Physics of Neutrino Mass

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Main theme of the talk

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OBSERVATIONS ?

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ABOUT NEW PHYSICS ?

Summary of what we now know

(B. Kayser's talk for details)

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➢ $\nu_i \neq 0;\ \theta_{ij} \neq 0$

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$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\alpha i} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

➢ $PMNS =$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} K$$

➢ $^{i\phi_1}, e^{i\phi_2})$

☞ $_{ij}$ **at** 3σ

➢ $^{2}2\theta_{12} \simeq 0.71 - 0.93$

confirms Mikheyev-Smirnov-Wolfenstein effect

➢ $^{2}2\theta_{23} \simeq 0.89 - 1.00$

➢ $_{13} \leq 0.23$

MASSES

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- $^{-3}$ eV$^2 \leq |\Delta m^2_{13}| \leq 3.3 \times 10^{-3}$ eV^2;
  (3σ)
- $^{-5}$ eV$^2 \leq |\Delta m^2_{12}| \leq 9.3 \times 10^{-5}$ eV^2;
  (3σ)
-

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1. ::NORMAL::$\rightarrow m_1 \ll m_2 \ll m_3$
   $\rightarrow \Delta m^2_{31} > 0$; $m_3 \simeq 0.05$ eV; $m_2 \simeq 0.009$ eV
2. ::INVERTED::$\rightarrow m_1 \simeq m_2 \gg m_3$
   $\rightarrow \Delta m^2_{31} < 0$; $m_1 \simeq m_2 \simeq 0.05$ eV
3. ::DEGENERATE:: $m_1 \simeq m_2 \simeq m_3 \rightarrow \Delta m^2_{31} > or < 0$

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that we stll do not know is:

Is neutrino its own antiparticle ? i.e. is $\nu = \bar{\nu}$

If $\nu = \bar{\nu}$, it is Majorana; otherwise Dirac

Overall mass scale

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and (ii) and absolute mass in case(iii)

: ☞

1. ^3H Decay end point: $\Sigma_i m_i^2 |U_{ei}|^2 \leq 2.2$ eV^2 (KATRIN expected to improve it to 0.2 eV)

2. Cosmology: $\Sigma m_i \leq 0.4$ eV (WMAP, SDSS: will be improved by Planck)

3. If neutrino Majorana i.e. $\nu = \bar{\nu}$, $\beta\beta_{o\nu}$ results imply: $\Sigma_i U_{ei}^2 m_i \leq 0.3 - 0.5$ eV (Expected improvement to 0.03 eV)

How many neutrinos?

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coupling to Z (active neutrinos $\nu_{e,\mu,\tau}$)

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(one or two) that do not couple to Z (sterile neutrinos ν_s), but mix with known neutrinos.
Masses ≈ 1 to a few eV.

➢ $_s$ from BBN (≤ 0.3
and WMAP ≈ 2

➢

formation.

University of Maryland

Prospects for discriminating between Dirac and Majorana neutrino

☞ ^{2}, $\beta\beta_{0\nu}$ **and KATRIN result can tell us a lot:**

there are 8 possibilities and in each case we learn something

$\beta\beta_{0\nu}$	Δm^2_{32}	KATRIN	Conclusion
yes	> 0	yes	Degenerate, Majorana
yes	> 0	No	Degenerate, Majorana or normal or heavy exchange
yes	< 0	no	Inverted, Majorana
yes	< 0	yes	Degenerate, Majorana
no	> 0	no	Normal, Dirac or Majorana
no	< 0	no	Dirac
no	< 0	yes	Dirac
no	> 0	yes	Dirac

Theoretical Implications: (first only three neutrinos)

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➢ $_\nu \ll m_{u,d,e}$?

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mixings ?

➢ $\frac{\Delta m^2_\odot}{\Delta m^2_A} \ll 1$ but $\gg \left(\frac{m_\mu}{m_\tau}\right)^2$ (for normal hierarchy)?

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standard model e.g. do they reveal any symmetries for leptons, quarks; any new forces, any new physical effects ?

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unification which unifies quarks and leptons (specially since there are so many differences) ?

A Primer on Fermion masses and mixings

☞ $\bar{\psi}_L\psi_R$ **in the Lagrangian**

If there are more fermions of the same kind, then

$$\mathcal{L}_{mass} = M_{ab}\bar{\psi}_{a,L}\psi_{b,R}$$

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- ➢ $_{ab}$= Mass matrix
- ➢
  $$U_L M U_R^\dagger = diag(m_1, m_2, \cdot, \cdot)$$
- ➢ $_{L,R}$ gives the mixings between different fermions, ψ_a and m_i are the actual masses e.g. for quarks, $U_{L,ab}$ contains the CKM mixings.
- ➢
  the mass matrix

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fermions: $\bar{\psi}_L \psi_R$ or $\psi_L^T C^{-1} \psi_L$ (or $L \leftrightarrow$ R)

➢ $^{i\alpha}\psi$, the

first mass is invariant whereas the second term is not;

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fermions and those with both kinds are called Majorana fermions

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symmetry: e.g. for $e, \mu, q..$, extra symmetry is $U(1)_{em}$; since $Q(\nu) = 0$, no such symmetry is there for ν

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small mass is easier for Majorana neutrino.

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Standard model

Glashow, Weinberg, Salam

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- $_c \times SU(2)_L \times U(1)_Y$
- $\begin{pmatrix} u_L \\ d_L \end{pmatrix}; \psi_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix};$

  Singlets: $u_R; \quad d_R; \quad e_R$

  Higgs: $H \equiv \begin{pmatrix} H^0 \\ H^- \end{pmatrix}$
- $_Y = h_u \bar{Q}_L H u_R + h_d \bar{Q}_L \tilde{H} d_R + h_e \bar{\psi}_L \tilde{H} e_R + h.c.$
- appropriately chosen form for the potential which gives $< H^0 >= v_{wk}$
- $_m \;=\; \bar{u}_{a,L} M^u_{ab} u_{b,R} + \bar{d}_{a,L} M^d_{ab} d_{b,R} + \bar{e}_{a,L} M^e_{ab} e_{b,R};$
-
- $_\nu = 0$ in the standard model

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fermions: $\bar{\psi}_L\psi_R$ or $\psi_L^T C^{-1}\psi_L$ (or $L \leftrightarrow$ R)

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$\nu_L^T C^{-1}\nu_L$ could be there

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exact symmetry, $B - L$

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hidden in the standard model that would give $m_\nu \neq 0$ **?**

We ignored gravity in our considerations

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➢ gravitational effects such as black holes or worm holes etc.

➢ standard model e.g. $(\psi_L H)^2/M_{P\ell}$;

➢ $_\nu \simeq \frac{v_{wk}^2}{M_{P\ell}} \sim 10^{-5}$ eV- clearly too small to explain atmospheric neutrino deficit.

Std model successful but unsatisfactory

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1. Not symmetric between quarks and leptons, even though weak interactions are

2. What is the origin of parity violation ?

3. Electric charge formula: $Q = I_{3L} + \frac{Y}{2}$;
   we know what is I_{3L}; what is Y- an adjustable parameter !!

4. Can neutrinos help us understand these issues better ?

Neutrino mass and Nature of new physics

☞ **$_R$ to the standard model**

➢ $_Y$: $h_\nu \bar{\psi}_L H \nu_R + h.c.$

➢ $_R = N_R$ is std model singlet, new term allowed by gauge invariance: $M_R N_R^T C^{-1} N_R + h.c.$

Important point: M_R breaks B-L symmetry

➢ $_L, N_R)$ system:

$$\begin{pmatrix} 0 & h_\nu v \\ h_\nu^T v & M_R \end{pmatrix}$$

➢ $_R \gg h_\nu v$, mass eigenvlaues have a heavy : $\longrightarrow$: M_R and a light set: $\mathcal{M}_\nu \simeq -\frac{h_\nu^2 v^2}{M_R}$. This implies $m_{\nu_i} \ll m_{u,d,e...}$.

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mass

Gell-Mann, Ramond, Slansky; Yanagida; Glashow; R. N. M., Senjanovic (1979)

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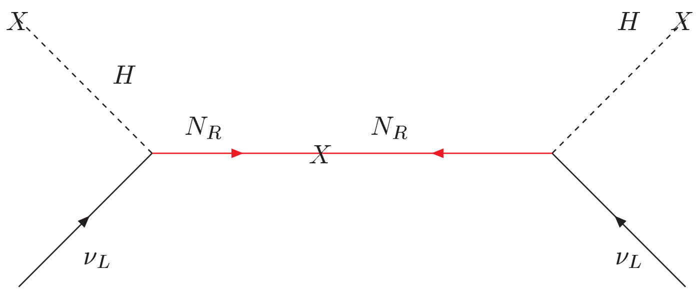

☞ $$\mathcal{L}_m(\nu, e) \; = \; \nu_L^T C^{-1} \mathcal{M}^\nu \nu_L + \bar{e}_L M^e e_R + h.c.$$

Diagonalize:

$$U^T \mathcal{M}^\nu U = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}; V_L M^e V_R^\dagger = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

Neutrino mixing matrix $U_{PMNS} \; = \; V_L^\dagger U$

Implications of Seesaw

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beta decay and other $\Delta L = 2$ processes;

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$M_{R,max} \simeq \frac{m_t^2}{\sqrt{\Delta m_A^2}} \simeq 10^{14} - 10^{15}$ GeV

M_R close to the conventional SUSY GUT scale !!

Could m_ν be the first indication of grand unification ?

Seesaw as a way to understand the origin of matter

☞ $\frac{n_B - n\bar{B}}{n_\gamma} \simeq 10^{-10}$**?**

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in RH neutrino couplings), then

➢ $_R \rightarrow \ell + H) - \Gamma(N_R \rightarrow \bar{\ell} + H) \neq 0 \rightarrow$ lepton asymmetry;

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asymmetry into baryon asymmetry.

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Why Seesaw is theoretically so appealing?

☞ $_R$ **to std model makes fermion spectrum quark-lepton symmetric.**

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under Parity

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \leftrightarrow \begin{pmatrix} u_R \\ d_R \end{pmatrix}; \quad \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \leftrightarrow \begin{pmatrix} N_R \\ e_R \end{pmatrix};$$

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1. Electroweak gauge group expands to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
2. weak interactions become parity conserving $\mathcal{L}_{wk} = \frac{g}{2\sqrt{2}}(\vec{W}_{\mu,L} \cdot \vec{J}^{\mu}_L + \vec{W}_{\mu,R} \cdot \vec{J}^{\mu}_R)$
3. Electric charge: $Q = I_{3L} + I_{3R} + \frac{B-L}{2}$ Involves all physical quantum numbers

Neutrino mass linked to parity violation

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1. Why are low energy weak int. V-A ?
2. Why $m_\nu \ll m_{u,d,e}$?

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➢ L
MASS

➢ $_L \times SU(2)_R \times U(1)_{B-L} \to G_{std} \to U(1)_{em}$

$$M_{\nu,N} = \begin{pmatrix} 0 & 0 \\ 0 & M_R \end{pmatrix} \to \begin{pmatrix} f v_L & h_\nu v \\ h_\nu^T v & f v_R \end{pmatrix} \text{ (SEESAW)}$$

➢ $_\nu \simeq f v_L - \frac{h_\nu^2 v^2}{f v_R}; \ (v_L \sim \frac{v_{wk}^2}{v_R})$
Strength of V+A currents $\propto \frac{1}{v_R^2}$;
as the scale of parity violation $v_R \to \infty$, $m_\nu \to 0$;

➢ $_\nu$ CONNECTED TO THE SUPPRESSION OF V+A currents

Implication of Parity for seesaw

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$$m_\nu \simeq f\frac{v_{wk}^2}{v_R} - \frac{h_\nu^2 v_{wk}^2}{f v_R}; \text{ (Type II seesaw)}$$

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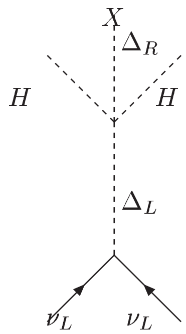

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➢ $\nu \simeq -\frac{h_\nu^2 v_{wk}^2}{f v_R}$ (Type I seesaw)

A simple pointer to Type II seesaw

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➢ $_\nu \sim h_e \sim$ hierarchical, Type I seesaw $\rightarrow$ $m_1 \ll m_2 \ll m_3$ (hierarchical) i.e.

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How to understand large mixings?

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➢ $\nu = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \rightarrow U_\nu = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$; Maximal mixing;

➢ $_{\odot}^{2} \ll \Delta m^2_A \rightarrow (a-b) \ll (a+b)$

➢ $\mathcal{M}_\nu = \sqrt{\Delta m^2_A}\begin{pmatrix} 1+\epsilon & 1 \\ 1 & 1 \end{pmatrix}$;

➢ possible $\nu_\mu \leftrightarrow \nu_\tau$ symmetry.

➢ HOW TO TEST FOR THIS SYM.?

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large solar and near maximal atmospheric

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➢ $$_\nu = \sqrt{\Delta m^2_A}\begin{pmatrix} d\epsilon^n & b\epsilon & a\epsilon \\ b\epsilon & 1+\epsilon & 1 \\ a\epsilon & 1 & 1+c\epsilon \end{pmatrix}; n \geq 1.$$

➢ $_{23}$ and large θ_{12} and small θ_{13}.

➢ $$\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}} \simeq \theta_{Cabibbo} \simeq \frac{1}{5}$$

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parameters?
$\beta\beta_{0\nu}$ measures d; unfortunately not very well for normal hierarchy WHAT ABOUT THE REST?

θ_{13} can provide very important information

☞ $_{13}$ **probes $\mu \leftrightarrow \tau$ symmetry and provides information about a, b, c in $\mathcal{M}_\nu$: Three cases**

➢ $_{13} = 0$

➢ predicts $\theta_{13} \simeq \epsilon^2 \simeq \frac{\Delta m^2_\odot}{\Delta m^2_A} \simeq 0.04$;

➢ $\theta_{13} \simeq \epsilon \simeq \sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}} \simeq 0.2$;

Conclusion

☞ $_{13}$ ia a measure of the extent of $\mu \leftrightarrow \tau$ symmetry in the neutrino mass matrix indicated by near maximal atmospheric mixing angle; zero to small to large,$\rightarrow$ exact to approximate to no symm.

☞ $_{13}$ is an important parameter to measure to understand the nature of new physics and new symmetries beyond the standard model.

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proposed to measure θ_{13}

Inverted Hierarchy

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➢ $\nu = \sqrt{\Delta m^2_A} \begin{pmatrix} d\epsilon & 1 & 1 \\ 1 & a\epsilon & b\epsilon \\ 1 & b\epsilon & c\epsilon \end{pmatrix}$

➢ $_{23}$ and large solar angle;

➢

$L_e - L_\mu - L_\tau$ symmetry; Thus we have a symmetry explanation of large mixing angles.

➢ $\frac{\Delta m^2_\odot}{\Delta m^2_A}$; solar mixing angle $- > d\epsilon \geq 0.5$; observable in $\beta\beta_{0\nu}$ decay.

➢

e.g. $a = c \rightarrow \theta_{13} = 0$ etc;

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departure from $L_e - L_\mu - L_\tau$ symmetry

☞ $_e - L_\mu - L_\tau$ **naturally explains**

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➢ $$_\nu = \sqrt{\Delta m^2_A}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$\rightarrow \Delta m^2_\odot = 0$ and $\theta_{12} \;=\; \frac{\pi}{4}$ (maximal)

➢ $\frac{\Delta m^2_\odot}{\Delta m^2_A} \ll 1$ because of small breaking of symmetry, which is also necessary to explain observed $\theta_{12} \simeq 33^0$

➢ $^2_{31} < 0.$

Can seesaw explain large neutrino mixings?

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  angles and 3 phases
- ➢
  parameters- grand unification, horizontal symmetry etc.
- ➢

Strategy for probing new physics

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SYMMETRIES AND NEW PHYSICS e.g evidence for $L_e - L_\mu - L_\tau$ symmetry would seriously argue against the idea of grand unification.

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considerations such as SUSY, Grand unification, string theory etc and study predictions for possible directions.

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m_ν and Grand unification

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hypothesis

➢ $_s, g_2, g_1$ into one coupling at a high scale ($10^{15} - 10^{16}$ GeV

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1. Raises the hope of explaining the free parameters of the standard model
2. Raises new problems: why $m_W \ll M_U$? i.e. why dont radiative corrections push m_W up to M_U ?
3. Solving this needs supersymmetry which removes infinities from Higgs mass;
4. Simple SUSY GUT gives coupling unification scale $M_U \sim 2 \times 10^{16}$ GeV

☞ $_R \simeq M_U$

➢

➢

due to higher symmetry of GUT theories which will reduce number of free parameters

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Another promise of SUSY GUT

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versa: $Q \leftrightarrow \tilde{Q}$**,...**

- $_p$ under which std model particles are even and their susy partners are odd.
- $_p$ odd particle is stable.
-
-
-

SO(10) SUSY GUT and neutrinos

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- ➢ $\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{L,R}$ into **16** dim. rep of SO(10)
- ➢ $_R$ needed for seesaw automatically
- ➢

  properties of asymptotic parity conservation.
- ➢
- ➢

  as a natural symmetry and gives a stable dark matter
- ➢

Breaking SO(10) down

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➢ $_{L} \times SU(2)_R \times SU(4)_c \rightarrow$ std model

➢

➢

Minimal SUSY SO(10) For Neutrinos with dark matter

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number of free parameters and predict masses

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ψ_a**16**- matter field
Higgs $\mathbf{10}(H), \mathbf{126}(\Delta) \oplus \mathbf{1\bar{2}6}(\bar{\Delta}), \mathbf{210}$ (only first two couple to matter by group theory

➢

$$\mathcal{L}_Y \;=\; h_{ab}\psi_a\psi_b H + f_{ab}\psi_a\psi_b\overline{\Delta}$$

➢ $_u, H_d)$ from H and another from $\overline{\Delta}$
All doublets can have vevs

➢

minus $M_Z \to$ total of 12 parameters.

➢

for quarks; 3 for charged leptons and 18 for the neutrino sector $\to$ a total of 31 parameters

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➢

angles;

➢

all but one neutrino masses and mixing angles predicted

➢

breaking sector; this can be included)

Babu, RNM (92); Bajc, Senjanovic, Vissani (2002); Goh, RNM, Ng (03)

Predictions of the minimal SO(10):

☞ $_b \simeq m_\tau$ **at the GUT scale due to radiative corrections**

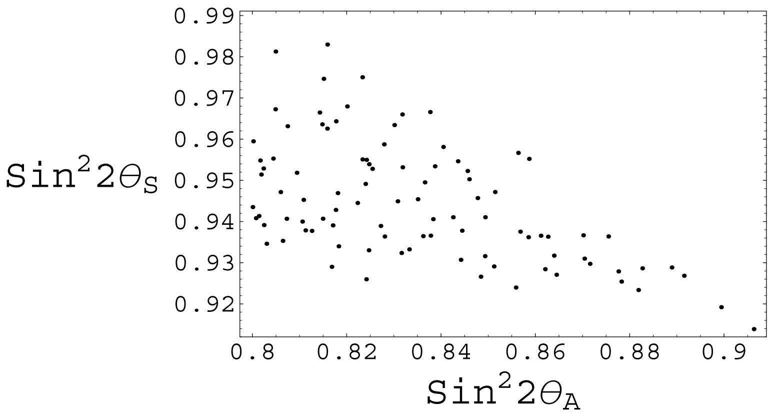


Figure 1: $sin^2 2\theta_{12}$ vrs $sin^2 2\theta_{23}$; scatter corresponds to different allowed quark mass values

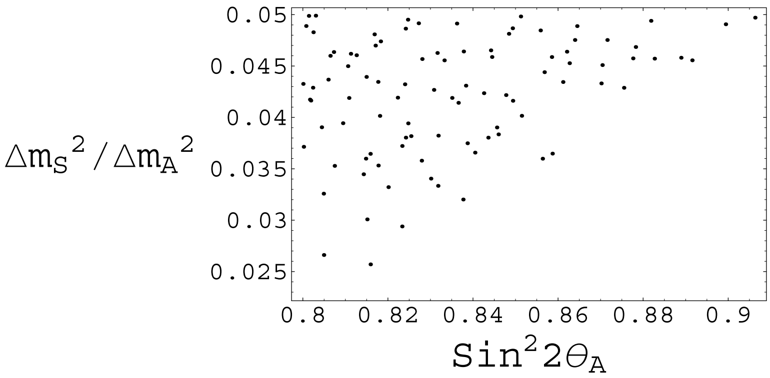
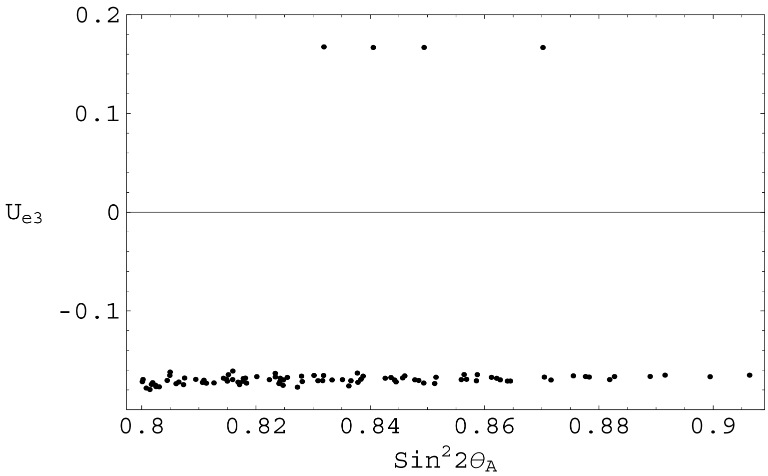


Figure 2: scatter corresponds to uncertainty in quark mass values

θ_{13}



☞ $_{e3} \equiv \theta_{13}$ **and just below the present upper limit: "high" value due to no $\mu \leftrightarrow \tau$ symmetry (see before)**

If MiniBoone confirms LSND

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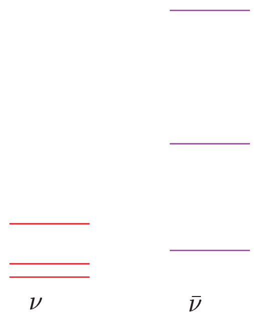
- ➢ ^{2}'s to explain solar, atm and LSND results; How can one accomplish this?
- ➢ $_\mu - \bar{\nu}_e$'s whereas solar is in $\nu_e - \nu_{\mu,\tau}$; so could it be that ν's have different masses from $\bar{\nu}$'s- that would give us room for a total of 4 Δm^2'



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invariance, which is one of the immediate implications of local Lorentz Inv field theory; B. KamLand which sees oscillations in $\bar{\nu}_e - \bar{\nu}_{\mu,\tau}$ disfavors this.

Sterile neutrinos

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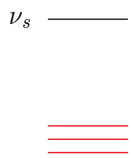
two) sterile ν_s with mass of order 1 to few eV; (2+2), (3+1) or (3+2) scenarios.



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➢ $\nu_s \leq 0.3$

➢ $\Sigma_i m_i \leq 0.4$ eV

➢

Theoretical challenge of the sterile neutrinos

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standard model would allow them to have arbitrary mass ?

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model-inspired by superstring theories

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visible sector $SU(3)_c \times SU(2)_L \times U(1)_Y$	mirror sector $SU(3)_c \times SU(2)_L \times U(1)_Y$
W, Z, γ, gluons $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ u_R, d_R $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ e_R, N_R	W, Z, γ, gluons $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ u_R, d_R $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ e_R, N_R

Implications of mirror models for neutrinos

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➢ $_L$ (mirror ν's) couple only to Z, W bosons and not the familiar Z, W bosons and are therefore candidates for sterile neutrinos ($\nu_s \equiv \nu$; three of them)

➢ $_s$ are light for the same reason (seesaw) that known $\nu_{e,\mu,\tau}$ are.

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breaking in the mirror sector

How to reconcile with very precise cosmological observations ?

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generate masses and mixings: needs a light boson (with eV mass)

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➢ $_{\nu} = 3$

➢ $_s$ annihilate and disappear by the time of recombination and do leave a strong imprint on CMB

Chacko, Hall, Oliver and Perelstein,2004

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scale seesaw

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conventional GUT, seesaw type theories

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Conclusions

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lunched

- important for our understanding of new physics beyond the standard model e.g.

- mechanism whereas Dirac will surely turn attention away from it!!

- $^2_{31}$ will clarify the mass pattern i.e. inverted vrs normal

- $_{13}$ measurement will tell us about any inherent $\mu \leftrightarrow \tau$ symmetry

- clarify our understanding of one of the fundamental mysteries of cosmology i.e. origin of matter

- SO(10) is the prime group for neutrinos and generically predicts Normal hierarchy;
  Again sign of Δm^2_{31} will be important for this

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confirms LSND with implications.