(Warped) XD gives
us a tool for understanding
& playing with subtle
strong dynamics issues
in phenomenological contexts!

- Central module of such "duality" is the AdS_{5D}/CFT_{4D} Correspondence.

"Anti-de Sitter"

review Aharony, Gubser, Maldacena, Ooguri, Oz '99

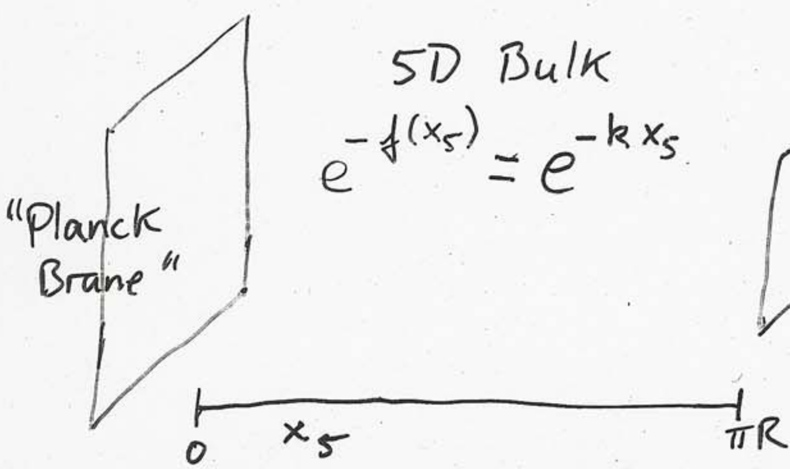
- The catch in this duality is that

$$\eta_{MN} \longrightarrow \begin{pmatrix} \eta_{\mu\nu}\, e^{-2f(x_5)} & 0 \\ 0 & -1 \end{pmatrix}$$

"warp factor" preserves 4D Poincaré symmetry

which makes calculations harder.

- Simplest warped compactification, used in phenomenological effective field theories



$$e^{-f(x_5)} = e^{-k x_5}$$

Randall-Sundrum I ('99) background solves 5D Einstein Equations supplemented with suitable bulk + brane cosmological constants

String realization: H. Verlinde '99 ; Giddings, Kachru, Polchinski ('...

Analogy — Inflating universe (dS spacetime)

$$\eta_{\mu\nu} \longrightarrow g_{\mu\nu} = \begin{pmatrix} +1 & 0 \\ 0 & -e^{Ht}\delta_{ij} \end{pmatrix}$$

"scale factor"

solve 4D Einstein Equations with positive cosmological constant. Space inflates in time.

ie. AdS_5 ≡ "inflation" of 4D spacetime in XD.

KK decomposition of bulk fields is more subtle (not sinusoidal).

General features:

- Low-lying KK excitations localize near IR brane
- Lowest mode shape is v. sensitive to boundary conditions & 5D spin, mass.

Eg. XD scalar

$$S_{5D} = \int d^4x \int dx_5 \sqrt{G} \left\{ G^{MN} \partial_M \phi \partial_N \phi - m_5^2 \phi^2 \right\}$$

$$\underset{\substack{AdS_5 \\ \text{background}}}{=} \int d^4x \int dx_5 \, e^{-4kx_5} \left\{ e^{2kx_5} \partial_\mu \phi \partial^\mu \phi - (\partial_5 \phi)^2 - m_5^2 \phi^2 \right\}$$

raised by $\eta_{\mu\nu}$

$$\Rightarrow -\partial_\mu \partial^\mu e^{-2kx_5} \phi + \partial_5 e^{-4kx_5} \partial_5 \phi - m_5^2 \phi e^{-4kx_5}$$
$$= \text{brane terms}$$

Consider a KK excitation with 4D mass, m_4,

$$m_4^2 e^{2kx_5} \phi + (\partial_5^2 - 4k\partial_5 - m_5^2)\phi$$
$$= \text{brane terms}$$

~~neglecting branes~~

$\cancel{(\partial_5^2 - 4k\partial_5 - m_5^2)\phi = 0}$

Change variables: $\tilde{x}_5 \equiv \dfrac{e^{kx_5}}{k}$, $\tilde{\phi} \equiv e^{-\frac{3}{2}kx_5} \phi$

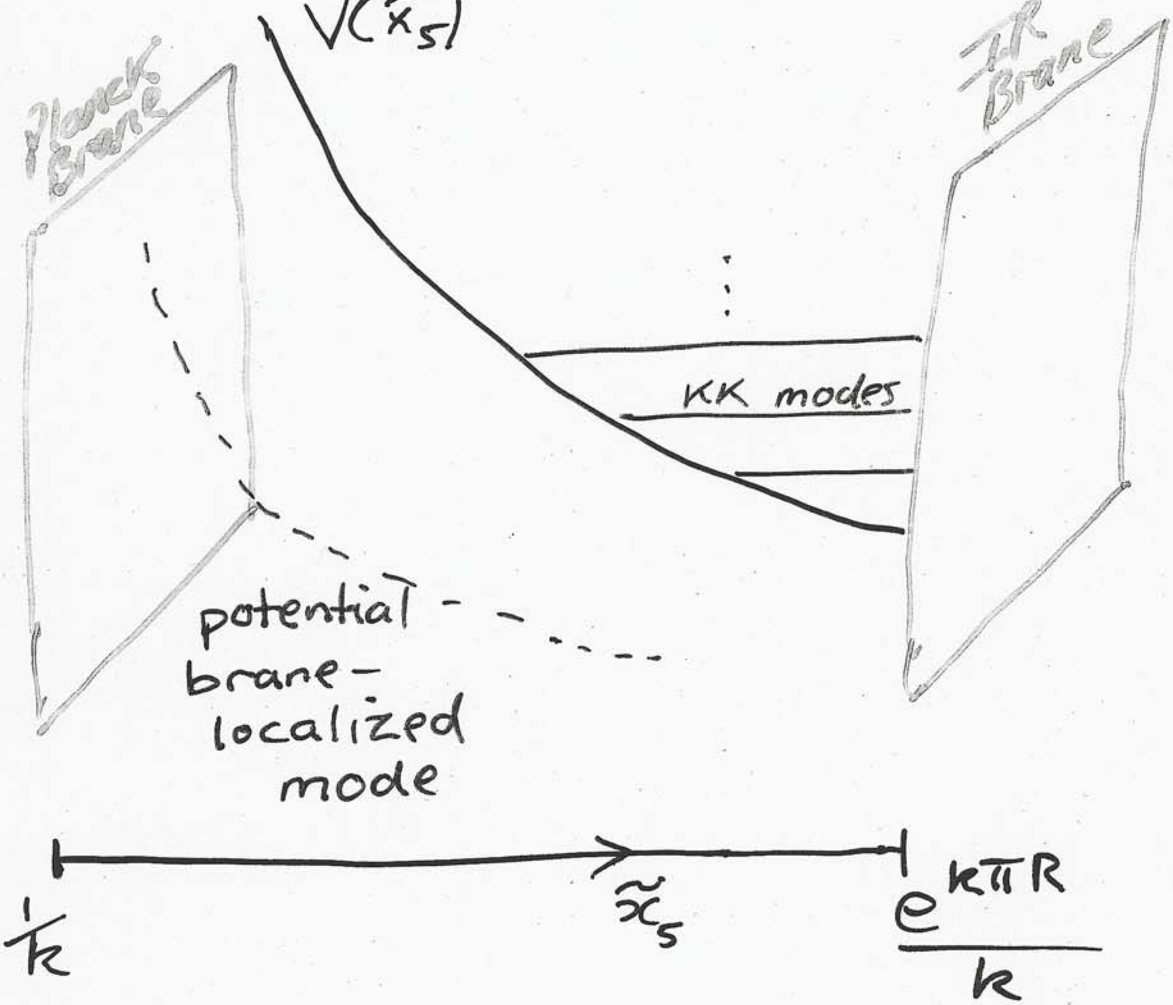
Schrodinger-like Equation in XD

$$-\partial_{\tilde{5}}^2 \tilde{\phi} + \frac{\frac{15}{4}k^2 + m_5^2}{\tilde{x}_5^2}\tilde{\phi} = m_4^2 \tilde{\phi}$$

+ brane terms

c/w $-\frac{1}{2m}\frac{d^2}{dx^2}\psi + V(x)\psi = E\psi$ in QM

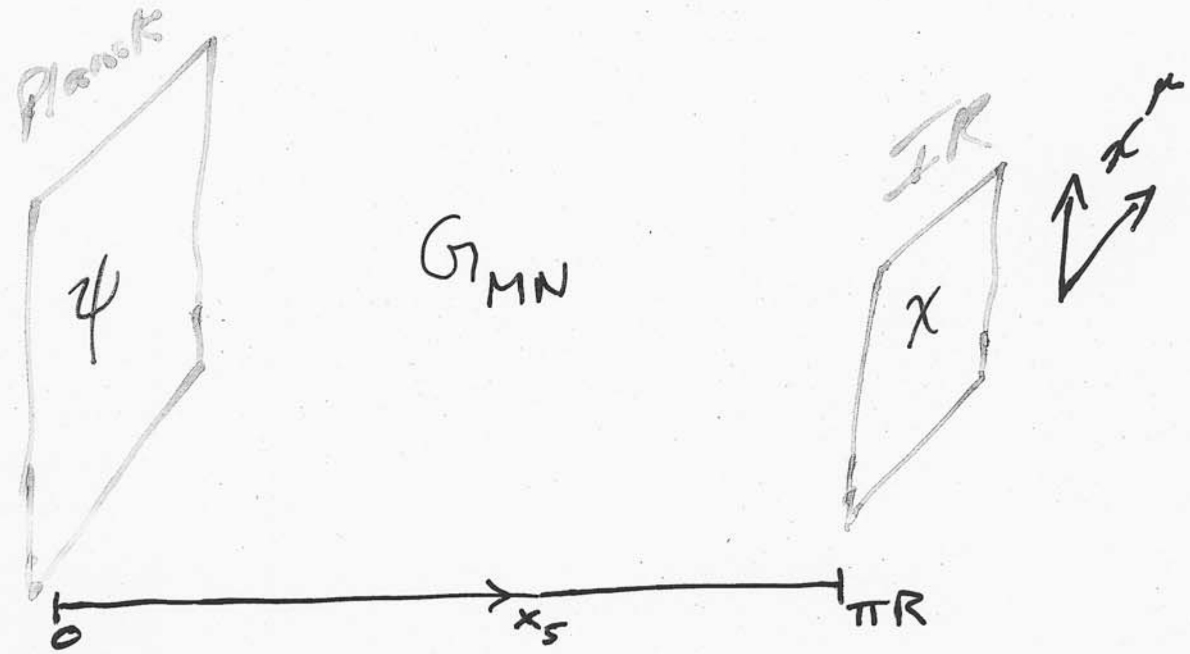
$\Rightarrow$



Warped Hierarchy

Randall, Sundrum '99

Consider scalars ψ localized to Planck brane, χ localized to IR brane:



In RS background,

$$ds^2 = e^{-2kx_5}\eta_{\mu\nu}dx^\mu dx^\nu - dx_5^2$$

$\Rightarrow$ a) $$ds^2\big|_{\text{Planck brane}} = \eta_{\mu\nu}dx^\mu dx^\nu$$ standard 4D Minkowski spacetime

b) $$ds^2\big|_{\text{IR brane}} = \underbrace{e^{-2k\pi R}\eta_{\mu\nu}}_{\text{Minkowski 4D in funny units}}dx^\mu dx^\nu$$

$\therefore$ on IR brane,

- "experimentalist's length" obtained by coordinate length $\times\, e^{k\pi R}$.
- "experimentalist's masses" obtained by $\underbrace{\underline{\text{Fundamental mass parameter}}}_{\substack{\text{This could be huge} \\ \sim 10^{18}\,\text{GeV} \\ \text{scalar mass parameter}}} \times\, e^{-\underline{k\pi R}}$.

This could be modest ~ 40

This could be TeV scalar <u>physical</u> mass

4D Gravity

4D general covariance $\Rightarrow$

4D massless graviton mode arises from $\eta_{\mu\nu} \longrightarrow g_{\mu\nu}(x)$ in RS background.

ie. $G^{(0)}_{\mu\nu}(x, x_5) = e^{-2kx_5}\, g_{\mu\nu}(x)$ Localized $\sim$ Planck brane!

$\therefore$ no large warp-factor rescaling of mass parameter,

$$G_{N_{\text{eff}}} \sim G_5\, k$$

Could be $10^{18}\,\text{GeV}$

Could be $(10^{18}\,\text{GeV})^{-3}$

measured $\dfrac{1}{(10^{18}\,\text{GeV})^2}$

Thus the $\dfrac{\text{Higgs (weak scale)}}{\text{Planck scale}}$ <u>hierarchy problem is solved!</u>

This scenario is dual to scenario where Higgs is a composite, of some 4D strong dynamics, compositeness scale $\sim$ TeV.

In original RS1, entire Standard Model localized on IR brane (χ's)

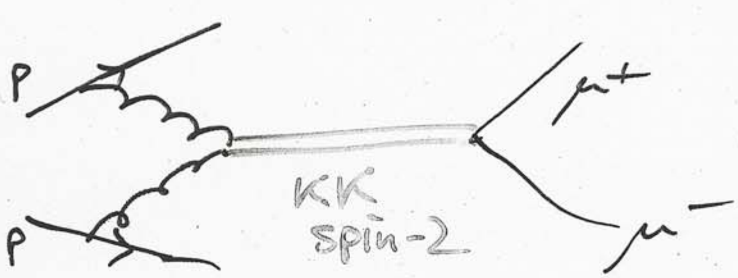
$\Rightarrow$ Low-lying "composites" $\equiv$ spin-2 KK gravitons

Because they are localized near IR brane,

$$m_{KK} \sim \underbrace{k}_{\sim 10^{18}\,\mathrm{GeV}} \; \underbrace{e^{-k\pi R}}_{\frac{\mathrm{TeV}}{10^{18}\,\mathrm{GeV}}} \sim \mathrm{TeV}$$

Coupling to SM:

$$G_{KK} \sim \underbrace{G_5 k}_{G_{N\,\mathrm{eff}}} \; \underbrace{e^{2k\pi R}}_{\left(\frac{10^{18}\,\mathrm{GeV}}{\mathrm{TeV}}\right)^2} \lesssim G_{\mathrm{Fermi}}$$

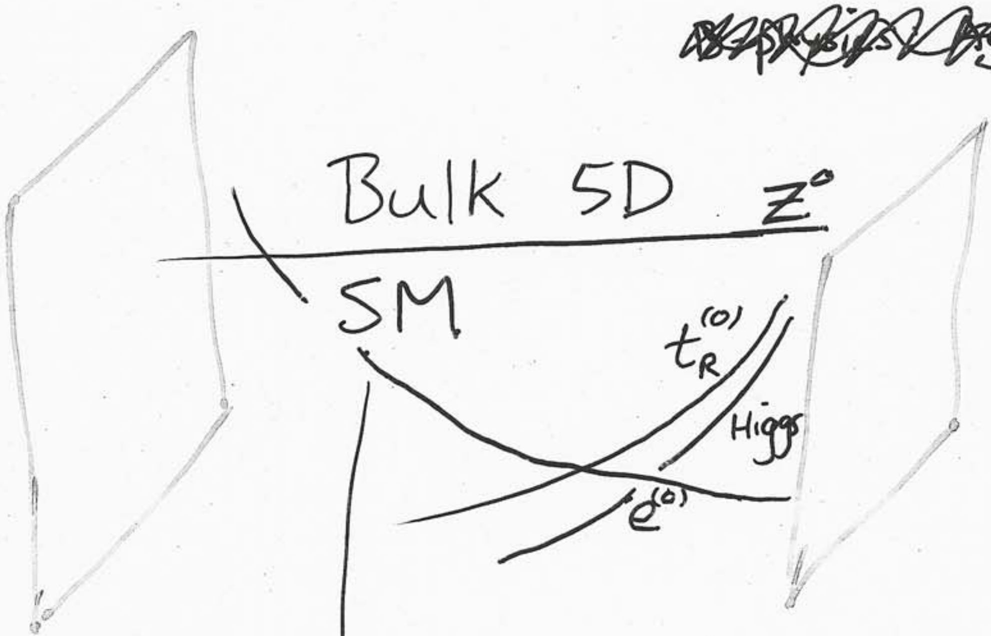


!

WARPED STANDARD MODEL

Agashe, Delgado, May, Sundrum '03

Agashe, Contino, Pomarol '04



→ enhanced to gauge group

$$\underbrace{SU(3)_{color} \times \underbrace{SO(5) \times U(1)_B}_{\supset SU(2)_L \times U(1)_Y} \times U(1)_L}_{\subseteq SO(11) \text{ for unification}}$$

Agashe, Contino, Sundrum '05

$$\text{Higgs} \equiv A_5^{\substack{\text{some of}\\ so(5)}}\,.$$

Addresses flavor structure, electroweak precision tests, gauge coupling unification in non-SUSY way.

is dual to 4D PARTIAL COMPOSITENESS

D.B. Kaplan '91

& COMPOSITE HIGGS

Georgi, D.B Kaplan '85

ie. Hierarchy Problem solved by

Higgs = TeV composite

Electron = Elementary fermion with tiny superposition of composite Fermion

t_R = TeV composite

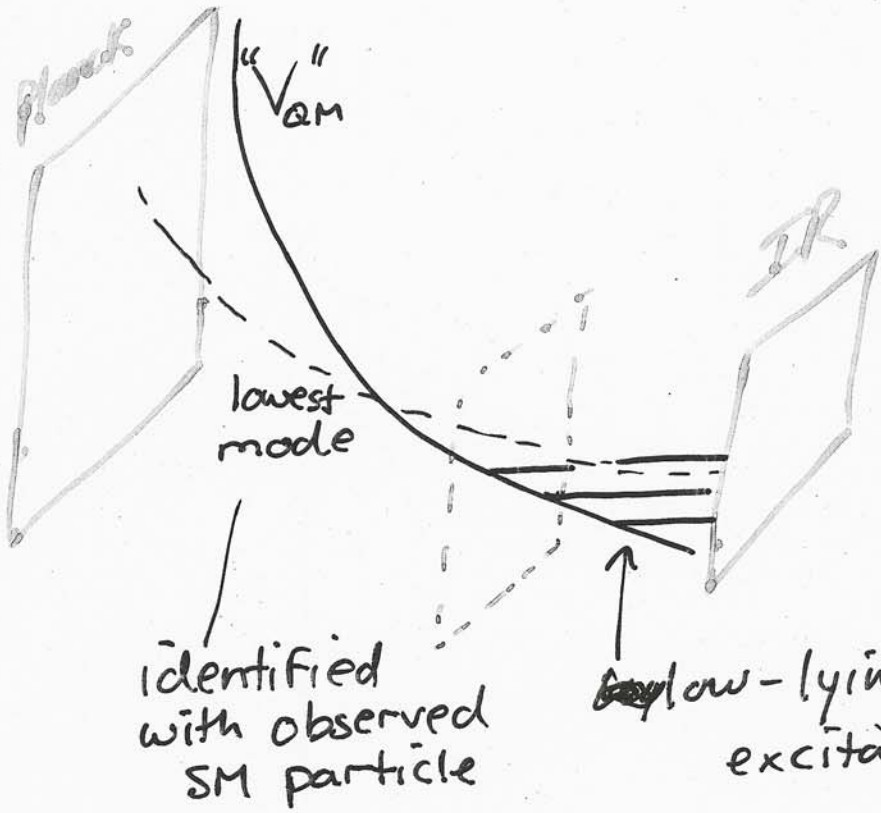
Z = Elementary gauge particle with moderate admixture of composite spin-1.

⋮

Degree of compositeness & large hierarchies arising from $\left(\frac{\text{TeV}}{M_{Pl}}\right)^{\gamma_i}$

anomalous dimensions of strong dynamics

MATCHING WARPED SPACETIME TO FLAT SPACETIME.



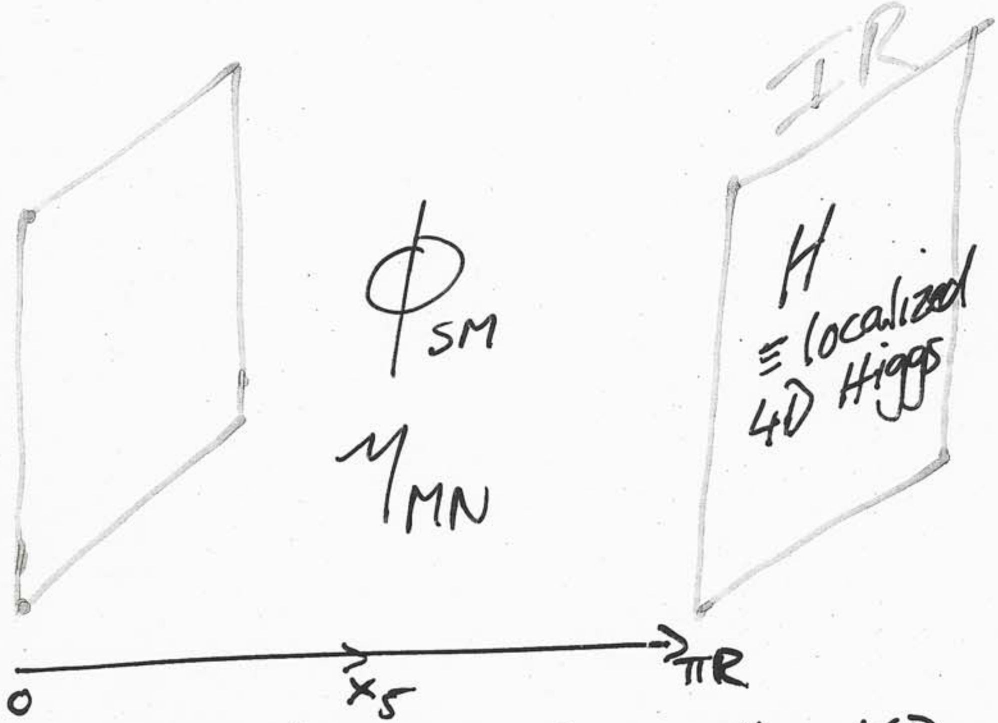
- New physics most accessible experimentally lives in small region of XD ~ IR brane,

<u>effectively</u> in a <u>smaller XD</u>
where <u>effects of warping are small.</u>

- But lowest mode is sensitive to full XD in matching 4D effective couplings etc. Must retain these effects.

Consider Flat XD with

Brane localized kinetic terms



c/w "Universal Extra Dimensions" Appelquist, Cheng, Dobrescu '00.

"Effective" Flat & smaller XD

$$S = \int d^4x \int_{-\pi}^{\pi} d\theta\, R \Big\{ \partial_M \phi\, \partial^M \phi$$

$$+ \delta(x_5 - \pi R) \big[\partial_\mu H \partial^\mu H - V(H) - \lambda H \phi\phi \big]$$

$$+ \delta(x_5)\, \kappa\, \partial_\mu \phi\, \partial^\mu \phi \Big\}$$

Free ϕ equation of motion:

$$\partial_\mu \partial^\mu \phi - \partial_5^2 \phi + \delta(x_5) K \partial_\mu \partial^\mu \phi = 0$$

~~4D momentum space~~

Look for KK modes, $p_\mu p^\mu = m_4^2$:

$$m_4^2 (1 + K\delta(x_5))\phi + \partial_5^2 \phi = 0$$

$m_4 = 0$: $\quad \phi(x,x_5) = \phi^{(0)}(x) / \sqrt{2\pi R + K}$

$m_4 > 0$: $\quad$ For large K, $m_4 = (n+\frac{1}{2})/R$,

$$\cancel{\phi(x,x_5) \doteq \frac{\sin(n+\frac{1}{2})\frac{x_5}{R}}{\sqrt{\pi R}} + \frac{2\sqrt{R/\pi}}{K(n+\frac{1}{2})}\cos(n+\tfrac{1}{2})\frac{x_5}{R}}$$

$$\phi(x,\theta) \doteq \phi^{(n)}(x)\left[\frac{\sin(n+\frac{1}{2})\theta}{\sqrt{\pi R}} + \frac{2\sqrt{R/\pi}}{K(n+\frac{1}{2})}\cos(n+\tfrac{1}{2})\theta\right]$$

4D effective theory $\ll 1/R$:

$$S = \int d^4x \left\{ (\partial_\mu \phi^{(0)})^2 + (\partial_\mu H)^2 - V(H) - \lambda_4 H \phi^{(0)} \phi^{(0)} \right\},$$

$$\lambda_{4\,eff} = \lambda / (2\pi R + K)$$

can fake extra volume of full warped spacetime + localization.

Fermion masses & mixings

captured by taking

λ, κ = matrix in flavor space.

$\kappa \gg R \equiv$ light fermions

$\lambda_{4\,\text{eff}} \ll 1$

$\kappa \lesssim R \equiv$ heavy fermions

off-diagonal entries lead to ~~[illegible]~~ CKM angles, phases

"GIM mechanism" suppressing FCNC's

Consider $\delta S_{\text{Bulk}} = \int d^4x \int d\theta R \, \dfrac{\bar{s} d \bar{s} d}{\Lambda^3}$

can mediate K-$\bar{K}$ mixing

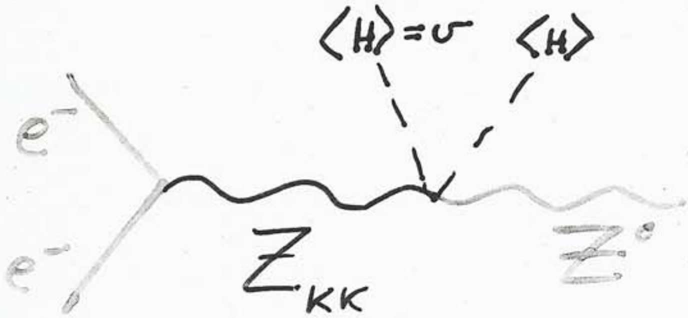
strange

down

$$\xrightarrow{E \ll 1/R} \int d^4x \, \frac{2\pi R \, \bar{s}^{(0)} d^{(0)} \bar{s}^{(0)} d^{(0)}}{\Lambda^3 \, (2\pi R + \kappa_{s,d})^2} \sim \frac{2\pi R}{\lambda^2 \Lambda^3} \left(\frac{m_s}{v}\right)^2 \bar{s}_0 d_0 \bar{s}_0 d_0$$

adequate suppression

Electroweak Precision Tests



Can be shown to be ~ contribution to Peskin-Takeuchi S.

For <u>simplicity</u> ignore brane kinetic terms :

$$\frac{\delta g_Z}{g_Z} \sim \frac{g_4^3 v^2}{g_4 m_{KK}^2} = \frac{(g_4 v)^2}{m_{KK}^2} \sim \left(\frac{m_Z}{m_{KK}}\right)^2$$

To ~~get~~ restrict to 10^{-3} effects (expt. bounds)

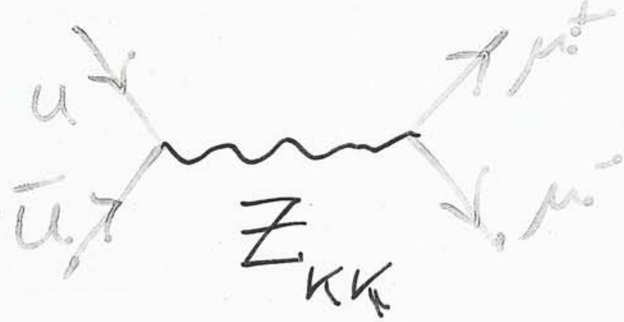
$$\Rightarrow \quad m_{KK} \gtrsim 3 \text{ TeV} \; !$$

+ <u>many</u> more exercises.

General rule is larger κ (lighter particles) more protected from KK-mediated corrections. In particular t & (by EW symmetry) b_L.

Agashe, Perez, Soni '04

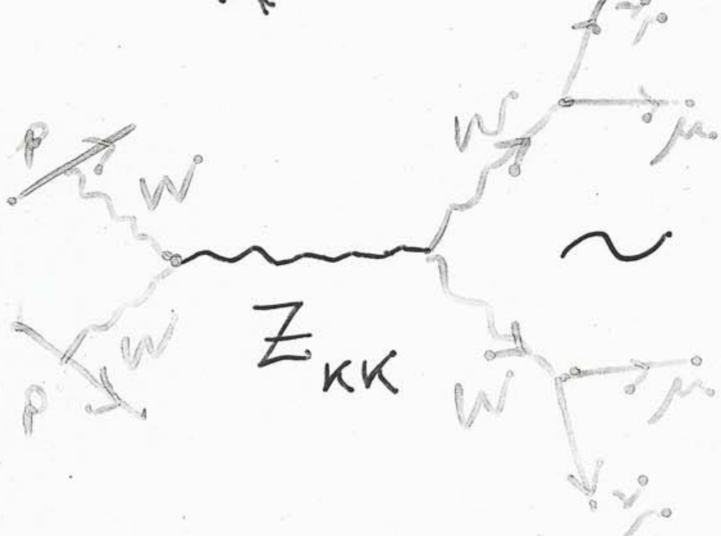
Challenges of K.K. Discovery with large κ_{fermions}



$$\sim \frac{g_5^2\, 2\pi R}{R^2}\,\frac{1}{2\pi R+\kappa_u}\,\frac{1}{2\pi R+\kappa_\mu}$$

$\propto$ Yukawa couplings

$\ll$



$$\sim \frac{g_5^2\, 2\pi R}{R^2}\,\frac{1}{(2\pi R+\kappa_W)^2}$$