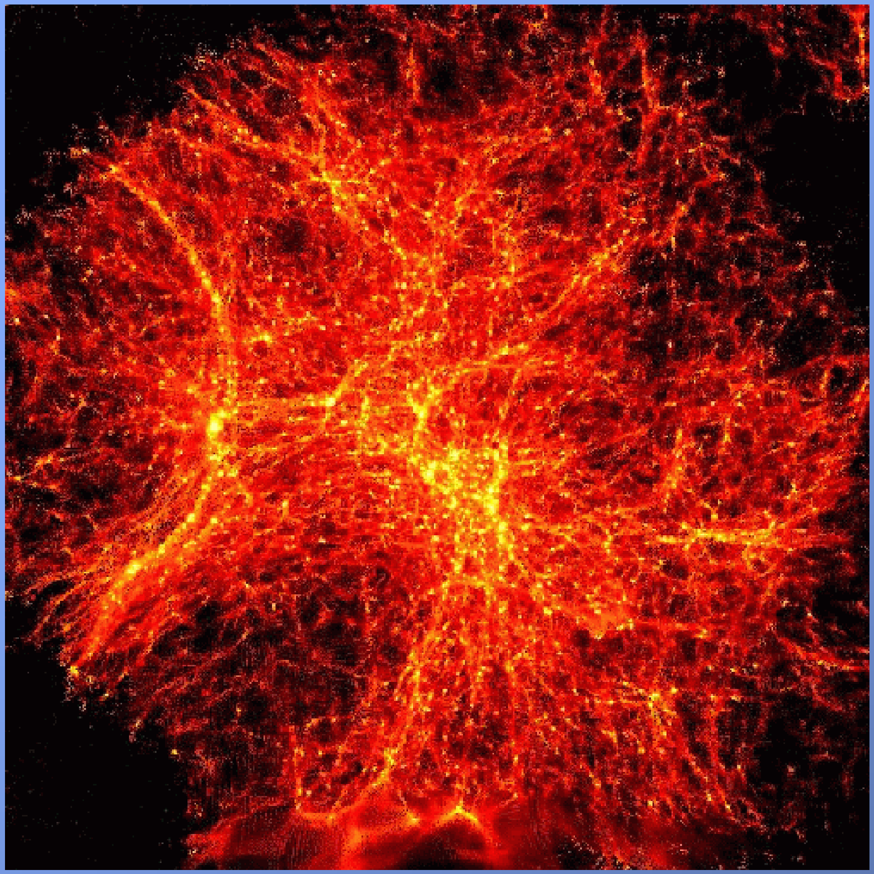


.... ad hoc

------- BSI

- Satisfies Lyman-alpha forest constraints
- May also help reduce core densities

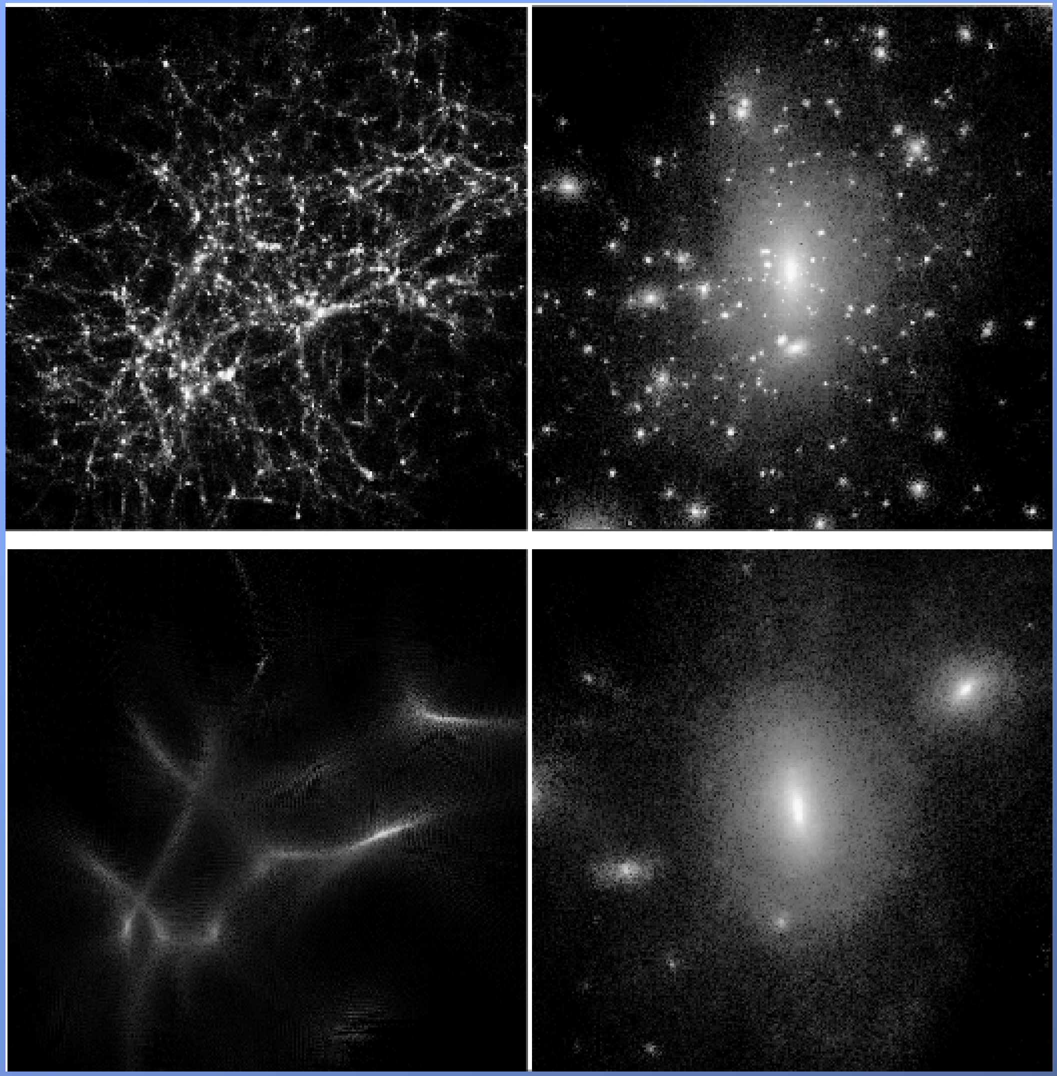
Moore et al show preservation of sub-clumps from turnaround to present, so can use extended Press-Schecter to estimate abundances



Turnaround

Today

Moore et al, 1999



Standard
CDM

Power
suppressed
on small scales

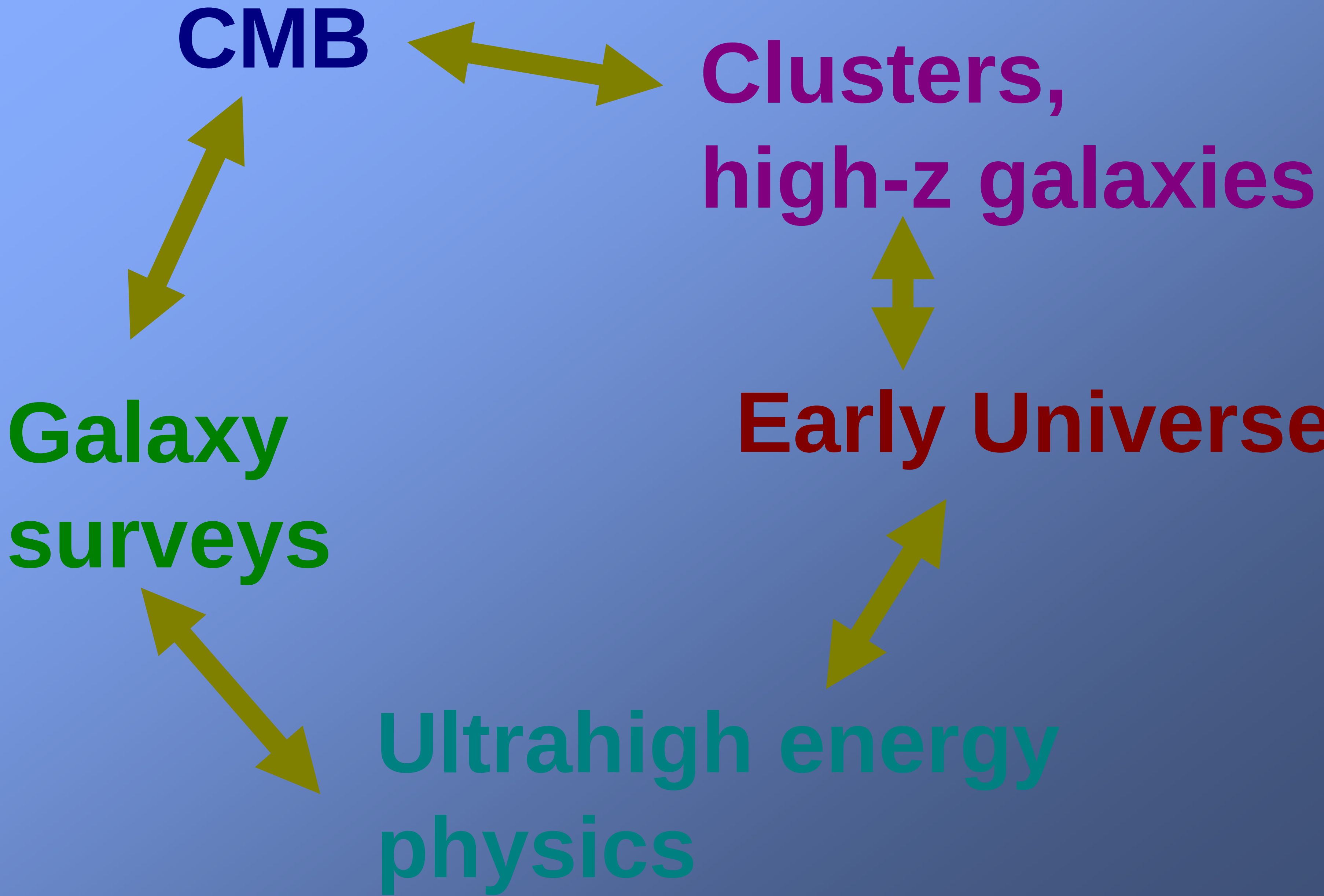
Turnaround Today

Moore et al, 2000

One more time...

CMB suggests we're on the right track with inflation.

Further tests possible with CMB polarization, large scale structure, and properties and abundances of extragalactic objects


CMB
Clusters,
high-z galaxies
Early Universe
Galaxy
surveys
Ultrahigh energy
physics