



The Gravity of Dark Energy

Gravity in the Quantum World and the Cosmos
XXXIII SLAC Summer Institute, August 2005
Rocky Kolb, Fermilab & Chicago

Precision cosmology

$$\Omega_{\text{tot}} = 1.02^{+0.02}_{-0.02}$$

$$w < -0.78 \text{ (95\% CL)}$$

$$\Omega_{\Lambda} = 0.73^{+0.04}_{-0.04}$$

$$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$$

$$\Omega_b = 0.044^{+0.004}_{-0.004}$$

$$n_b = 2.5 \times 10^{-7} \text{ cm}^{-3}$$

$$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$$

$$\Omega_m = 0.27^{+0.04}_{-0.04}$$

$$\Omega_v h^2 < 0.0076 \text{ (95\% CL)}$$

$$m_\nu < 0.23 \text{ eV (95\% CL)}$$

$$T_{\text{cmb}} = 2.725^{+0.002}_{-0.002} \text{ K}$$

$$n_\gamma = 410.4^{+0.9}_{-0.9} \text{ cm}^{-3}$$

$$\eta = 6.1 \times 10^{-10}$$

$$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$$

$$\sigma_8 = 0.84^{+0.04}_{-0.04} \text{ Mpc}$$

$$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$$

$$A = 0.833^{+0.086}_{-0.083}$$

$$n_s = 0.93^{+0.03}_{-0.03}$$

$$dn_s/d \ln k = -0.031^{+0.016}_{-0.018}$$

$$r < 0.71 \text{ (95\% CL)}$$

$$z_{\text{dec}} = 1089^{+1}_{-1}$$

$$\Delta z_{\text{dec}} = 195^{+2}_{-2}$$

$$h = 0.71^{+0.04}_{-0.03}$$

$$t_0 = 13.7^{+0.2}_{-0.2} \text{ Gyr}$$

$$t_{\text{dec}} = 379^{+8}_{-7} \text{ kyr}$$

$$t_r = 180^{+220}_{-80} \text{ Myr (95\% CL)}$$

$$\Delta t_{\text{dec}} = 118^{+3}_{-2} \text{ kyr}$$

$$z_{\text{eq}} = 3233^{+194}_{-210}$$

$$\tau = 0.17^{+0.04}_{-0.04}$$

$$z_r = 20^{+10}_{-9} \text{ (95\% CL)}$$

$$\theta_A = 0.598^{+0.002}_{-0.002}$$

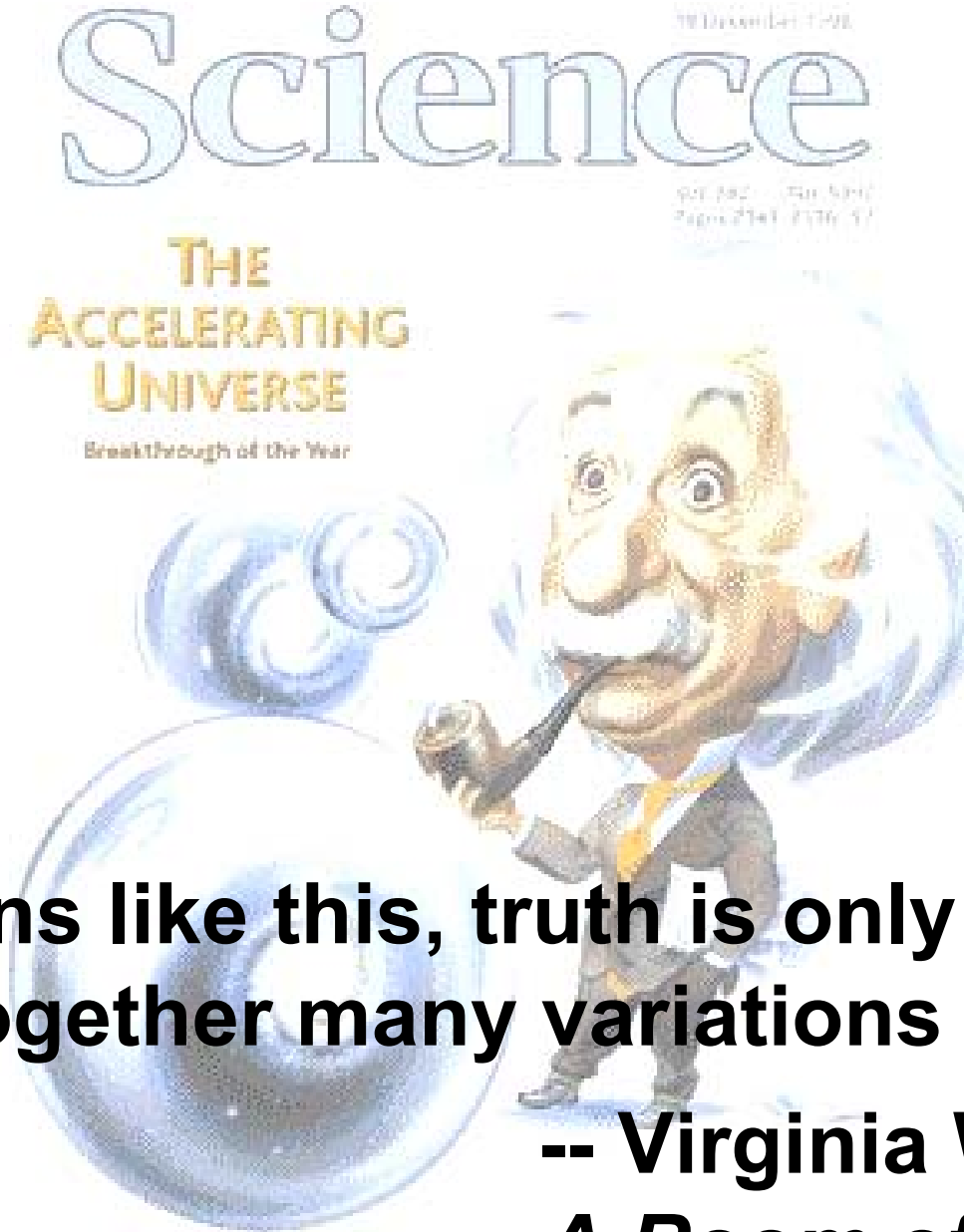
$$d_A = 14.0^{+0.2}_{-0.3} \text{ Gpc}$$

$$l_A = 301^{+1}_{-1}$$

$$r_s = 147^{+2}_{-2} \text{ Mpc}$$



What is the nature of dark energy?



“In questions like this, truth is only to be had by laying together many variations of error.”

**-- Virginia Woolf
*A Room of Ones Own***

Big-Bang

Robertson-Walker metric

$a(t)$ = cosmic scale factor

$k = 0, \pm 1$

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

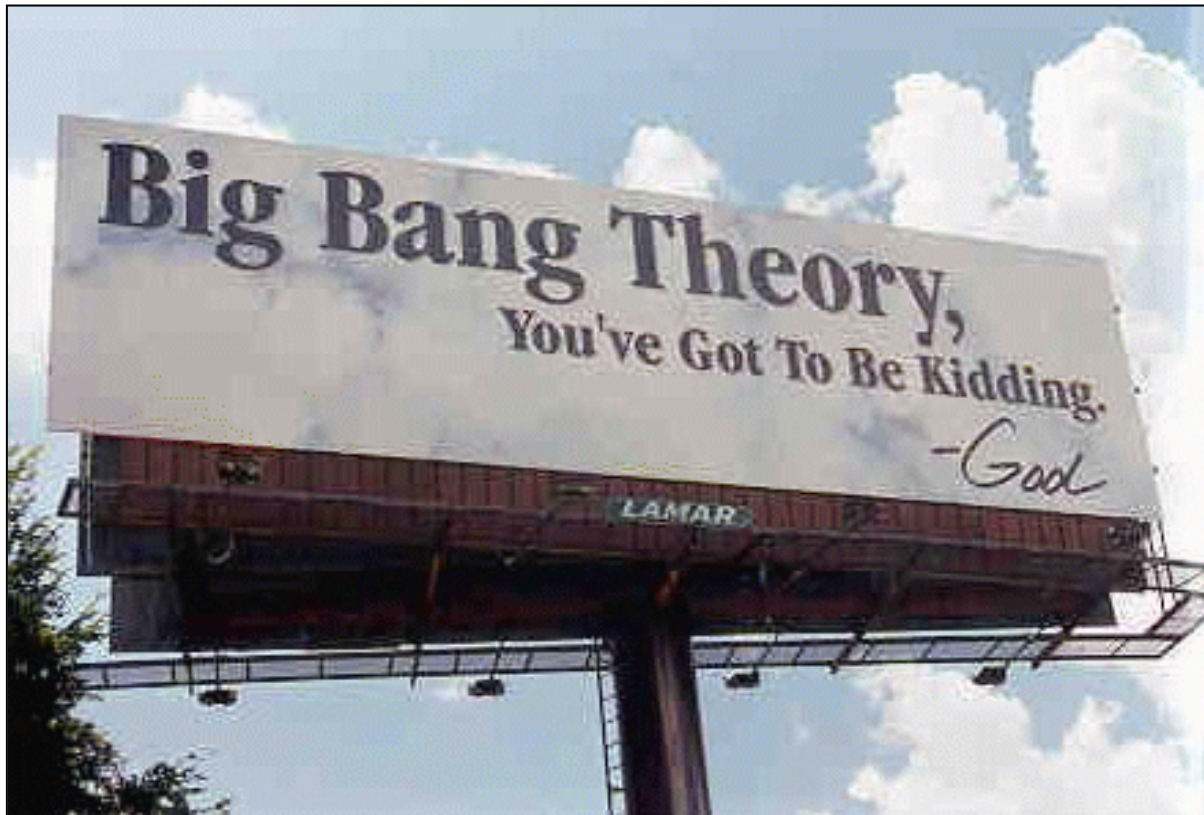
$$\leftarrow G_{\mu\nu} = 8\pi G T_{\mu\nu} \rightarrow$$

Perfect-fluid stress tensor

ρ = energy density

p = pressure

$T^\mu_\nu = \text{diag}(\rho, p, p, p)$



Field equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho \quad H \equiv \frac{\dot{a}}{a} = \text{expansion rate}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) \quad q \equiv -\frac{\ddot{a}}{a} \frac{1}{H^2} = \text{deceleration parameter}$$

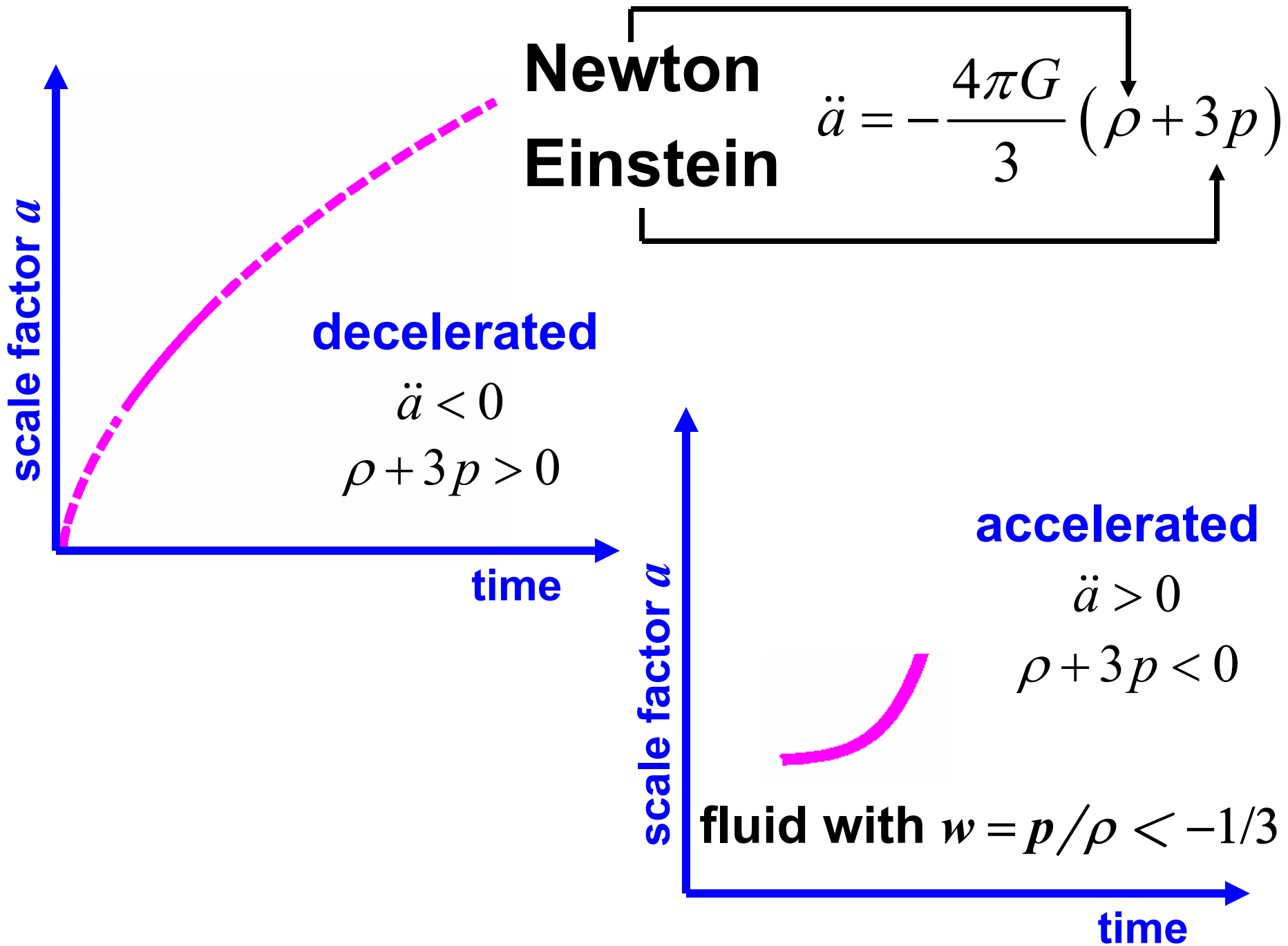
$$T^{\mu\nu}_{;\nu} = 0 \quad \rho \propto a^{-3(1+w)} \quad w = p/\rho$$

$T^{\mu\nu}$: fluids with different w

radiation: $p_R = \rho_R/3$ $w = 1/3$ $\rho_R \propto a^{-4}$

matter: $p_M = 0$ $w = 0$ $\rho_M \propto a^{-3}$

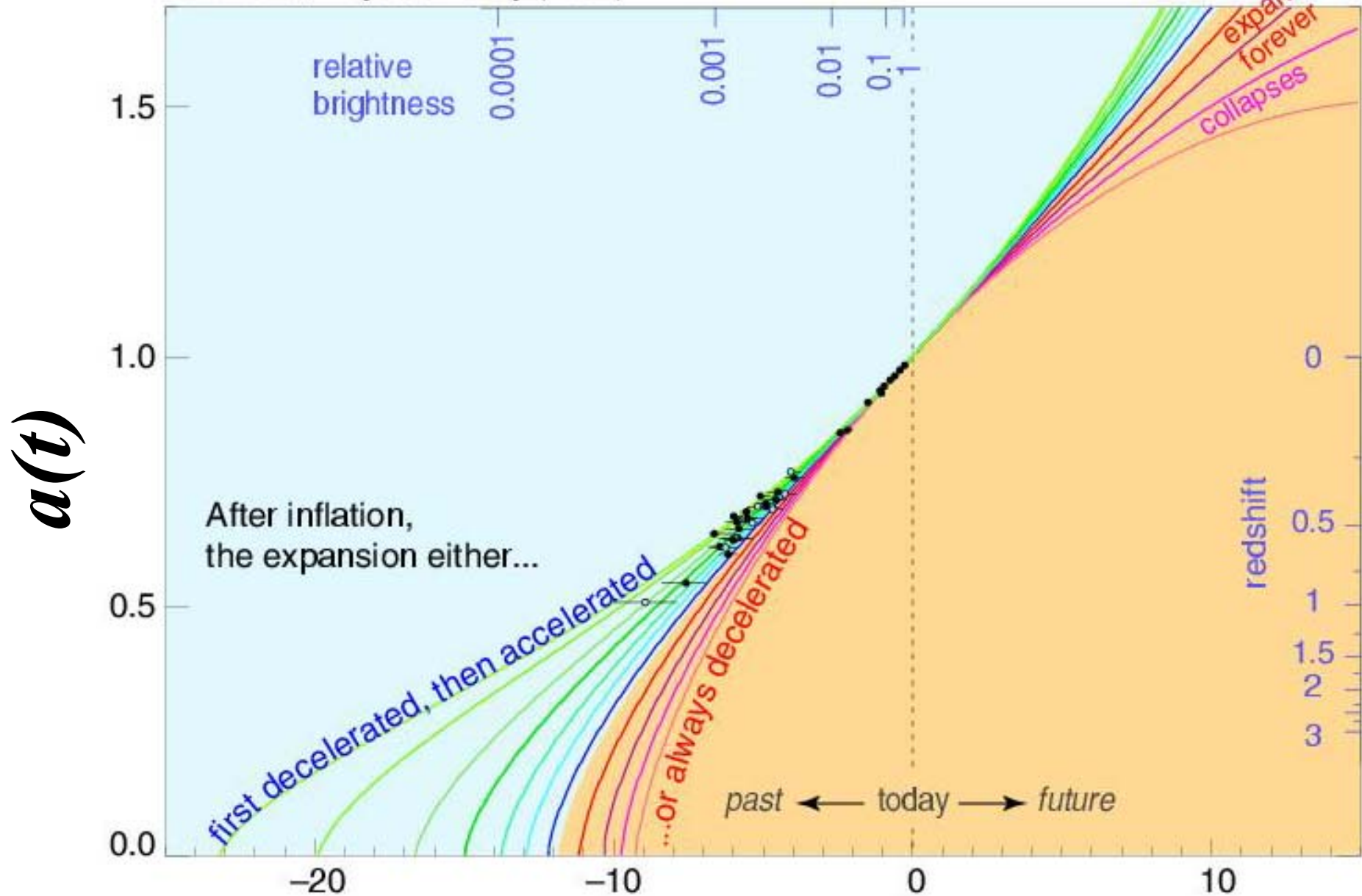
vacuum: $p_\Lambda = -\rho_\Lambda$ $w = -1$ $\rho_\Lambda \propto a^0$



Dynamics → Evolution

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho \quad \rho = \rho_M(a) + \rho_R(a) + \rho_\Lambda(a) + \dots$$

- $a(t)$, $H(t)$, depend on matter/energy content
- $a(t)$ measurable via redshift $1 + z \equiv a(0)/a(t)$
- Redshift z is a proxy for time or scale factor



Time (billions of years from today)

Evolution of $H(z)$ is a key quantity

Many observables based on the comoving distance r

$$\left. \begin{array}{l} \sin^{-1} r(z) \\ r(z) \\ \sinh^{-1} r(z) \end{array} \right\} = \int_0^z \frac{dz'}{H(z')} \equiv R(z)$$

- Luminosity distance

$$\text{Flux} = \text{Luminosity} / 4\pi d_L^2$$

$$d_L(z) \propto R(z)(1+z)$$

- Angular diameter distance

$$\text{Angular diameter} = \text{Physical size} / d_A$$

$$d_A(z) \propto \frac{R(z)}{(1+z)}$$

- Comoving volume element

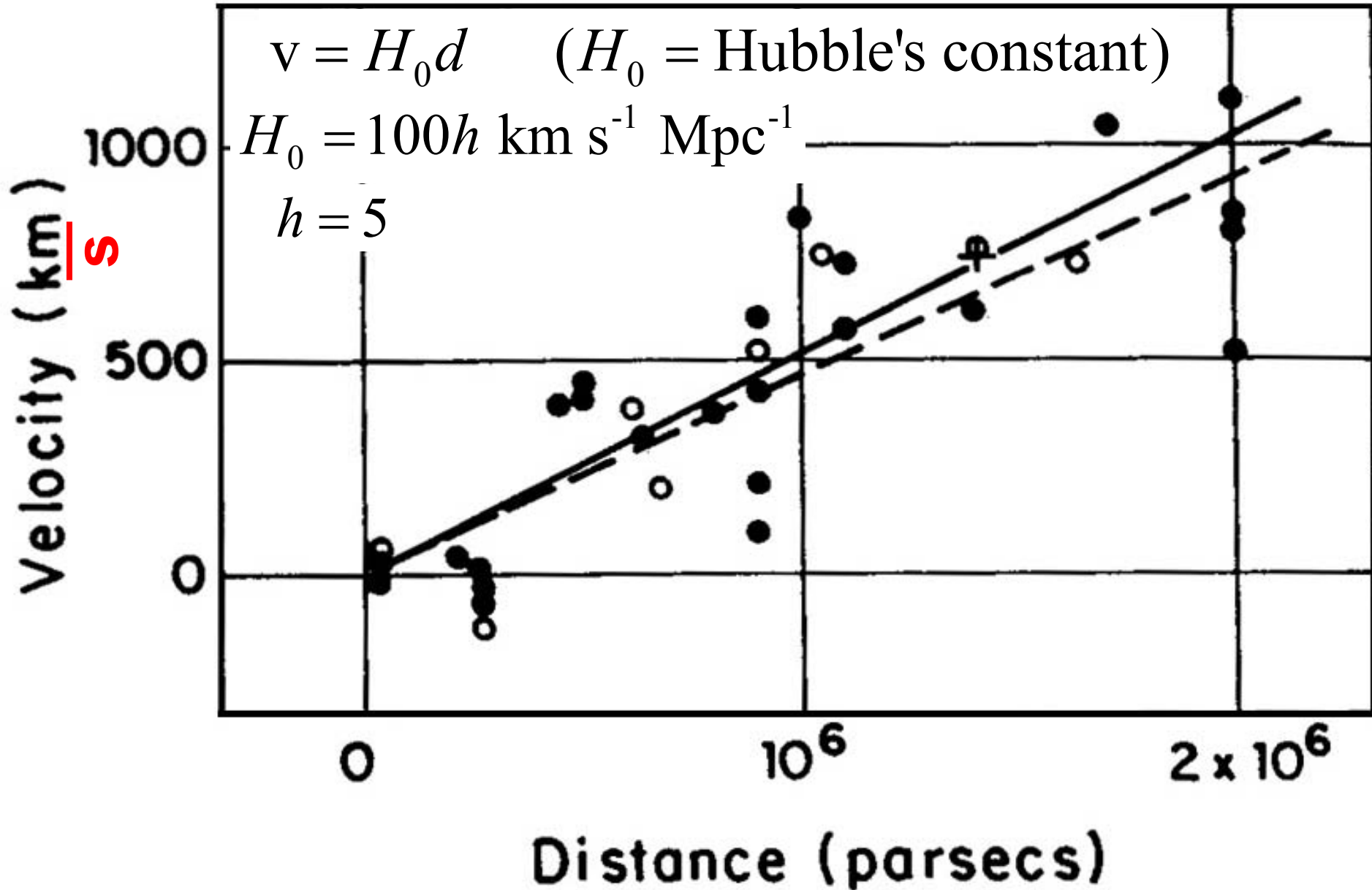
$$N \propto V^{-1}(z)$$

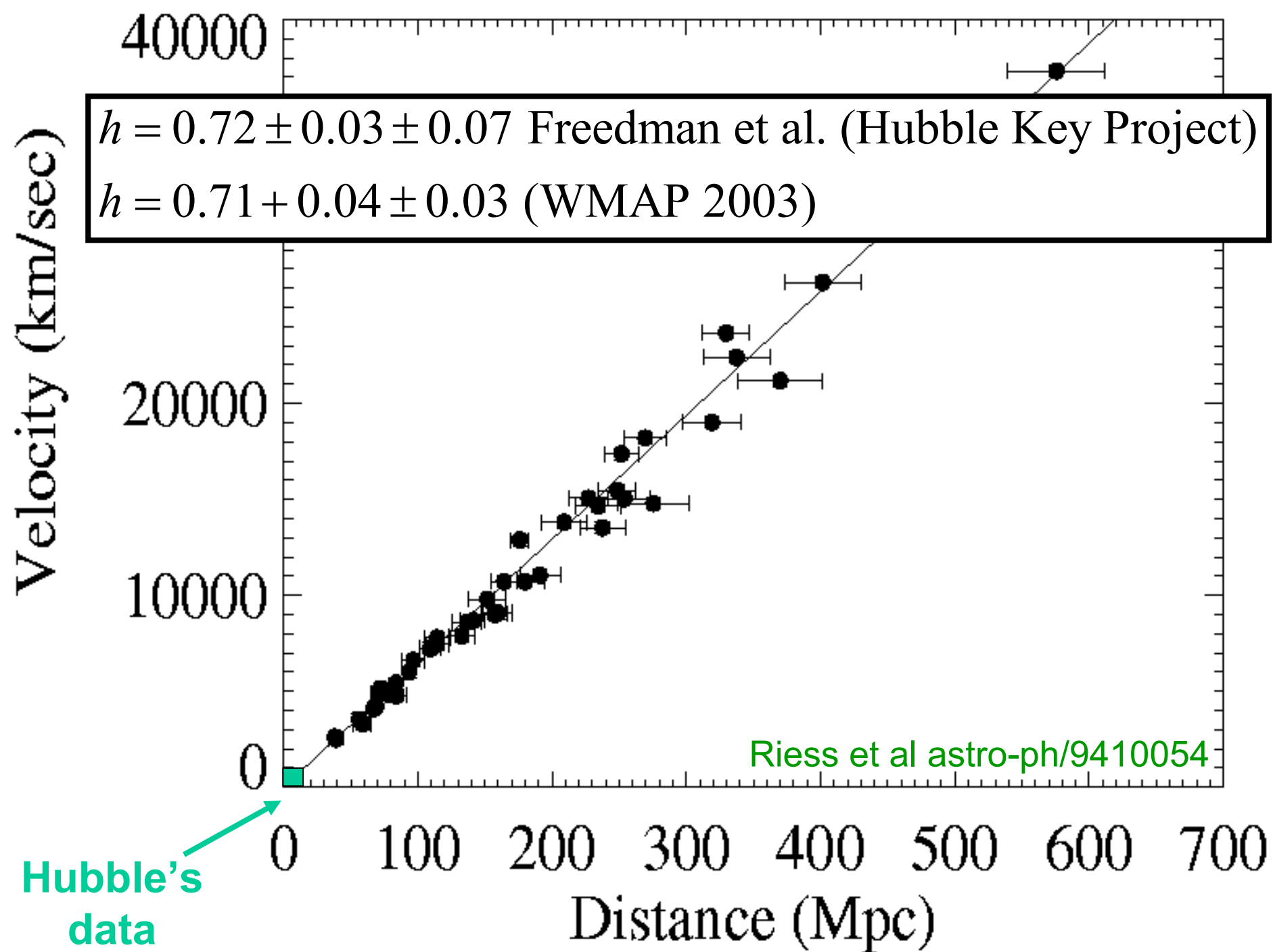
$$\frac{dV(z)}{dz d\Omega} \propto \frac{R^2(z)}{H(z)}$$

- Age of the universe

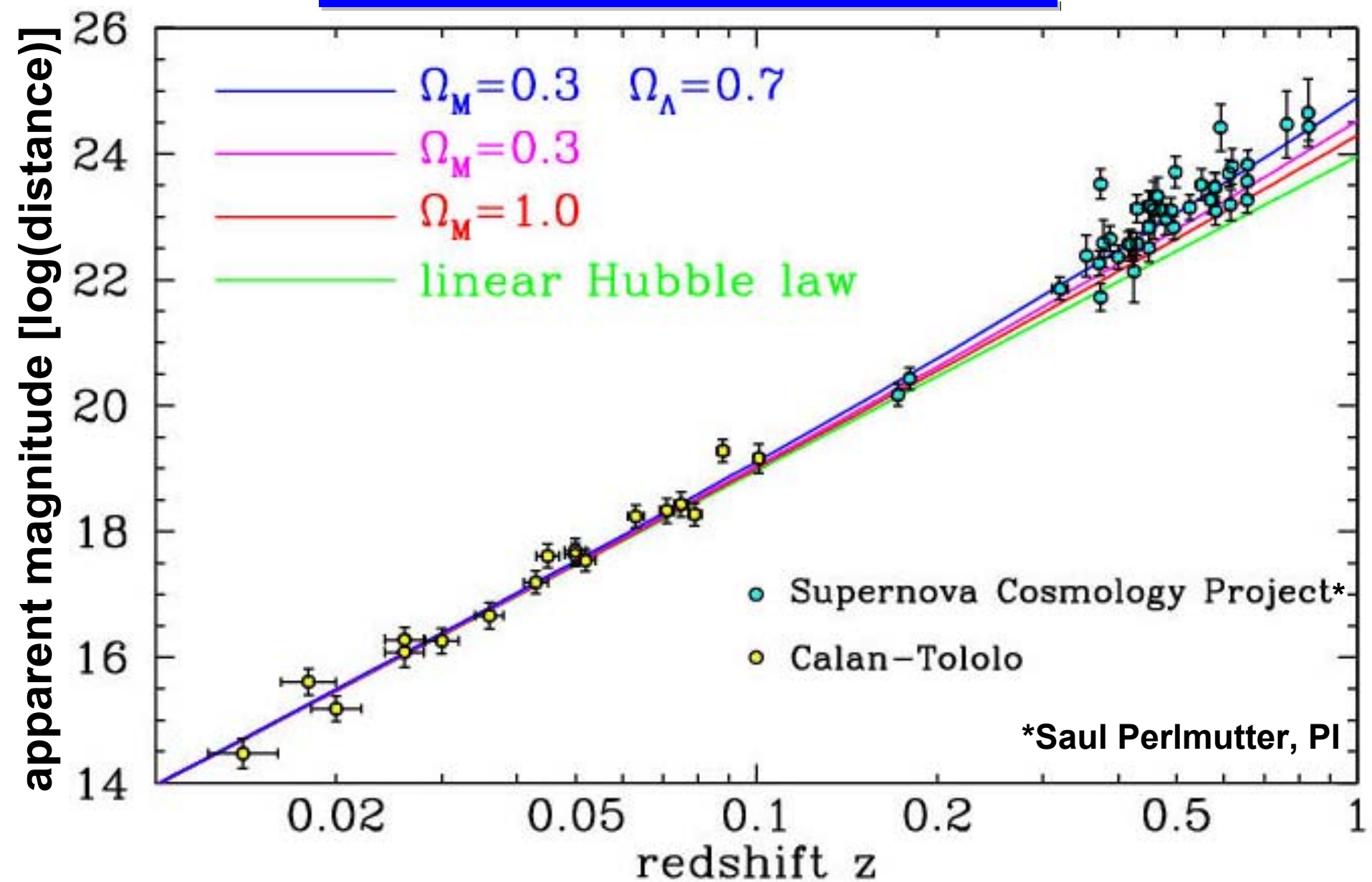
$$t(z) \propto \int_0^z \frac{dz'}{(1+z')H(z')}$$

Hubble's Discovery Paper - 1929

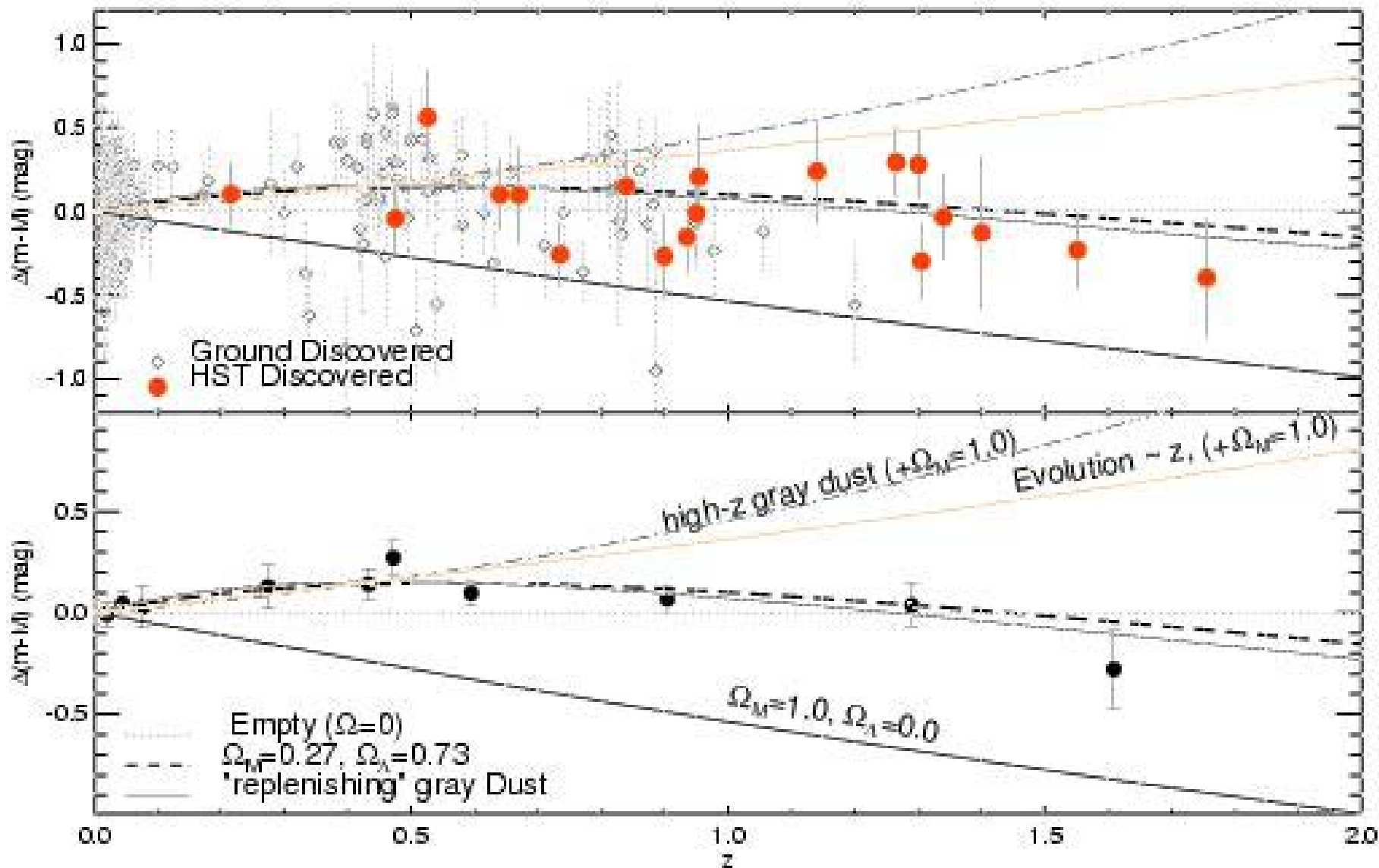




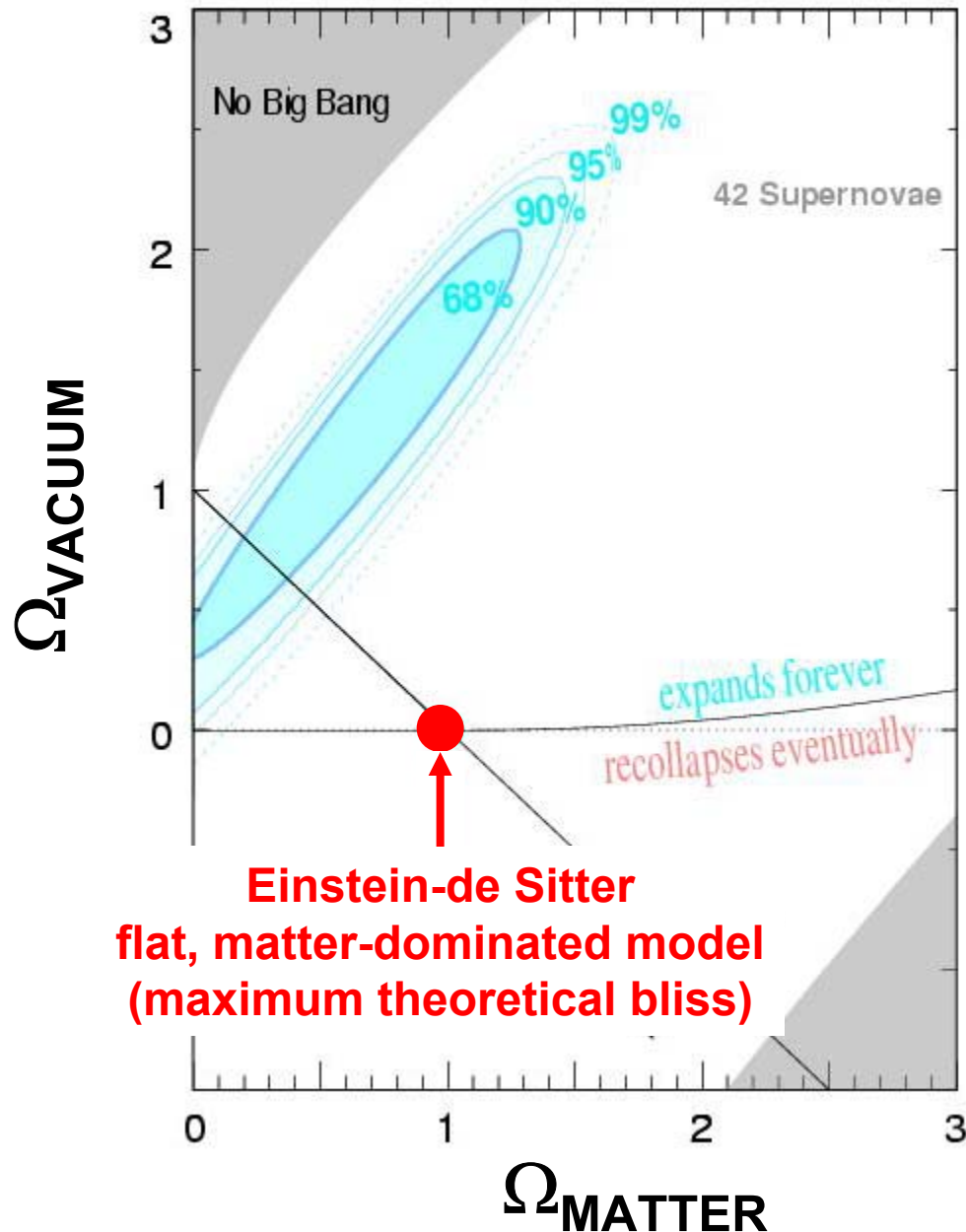
Type Ia supernova



deceleration \Leftrightarrow acceleration transition at $z = 0.46 \pm 0.13$



Supernova Cosmology Project
Perlmutter *et al.* (1998)



High- z SNeIa are fainter than expected in an Einstein-deSitter model

cosmological constant, or ...some changing non-zero vacuum energy, or ... or some unknown systematic effect(s)

The case for Λ :

- 1) Hubble diagram $d_L(z)$
- 2) subtraction

Subtraction

$$\Omega_i \equiv \rho_i / \rho_C$$

$$\rho_C \equiv 3H_0^2 / 8\pi G$$

dynamics

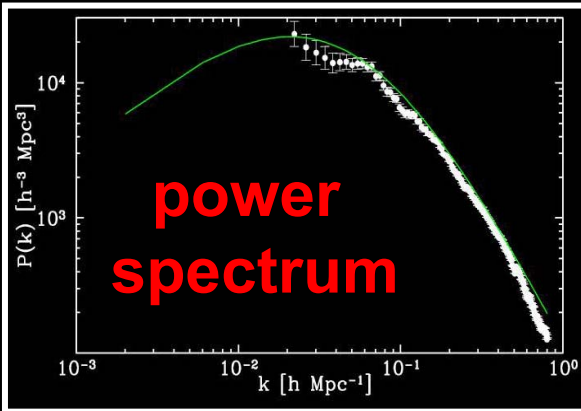
lensing

x-ray gas

cmb

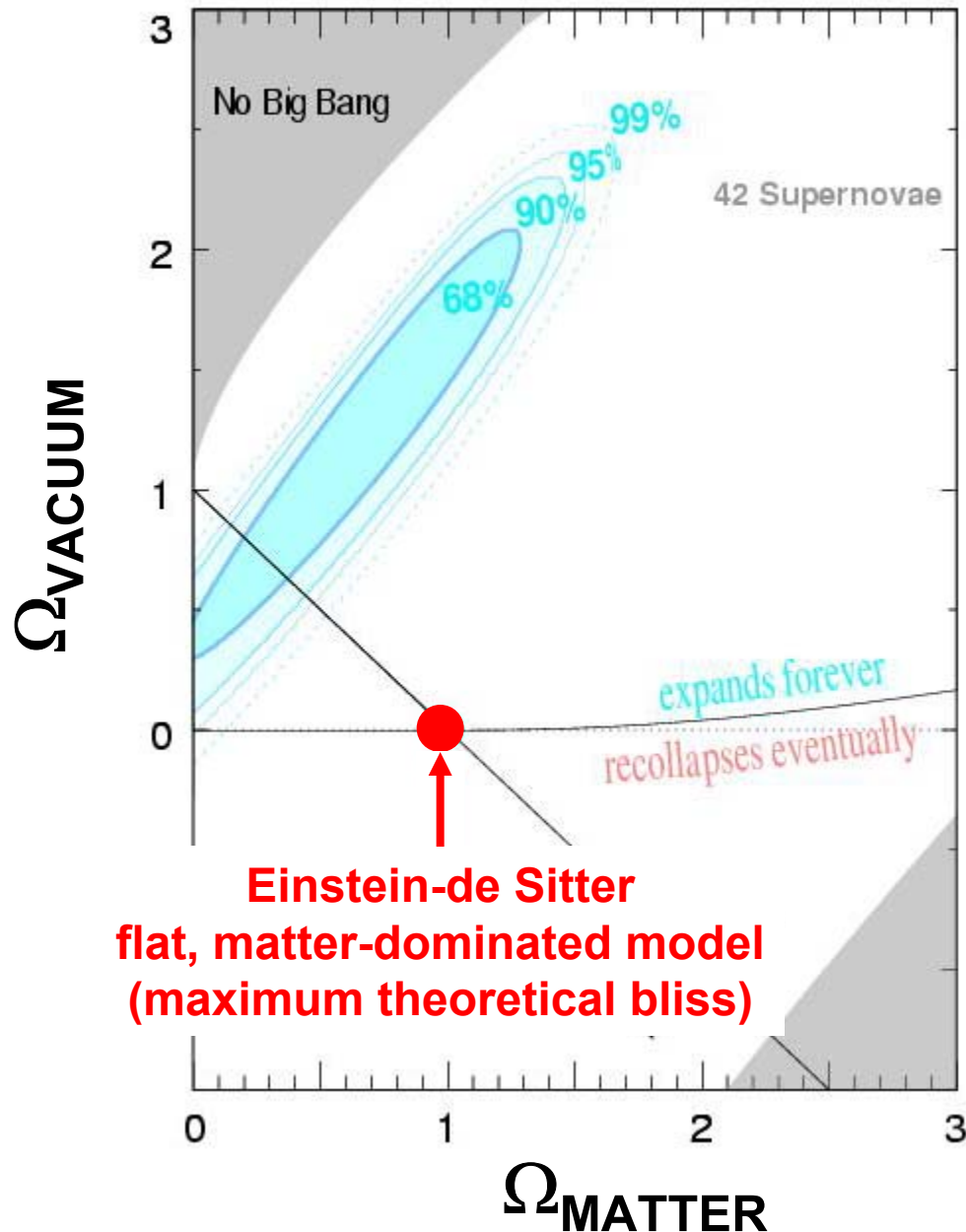
simulations

power
spectrum



$$\Omega_{\text{TOTAL}} = 1 \text{ (CMB)}, \quad \Omega_M = 0.3, \quad 1 - 0.3 = 0.7$$

Supernova Cosmology Project
Perlmutter *et al.* (1998)



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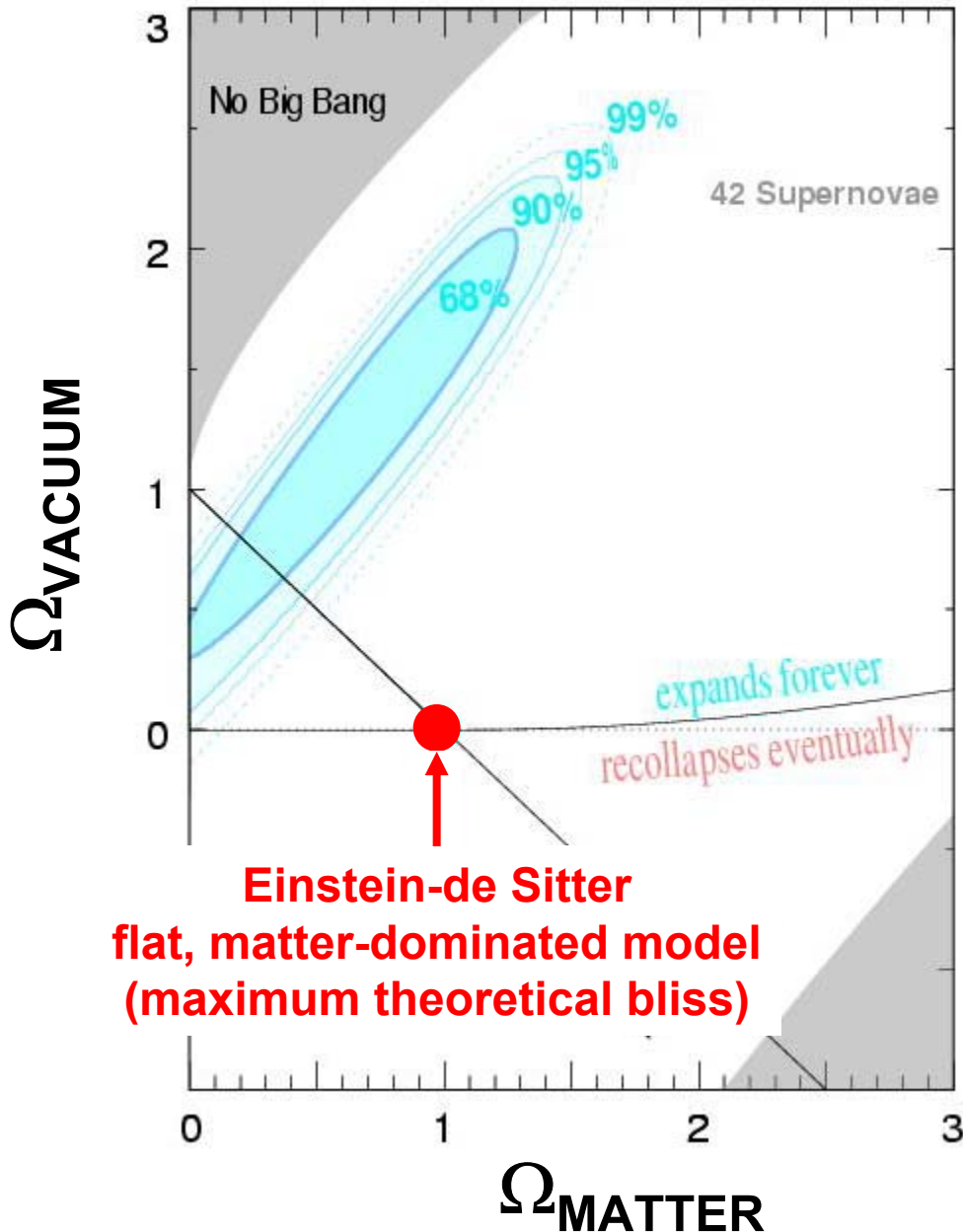
3) age of the universe

t_0 : age of the universe

size challenged star

- ~~white dwarf star cooling~~ 11 ± 2 Gyr
- nucleocosmochronology 12.6 ± 3 Gyr
- globular cluster evolution 13.5 ± 2 Gyr

$H_0 = 70$	Ω_M	Ω_Λ	t_0 (Gyr)
Flat	1.0	0	9.3
Open	0.3	0	12
Open	0.2	0	14
Flat	0.3	0.7	13.5
Flat	0.2	0.8	15



**Einstein-de Sitter
flat, matter-dominated model
(maximum theoretical bliss)**

**High-z SNeIa are fainter
than expected in an
Einstein-deSitter model**

**...some changing non-zero
vacuum energy, or
... or some unknown
systematic effect(s)**

The case for Λ :

1) Hubble diagram $d_L(z)$

2) subtraction

$$\Omega_{\text{TOTAL}} = 1$$

$$\Omega_M = 0.3$$

$$1 - 0.3 = 0.7$$

3) age of the universe

4) structure formation

Einstein's *Biggest Blunder?*

1917 Einstein proposes
cosmological constant

1929 Hubble discovers
Expansion of the universe

1934 Einstein calls it
“my biggest blunder”

1998 Astronomers find
evidence for it



Cosmological constant

Mass density of space:

$$\rho_{\Lambda} \simeq 10^{-30} \text{ g cm}^{-3} \simeq (10^{-4} \text{ eV})^4 = (10^{-3} \text{ cm})^{-4}$$
$$\Lambda = 8\pi G \rho_{\Lambda} = (10^{29} \text{ cm})^{-2} = (10^{-33} \text{ eV})^2$$

The unbearable lightness of nothing!

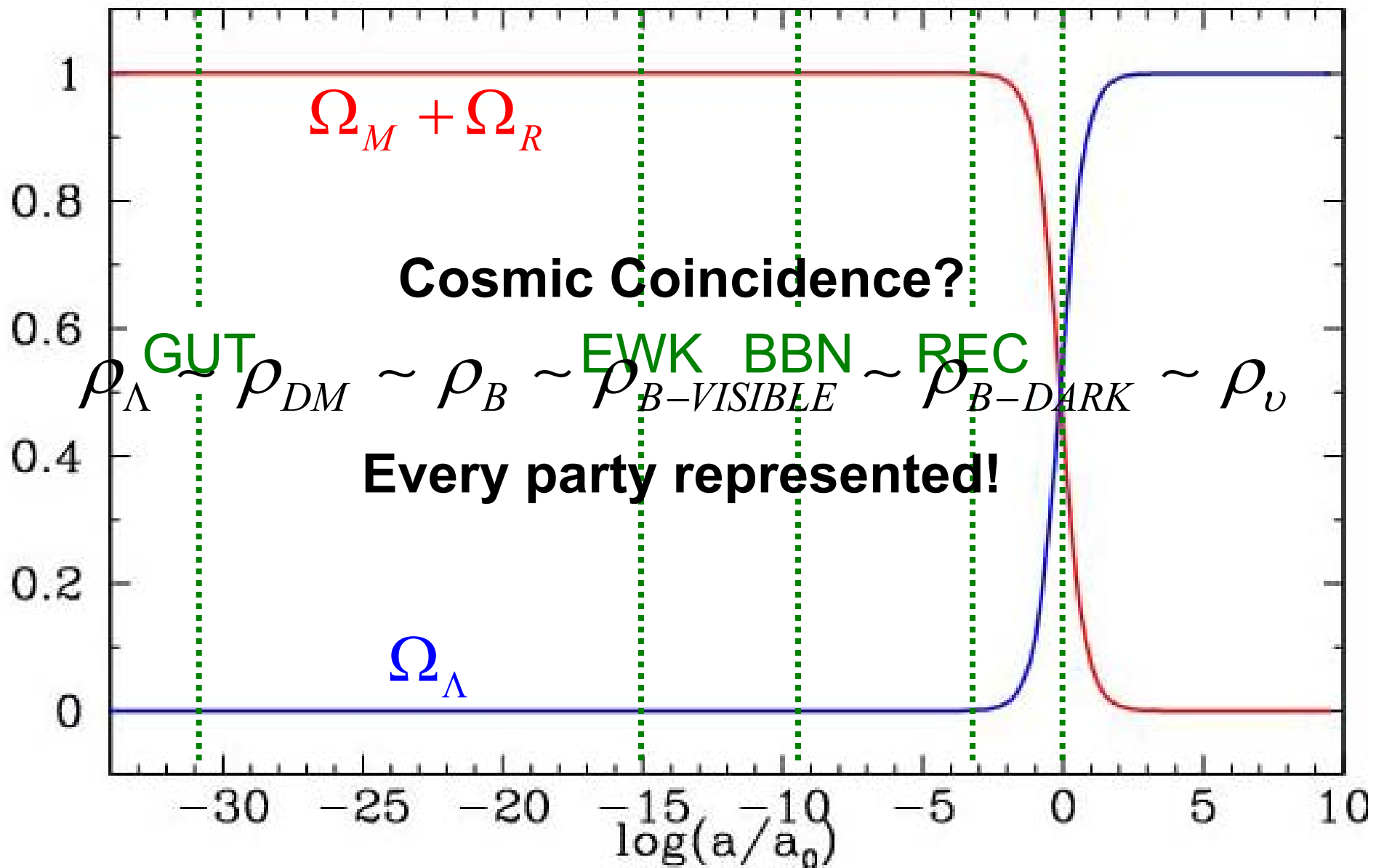
Cosmo-illogical constant?

Numerology:

$$\rho_V = M_W^4 \exp(-2/\alpha) \quad \rho_V = M_{\text{SUSY}}^8 / M_{\text{Pl}}^4$$

$$m_{\nu} = 10^{-3} \text{ eV} \quad R_5 = 10^{-4} \text{ cm}$$

Cosmic coincidence



Dark energy depression?

- 1. Alcohol***
- 2. Drugs***
- 3. Anthropic principle***
- 4. Creative theories**
- 5. Hard experimental work**
- 6. Observational direction**

*** Therapy, medication, and twelve-step programs available.**

Take sides!

Field equation: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$

Perfect fluid stress tensor: $T^{\mu}_{\nu} = \text{diag}(\rho, -p, -p, -p)$

“I found it very ugly indeed that the field law of gravitation should be composed of two logically independent terms connected by addition. About the justification of such feelings concerning logical simplicity it is difficult to argue. I cannot help to feel it strongly and I am unable to believe that such an ugly thing should be realized in nature.”

Einstein in a letter to Lemaitre, Sept. 26, 1947

Modern view: “It belongs on the right-hand side, and has many contributions.”

$$\tilde{T}^{\mu}_{\nu} = \text{diag}(\rho_{\Lambda}, \rho_{\Lambda}, \rho_{\Lambda}, \rho_{\Lambda}) \quad \rho_{\Lambda} = \Lambda/8\pi G$$

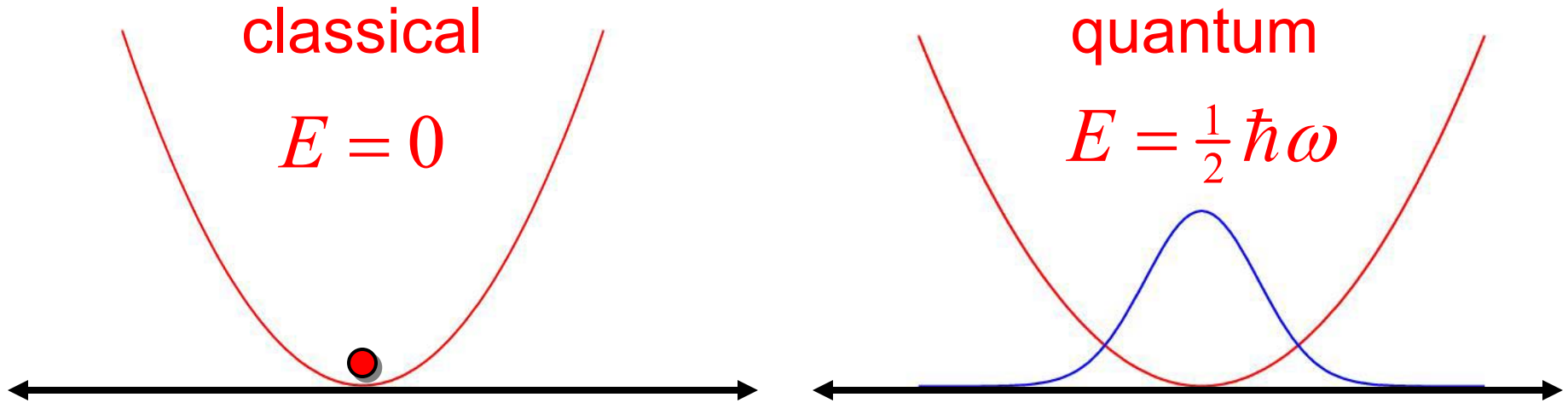
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G (T_{\mu\nu} + \tilde{T}_{\mu\nu})$$

Take sides!

- Something is established – Λ CDM too good to ignore
 - SNIa
 - Subtraction
 - Age
 - Large-scale structure
 - ...
- } $H(z)$ not given by Einstein–de Sitter cosmological model
- Dark energy (right-hand side of Einstein equations)?
 - Is it “just” a cosmological constant?
 - If not cosmological constant, what is dynamics?
 - interpretation of $w = \rho/p$?
 - Gravity (left-hand side of Einstein equations)?
 - Beyond Einstein (non-GR: branes, etc.)
 - (Just) Einstein (Back reaction of inhomogeneities)

Quantum uncertainty

Fourier modes of all fields are harmonic oscillators with a zero-point energy

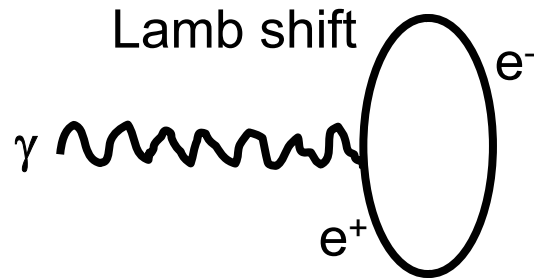


$$\rho = \sum_{\text{all particles}} \pm \int d^3 k \sqrt{k^2 + m^2}$$

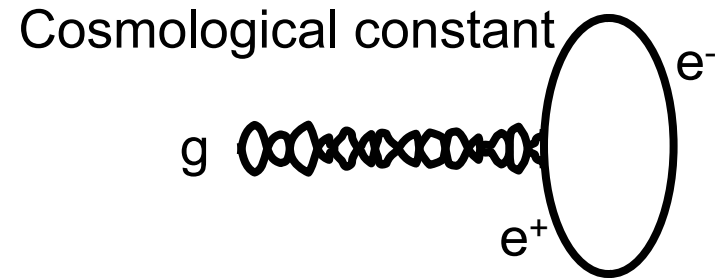
Quantum uncertainty

$$\rho = \sum_{\text{all particles}} \pm \int d^3k \sqrt{k^2 + m^2} \simeq \sum_{\text{all particles}} \pm \int^{\Lambda_C} d^3k k^4$$

Photons couple to virtual particles



Gravitons couple to virtual particles



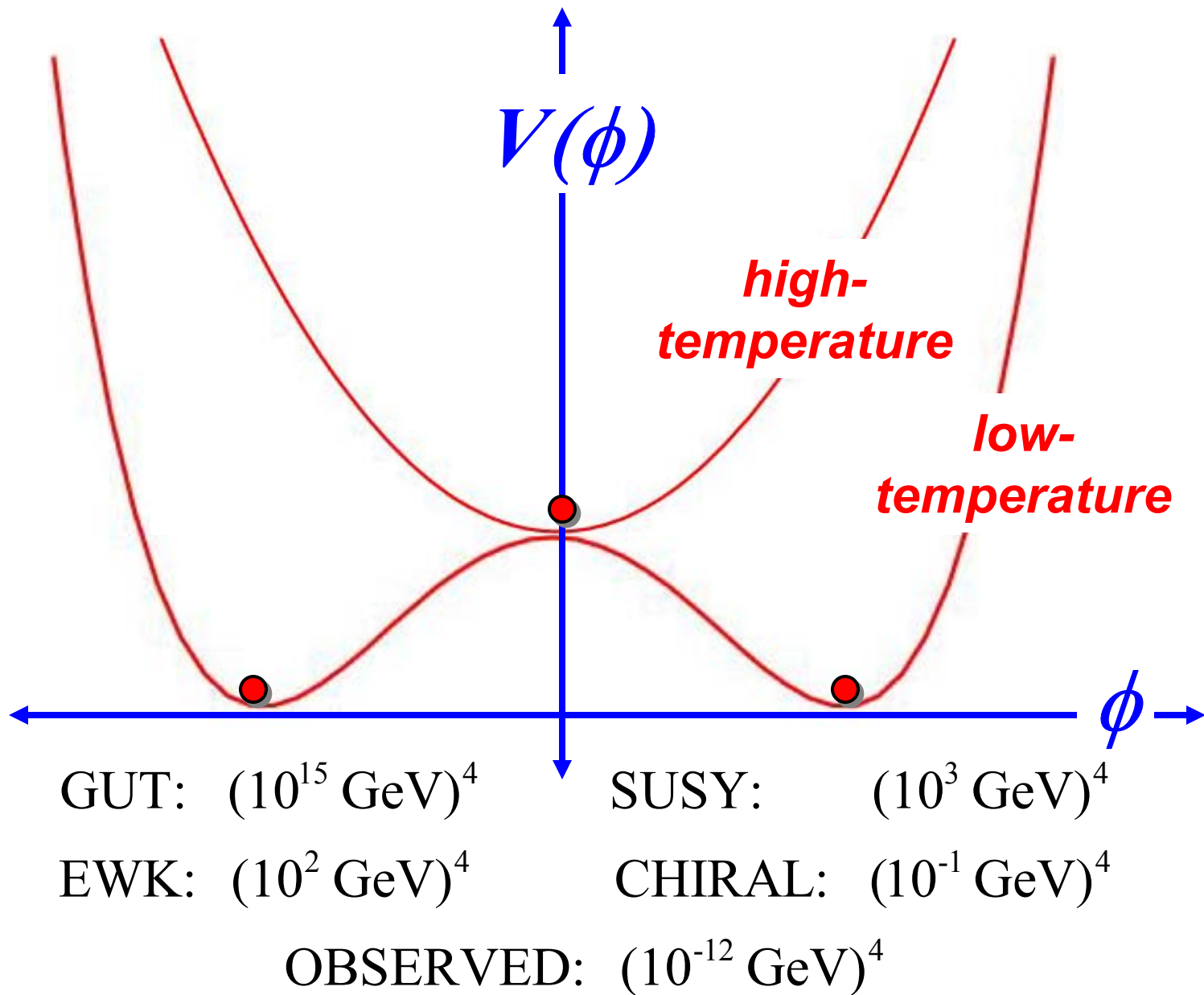
$$\Lambda_C = \infty : \quad \rho_\Lambda = \infty^4 \quad = \text{bad prediction}$$

$$\Lambda_C = M_{Pl} : \quad \rho_\Lambda = M_{Pl}^4 = (10^{28} \text{ eV})^4$$

$$\Lambda_C = M_{SUSY} : \quad \rho_\Lambda = M_{SUSY}^4 = (10^{12} \text{ eV})^4$$

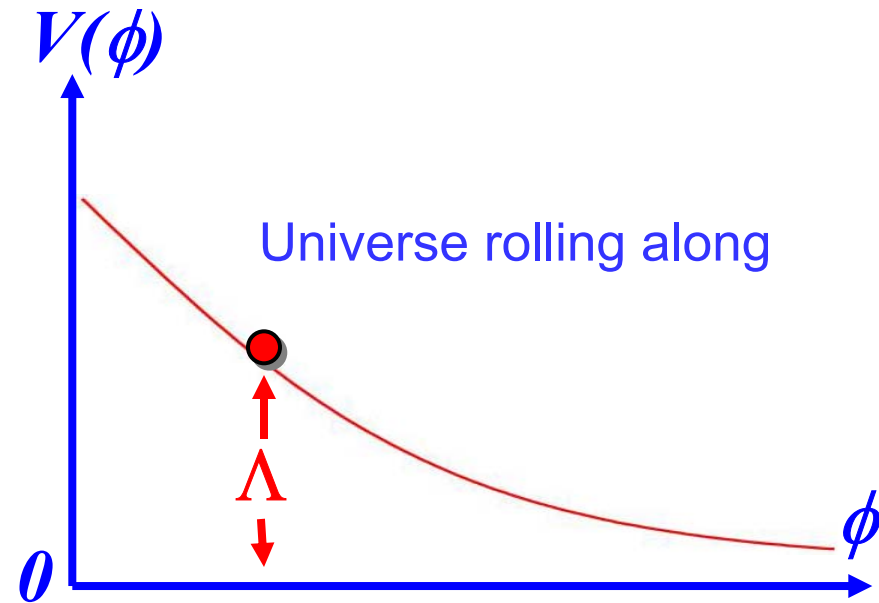
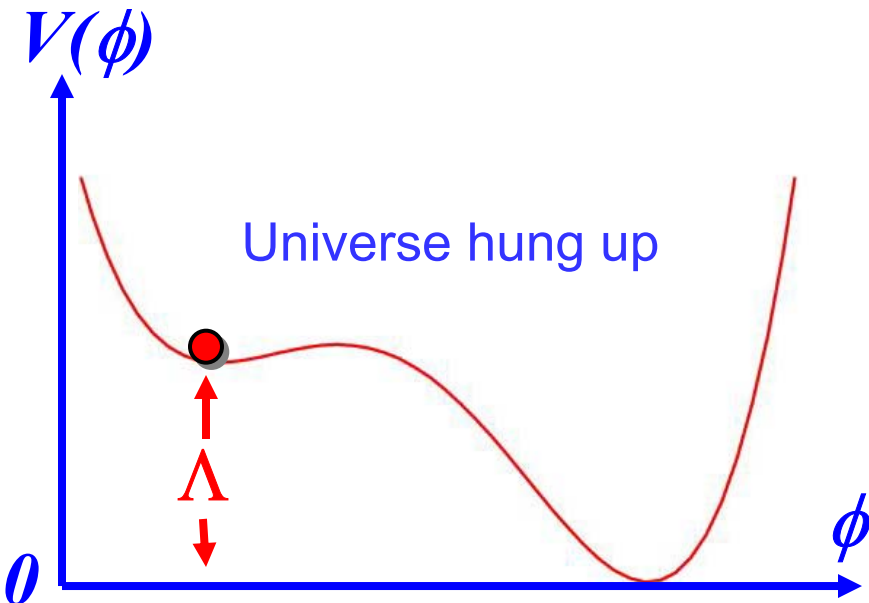
$$\Lambda_C = 10^{-3} \text{ eV} : \quad \rho_\Lambda = \text{Observed}$$

Spontaneous symmetry breaking



WD-40

- Many possible contributions.
- Why then is total so small?
- Perhaps unknown dynamics sets global vacuum energy equal to zero.....*but we're not there yet!*



WD-40

- **But *why now?***
- **Tracker potentials, $e^{-\phi}$, ϕ^{-n} , ... relate dark energy to other contributions.** Wetterich; Ratra & Peebles; ...

Field equation for tracker: $\ddot{\phi} + 3H\dot{\phi} + dV/d\phi = 0$

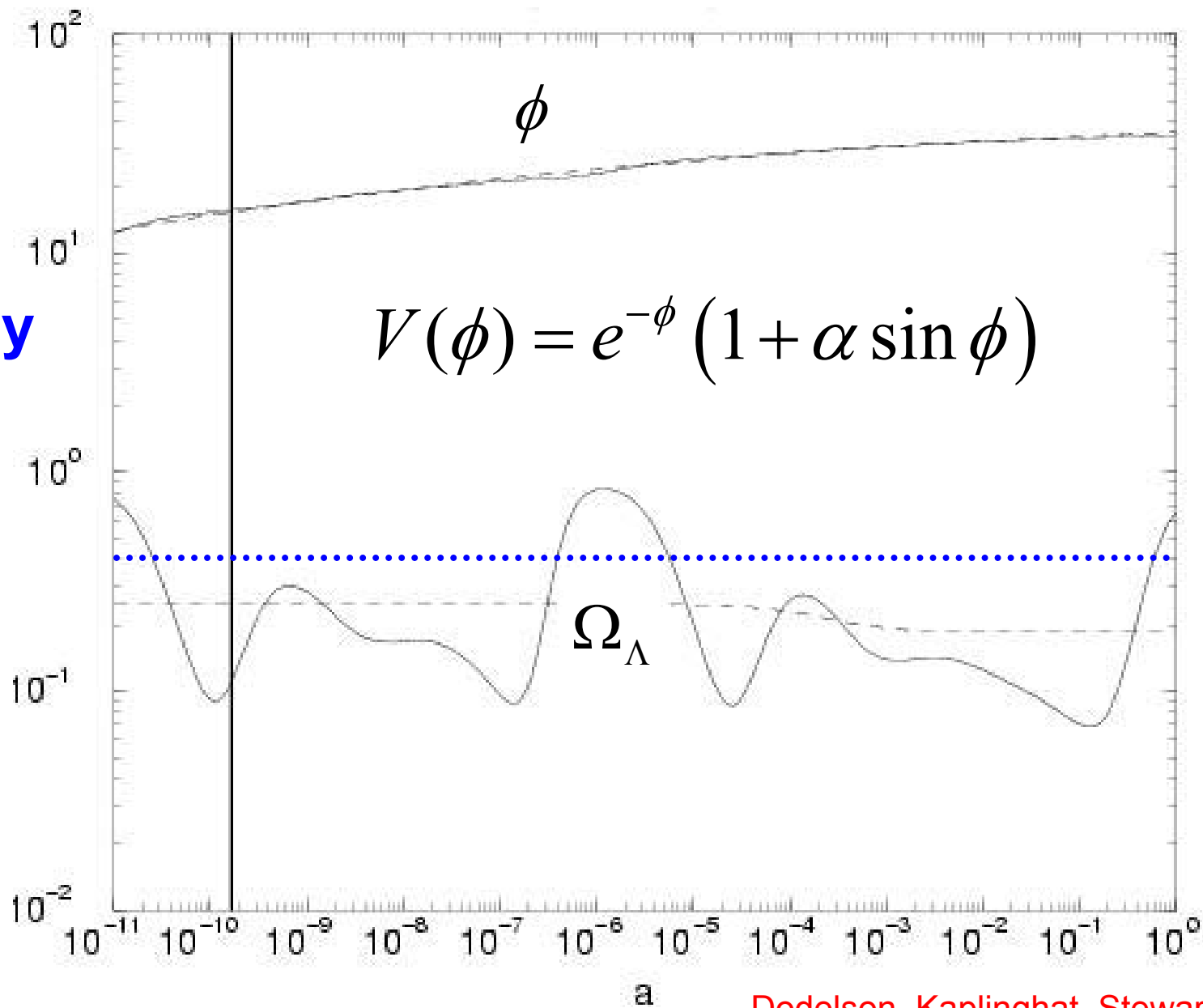
$$H^2 = \frac{8\pi G}{3} (\rho_\phi + \rho_M + \dots)$$

Can have ρ_ϕ track (stalk) ρ_M - effective mass of the field decreases as H : close but not final answer

- ***Why now?.....* “close to matter domination”**
- ***Why now?.....* “it’s just that time”**

WD-40

dark energy
 \neq destiny



WD-40

- “Quintessence” models have very light scalar fields
- Long-range interactions? Carroll 1998

Entertaining conjecture

Now entertain conjecture of a time
When creeping murmur and the poring dark
Fills the wide vessel of the universe.

— *Shakespeare, King Henry Vth*

All evidence for dark energy (creeping murmur) is indirect!

$$\begin{array}{l} \text{SNIa} \\ \text{Age} \\ \text{LSS} \end{array} : \int \frac{dz}{H(z)}$$

- Observed time evolution of H does not fit Einstein–de Sitter.
- We infer the existence of dark energy!
- Could Friedmann equation be modified-and no dark energy?

Modifying the left-hand side

- Braneworld modifies Friedmann equation Binetruy, Deffayet, Langlois
- Friedmann equation modified today Freese & Lewis
$$H^2 = A\rho \left[1 + \left(\rho / \rho_{\text{cutoff}} \right)^{n-1} \right]$$
- Gravitational force law modified at large distance Deffayet, Dvali & Gabadadze
Five-dimensional at cosmic distances
- Tired gravitons Gregory, Rubakov & Sibiryakov; Dvali, Gabadadze & Porrati
Gravitons metastable - leak into bulk
- Gravity repulsive at distance $R \approx \text{Gpc}$ Csaki, Erlich, Hollowood & Terning
- $n=1$ KK graviton mode very light, $m \approx (\text{Gpc})^{-1}$ Kogan, Mouslopoulos, Papazoglou, Ross & Santiago
- Einstein (Hilbert) got it wrong Carroll, Duvvuri, Turner, Trodden
$$S = (16\pi G)^{-1} \int d^4x \sqrt{-g} \left(R - \mu^4 / R \right)$$
- Backreaction of inhomogeneities Räsänen; Kolb, Matarrese, Notari & Riotto, Notari; Kolb, Matarrese & Riotto

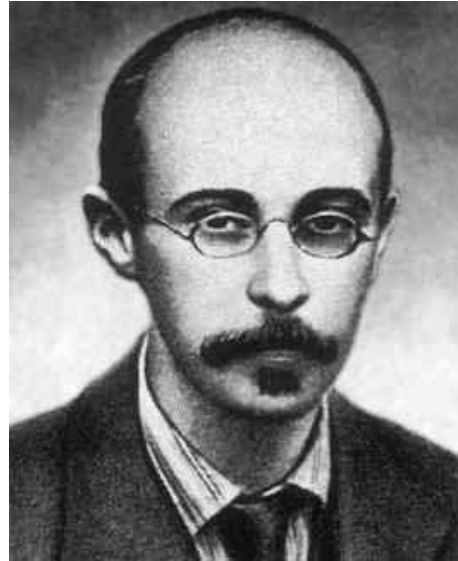
Braneless cosmology

Old Friedmann law:

$$G_{00} = M_{Pl}^{-2} T_{00}$$

$$3H^2 = M_{Pl}^{-2} \rho$$

Friedmann (1921)

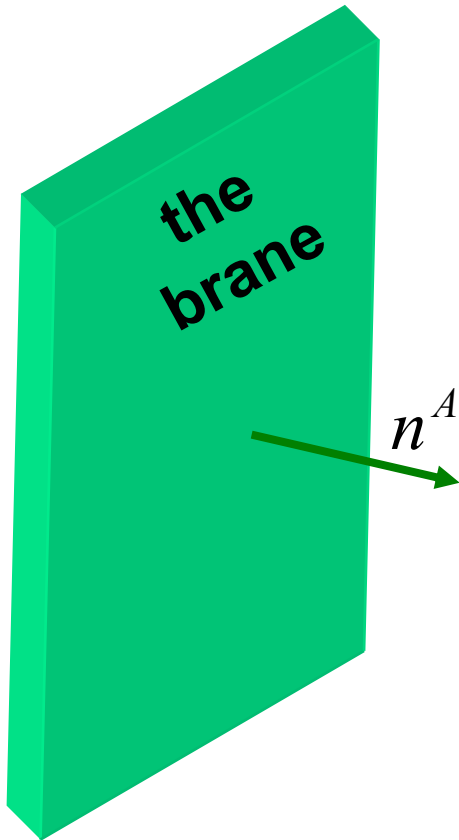


SN Ia evidence
for dark energy:

$$\int \frac{dz}{H(z)}$$

Brane cosmology

- Israel junction condition (Israel 1966)



- n^A : unit vector normal to the brane
- $h_{AB} = g_{AB} - n_A n_B$: the induced metric
- $\kappa_{AB} = h_A^C \nabla_C n_B$: the extrinsic curvature

$$[\kappa_{\mu\nu}] = -M_*^{-3} T_{\mu\nu}^{BRANE}$$

[...] = discontinuity across the brane

$$a'' = \langle a'' \rangle + [a'] \delta(y)$$

discontinuity in second derivative of scale factor

Braneless cosmology

Old Friedmann law:

$$G_{00} = M_{Pl}^{-2} T_{00}$$
$$3H^2 = M_{Pl}^{-2} \rho$$

Friedmann (1921)



SN Ia evidence
for dark energy:

$$\int \frac{dz}{H(z)}$$

Braneful cosmology

New Friedmann law:
Israel jump conditions

Binetruy, Deffayet, Langlois (2000)

$$3H^2 = \frac{\Lambda}{2} + \frac{M_*^{-6}}{12} \rho^2 + \frac{c}{a^4(t, y=0)}$$

Brane cosmology

- New Friedmann law Binetruy, Deffayet, Langlois (2000)

$$3H^2 = \frac{\Lambda}{2} + \frac{M_*^{-6}}{12} \rho^2 + \frac{c}{a^4(t, y=0)}$$

- Possible solution Randall & Sundrum (2000)

Introduce a tension σ on the brane $\rho \rightarrow \rho + \sigma$

$$3H^2 = \underbrace{\left(\frac{\Lambda}{2} + \frac{M_*^{-6}}{12} \sigma^2 \right)}_{\text{cosmological constant (cancels?)}} + \underbrace{\frac{M_*^{-6}}{6} \sigma \rho}_{\frac{M_*^{-6}}{18} \sigma = \frac{8\pi G}{3}} + \underbrace{\frac{M_*^{-6}}{12} \rho^2 + \frac{c}{a^4(t, y=0)}}_{\text{unconventional corrections}}$$

cosmological
constant
(cancels?)

$$\frac{M_*^{-6}}{18} \sigma = \frac{8\pi G}{3}$$

Friedmann
equation

unconventional
corrections

Inhomogeneities

Most conservative approach

(no new fields, gravity, extra dimensions)

Universe is inhomogeneous & anisotropic → smooth

- Matter smoothing is straightforward (particles → fluid), space-time metric smoothing not straightforward!
- Suppose A 's equations apply on some small inhomogeneous/anisotropic scale
- Smooth on some larger scale
- Smoothing and evolution (going to field equations) do not commute
- Einstein tensor computed from smoothed metric is not the same as Einstein tensor computed from smoothed stress-energy
- Difference is a new term that enters Friedmann equation
- New term need not obey energy conditions!

Inhomogeneities

- The Universe is inhomogeneous
- One can define an average density $\langle \rho \rangle$
- The expansion rate of an inhomogeneous universe of average density $\langle \rho \rangle$ is NOT! the same as the expansion rate of a homogeneous universe of average density $\langle \rho \rangle$!
- We deduce dark energy because we are comparing to the wrong model universe
- Acceleration is a pure GR effect

Inhomogeneities

Kolb, Matarrese, Notari & Riotto

- $H(z)$ in a perturbed FLRW model:

$$G_{\mu\nu}(\vec{x}, t) = G_{\mu\nu}^{\text{FLRW}}(t) + \delta G_{\mu\nu}(\vec{x}, t)$$

$$G_{00}^{\text{FLRW}}(t) + \delta G_{00}(\vec{x}, t) = \kappa^2 T_{00}(\vec{x}, t)$$

$$3(\dot{a}/a)^2 = \kappa^2 (\langle \rho \rangle - \kappa^{-2} \langle \delta G_{00} \rangle)$$

- $(\dot{a}/a)^2$ is not $\kappa^2 \langle \rho \rangle / 3$.
- \dot{a}/a is not even the expansion rate.

Inhomogeneities

- For a general fluid, four velocity $u^\mu = (1, \vec{0})$.
(Local observer comoving with energy flow)
- For irrotational dust, work in synchronous and comoving gauge

$$ds^2 = -dt^2 + h_{ij}(\vec{x}, t) dx^i dx^j$$

- Velocity gradient tensor

$$\Theta^i_j = u^i_{;j} = \frac{1}{2} h^{ik} \dot{h}_{kj} = \Theta \delta^i_j + \sigma^i_j$$

- Θ is the volume-expansion factor and σ^i_j is the shear
($\Theta = 3H$ and $\sigma^i_j = 0$ in the homogeneous case)

Inhomogeneities

- Local deceleration parameter positive (no-go theorem):

$$q = -\frac{(3\dot{\Theta} + \Theta^2)}{\Theta^2} = 6(\sigma^2 + 2\pi G\rho) \geq 0$$

Hirata & Seljak; Flanagan; Giovannini;
Alnes, Amarzguioui & Gron

- However must course-grain over some finite domain:

$$\langle F \rangle_D = \frac{\int_D \sqrt{h} F d^3x}{\int_D \sqrt{h} d^3x}$$

- Evolution and smoothing do not commute:

$$\langle F \rangle_D^\bullet - \langle F^\bullet \rangle_D = \langle F\Theta \rangle_D - \langle \Theta \rangle_D \langle F \rangle_D$$

$$\langle \Theta \rangle_D^\bullet = \langle \Theta^\bullet \rangle_D + \langle \Theta^2 \rangle_D - \langle \Theta \rangle_D^2 \geq \langle \Theta^\bullet \rangle_D$$

- Invalidates *no-go* theorem

Buchert & Ellis

Kolb, Matarrese & Riotto
astro-ph/0506534

Inhomogeneities

Kolb, Matarrese & Riotto
astro-ph/0506534

Buchert & Ellis

- Define an effective scale factor:

$$a_D \equiv (V_D / V_{D0})^{1/3} \quad V_D = \int_D d^3x \sqrt{h}$$

- Course-grained Hubble rate:

$$H_D = \frac{\dot{a}_D}{a_D} = \frac{1}{3} \langle \Theta \rangle_D$$

- Effective evolution equations:

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G}{3} (\rho_{\text{eff}} + 3p_{\text{eff}}) \quad \rho_{\text{eff}} = \langle \rho \rangle_D - \frac{Q_D}{16\pi G} - \frac{\langle R \rangle_D}{16\pi G} \quad \text{not described by a simple } p = w \rho$$

$$\left(\frac{\dot{a}_D}{a_D} \right)^2 = \frac{8\pi G}{3} \rho_{\text{eff}} \quad p_{\text{eff}} = -\frac{Q_D}{16\pi G} + \frac{\langle R \rangle_D}{48\pi G}$$

- Kinematical back reaction: $Q_D = \frac{2}{3} \left(\langle \Theta^2 \rangle_D - \langle \Theta \rangle_D^2 \right) - 2 \langle \sigma^2 \rangle_D$

Inhomogeneities

- Does this have anything to do with our universe?
- Have to go to non-perturbative limit
- Toy model proof of principle: Tolman-Bondi dust model
(Nambu and Tanimoto (gr-qc/0507057))

How do we sort it out?

- Something is established- Λ CDM too good to ignore
 - SN Ia
 - Subtraction
 - Age
 - Large-scale structure
 -
- Left-hand side or right-hand side?

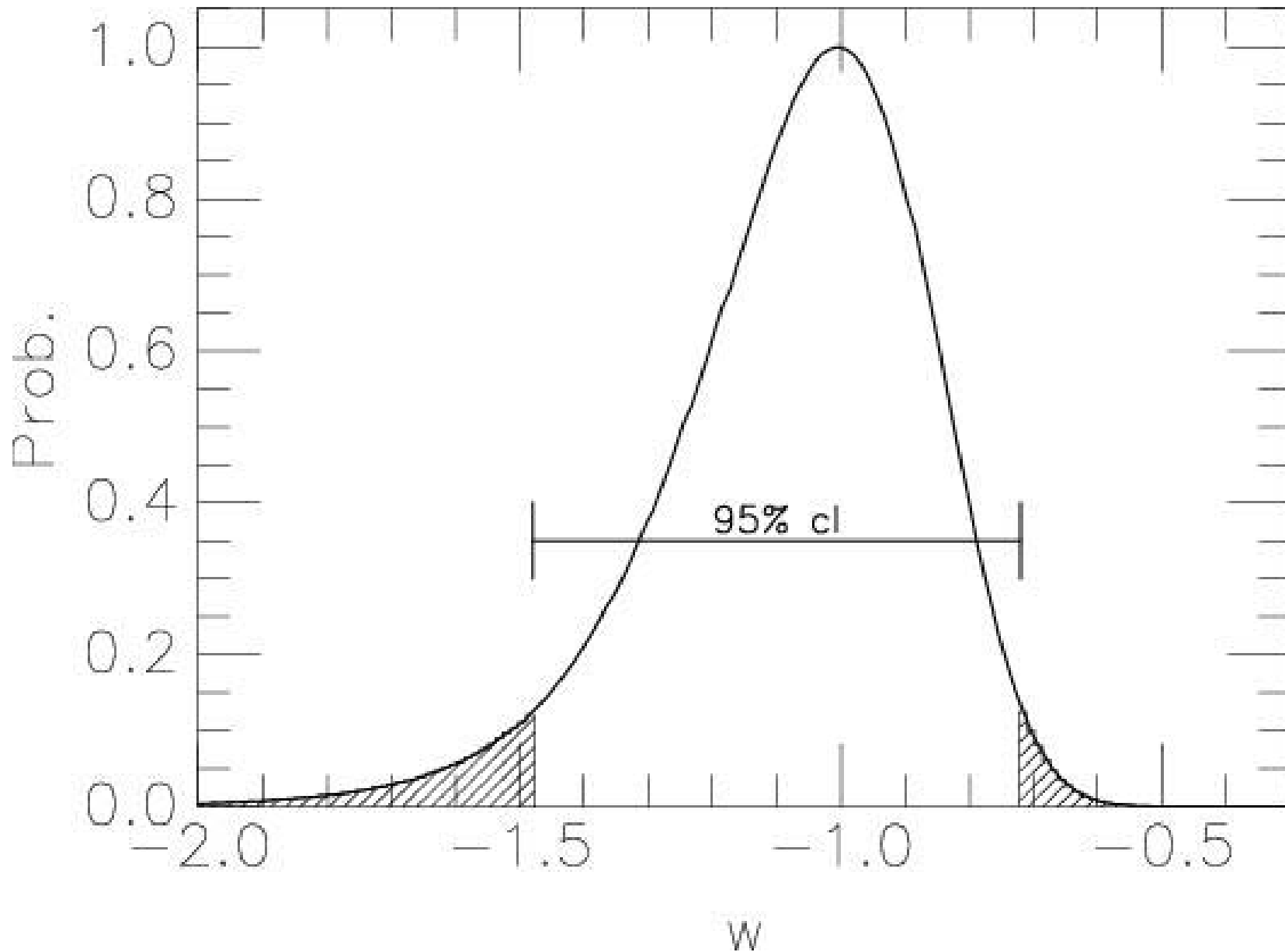
Left-hand side:

- Growth of structure
- New gravity?
 - solar-system effects
 - short-range effects
 - branes (accelerator effects)
- Inhomogeneities?

Right-hand side:

- $w = -1$
 - “just” Λ ?
- $w \neq -1$
 - what is dynamics?
- Scalars
 - long-range forces?

For now...parameterize



High- z supernova team

$w < -1$?

- **null dominant energy condition: energy doesn't propagate outside the light cone**

$$|p| \leq |\rho| \Rightarrow -\rho \leq p \leq \rho$$

- **model with $w < -1$: negative kinetic energy scalar field**

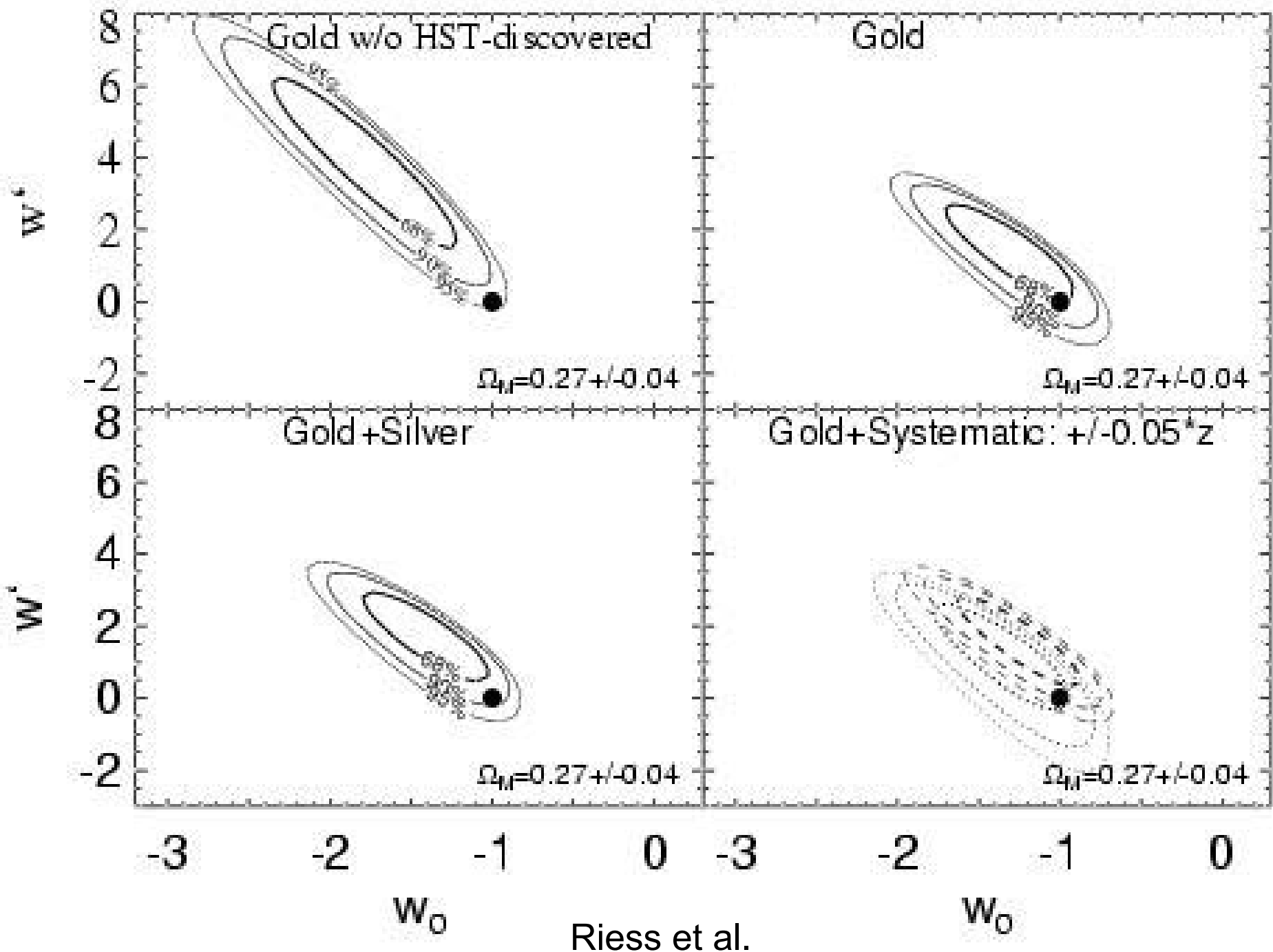
$$L = -\dot{\phi}^2 - \exp(-\phi^2)$$

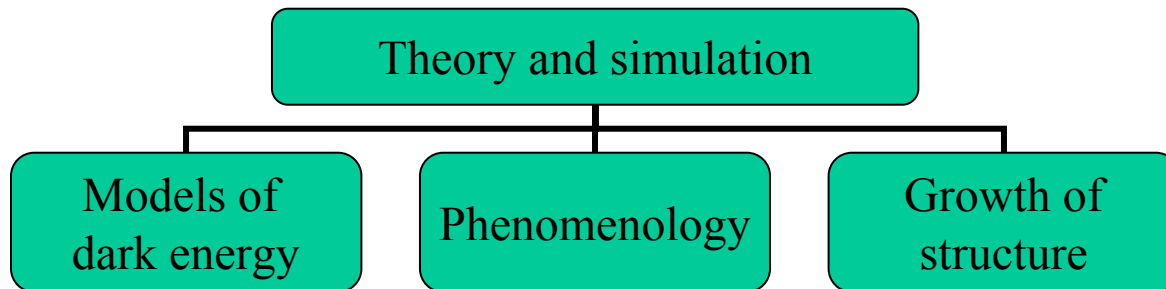
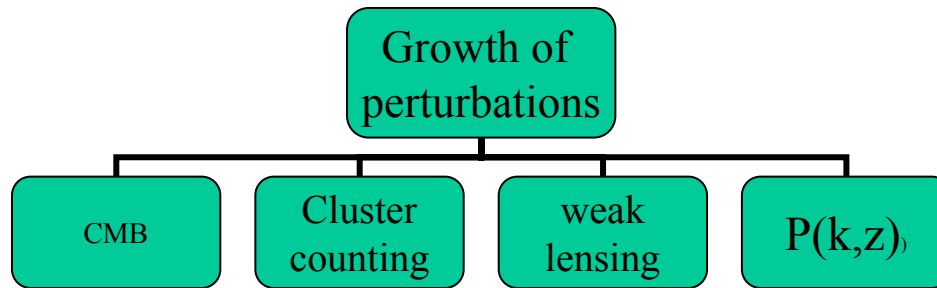
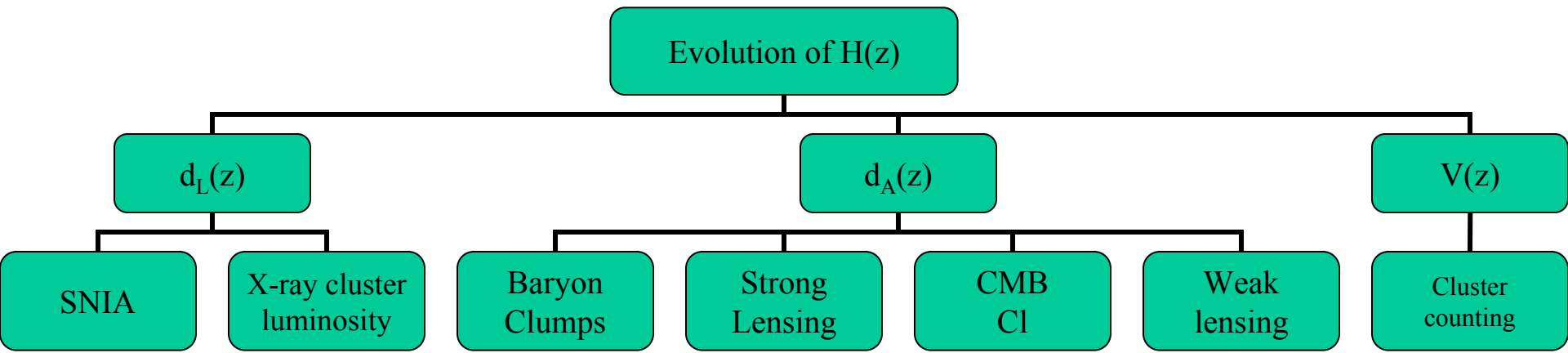
- **instability cured with higher derivative terms?**

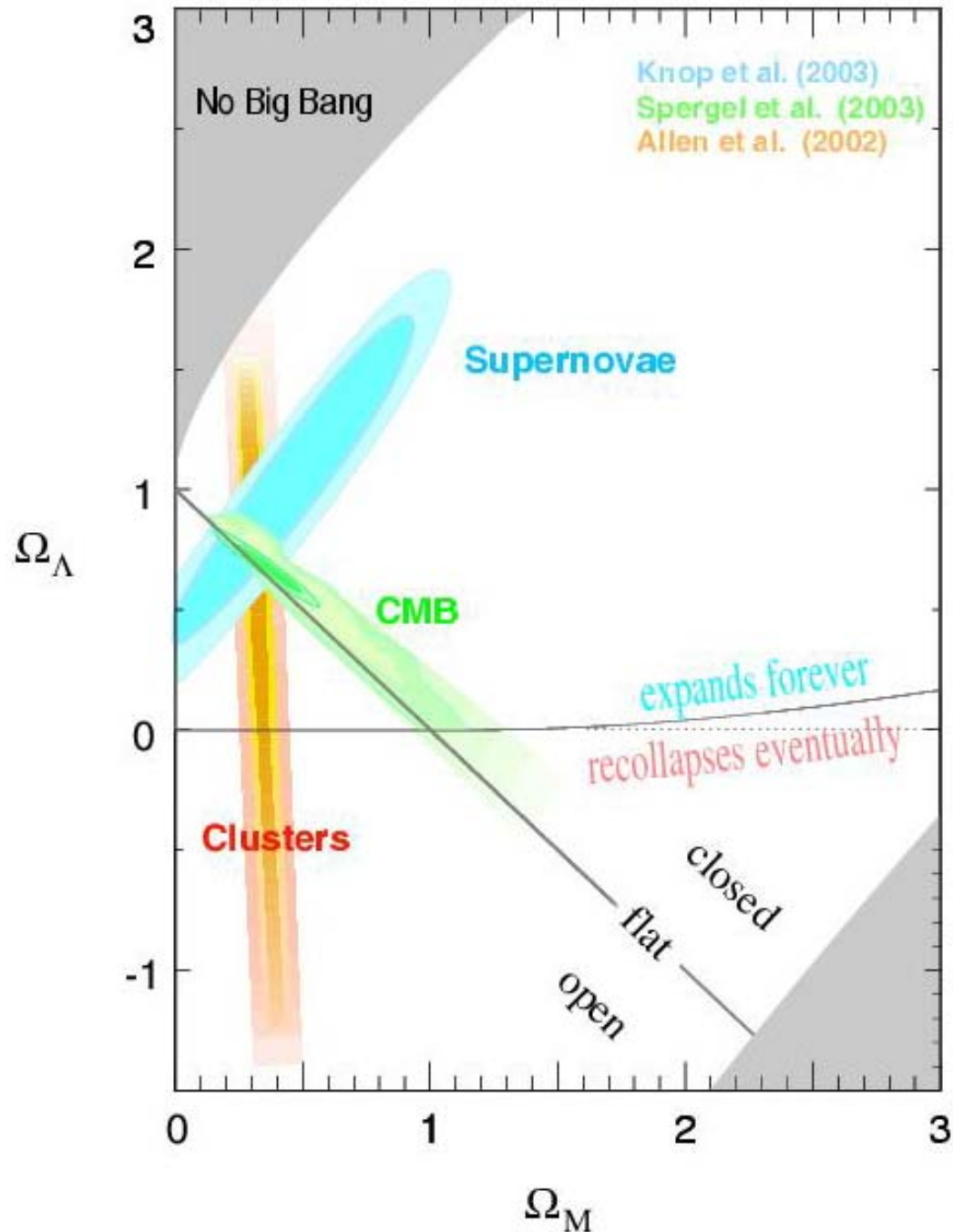
Caution in interpretation

Always read the fine print:

- **Astrophysical systematic errors**
- **What are the model assumptions?**
 - **$w = \text{constant?}$** w', w_a
 - **assume Ω_A ?**
- **What are the priors?**
 - $\Omega_M, \Omega_B, H_0, \dots$







- Don't focus on any one particular error contour
- Focus on fact that error contours for different methods are not parallel

Aether of the 21st century?

- **It's an infrared issue!**
- **It's an ultraviolet issue!**



The Gravity of Dark Energy

Gravity in the Quantum World and the Cosmos
XXXIII SLAC Summer Institute, August 2005
Rocky Kolb, Fermilab & Chicago