The Plan

• Yesterday:
  ★ Quick review of the Linear Power Spectrum and Growth of Fluctuations in the Linear Regime
  ★ Basics of Non-Linear Structure Formation; Spherical Collapse
  ★ Abundance of Dark Matter Halos (The “Mass Function”)

• Today:
  Growth, and Structure of Dark Matter Halos
  Dark Matter Substructure
  Clustering of Dark Matter, Halos, & Galaxies
via Lactea simulation, Diemand et al

Millenium Simulation, Springel et al
simulating the Universe

• choose a cosmological model ( $\Omega_m, \Omega_\Lambda, \Omega_b, h, n$), dark matter, etc)
• choose a computational set up (box size, dynamic range, what physics to include)
• find the linear $P(k)$
• set up a random or constrained realization of $P(k)$ in the linear regime ($200<z<30$) in the chosen box
• find yourself a computer. the bigger the better!
• follow the evolution of dark matter using particle N-body methods
• optionally, follow the evolution of the gas by numerically solving hydrodynamic equations
• optionally, add sink and source terms to hydro equations, modeling heating and cooling of the gas, star formation, etc.. (“subgrid physics”)
• evolve to the redshift of interest
evolution of the matter power spectrum

\[ \Delta^2(k) = \frac{k^3 P(k)}{2\pi^2} \]

largest scales are still in the linear regime

finite volume box; large modes have noise

Springel et al 2005
**evolution of dark matter clustering**

- evolves rapidly with redshift
- 2PCF not a power law; has a feature at the scale of halos
- evolution is a strong function of matter density and dark energy

Colin et al 1999
halo formation in peaks

first sites of halo formation

first sites of snowfall

Gaussian fluctuations on various scales

\[ \delta_c \]

Enhanced "Peaks"

Large Scale "Background"
halo bias

- if halos are formed without regard to the underlying density, then
  \[
  \frac{\delta n_h}{n_h} = \frac{\delta \rho}{\rho}
  \]

- but spherical collapse model indicates that the probability of forming a halo depends on the initial density field: large scale density acts as a background enhancement

- halos are “biased” tracers of the background dark matter field. bias can be calculated from spherical collapse and the form of the mass function
  \[
  \frac{\delta n_M}{n_M} = [1 + b(M)] \delta
  \]
halo bias

• the relative abundance of halos in dense regions compared to halos in the background is

\[
\delta^L_{\text{halo}} = \frac{\mathcal{N}(M|\delta_0, S_0)}{(dn(M)/dM)V_0} - 1
\]

\[
\nu \equiv \delta_c / \sigma(M)
\]

• to first order,

\[
\delta^L_{\text{halo}} = \frac{\nu^2 - 1}{\delta_c} \delta_o,
\]

\[
\delta_{\text{halo}} = \left(1 + \frac{\nu^2 - 1}{\delta_c}\right)\delta 
\equiv b_h\delta.
\]

• for Press-Schechter mass function,

\[
n_M \propto \nu \exp(-\nu^2/2)
\]

\[
b(M) = 1 + \frac{\nu^2 - 1}{\delta_c}
\]

• improved by Sheth-Torman mass function

\[
f^f(\nu) = A \left(1 + \frac{1}{\nu^p}\right) \left(\frac{\nu}{2}\right)^{1/2} e^{-\nu/2}\sqrt{\pi},
\]

\[
b(M) = 1 + \frac{a\nu^2 - 1}{\delta_c} + \frac{2p}{\delta_c[1 + (a\nu^2)^p]}
\]

in general a given model should simultaneously give \(b(M)\) and \(n(M)\)
halo bias in simulations

\[ \xi_h = b^2 \xi_{DM} \]

Hu & Kravtsov 2002

\[ \langle\xi_h\rangle = \langle\xi_{DM}\rangle \]

Seljak & Warren 2004

b > 1 : bias
b < 1 : anti-bias

see also
Mo & White 1996; Sheth & Tormen 1999,
Sheth, Mo & Tormen 2001, etc.
\[ \nu \equiv \frac{\delta_c}{\sigma(M)} \]

\[ \sigma(M_*) = \delta_c \]

\[ \langle M \rangle = \bar{\rho}V = \frac{4\pi}{3}R^3 \bar{\rho} \]

\[ \sigma^2(M) = \frac{\langle (M - \langle M \rangle)^2 \rangle}{\langle M \rangle^2} = \frac{\langle \delta M^2 \rangle}{\langle M \rangle^2} \]

\[ = \frac{1}{2\pi^2} \int P(k)W^2(kR)k^2 dk \]

\[ W(kR) = \frac{3|\sin(kR) - kR \cos(kR)|}{(kR)^3} \]

high peaks are rarer and more clustered
halo merger histories

- can extract merger histories from simulations
- or get merger histories analytically from “Extended Press-Schechter”
- PS: mass functions
- EPS: predicts the probability of having a halo of mass $M$ with progenitor $M_1$. 

Wechsler et al 2002
average MAH

\[ M > 1.4 \times 10^{12} \text{M}_\odot / h \]

\[ M > 3 \times 10^{13} \text{M}_\odot / h \]

\[ M(z) = M_0 e^{-S_a_c z} \]

individual MAH

Wechsler et al. 2002

Diemand et al. 2006
formation time is related to the mass fluctuation spectrum.

formation time is measured here as the time when it had a given accretion rate, but this is equivalent to a formation time defined as the time when $FM = M^*$

scatter is $0.13 \text{ in log}_{10} a_c$

$\sigma[M \ast (a)] = \delta_c$.
how is the mass accreted?

most mass accreted in halos 
\(~ 10-30\% \) of the host

**Figure 12.** The fraction of mass accreted in haloes of mass \( \Delta M \) by haloes of given current mass \( M_0 \), since time \( t_i \), for power-law power spectra with \( n = -2 \) (solid curve), \( n = -1 \) (dotted), \( n = 0 \) (short-dashed) and \( n = 1 \) (long-dashed). Curves are obtained from Monte Carlo simulations of halo merger histories, with the parameters the same as in Fig. 9.

Lacey & Cole 1993
Merger rate of DM halos

merger rate goes as $(1+z)^3$

Gottloeber et al 2000
formation history summary

- HSF implies small halos form first and merge into bigger halos
- halo merger rate declines with \( z \).
halo density profiles


Also:
Dubinsk 1991,
Moore et al 1999
Fukushige & Makino 1999
Klypin 2001
Bullock 2001
Jing & Suto 2001
Power et al 2003
Navarro et al 2004
Maccio et al 2006
Neto et al 2007...

dark matter halos in N-body simulations found to have a roughly ‘Universal’ density profile
halo density profiles

- roughly self-similar form:
  \[ \rho(r) \sim \frac{1}{r^3} \]

- convenient parameterization:
  \[ \rho \left( r \right) = \frac{\delta_c}{\rho_{\text{crit}}} \left( \frac{r}{R_s} \right) \left( 1 + \frac{r}{R_s} \right)^2 \]

Navarro, Frenk & White 1996, 1997 (NFW)

- concentration parameter:
  \[ C_{\text{vir}} \equiv R_{\text{vir}} \]

* virial radius with respect to the background density; \( \Delta_{\text{vir}} = 337 \) at \( z = 0 \)

\[ M_{\text{vir}} \equiv \frac{4\pi}{3} \Delta_{\text{vir}} \rho_b R_{\text{vir}}^3 \]

\[ \rho(r) \sim \frac{1}{r^\alpha} \quad \alpha \sim 0.7 - 1.5 \]

* large radius
* small radius

note: concentrations depend on definition of the density threshold used to define halos
\[ V_c(r) = 4\pi G \rho_s r_s^3 \frac{f(r)}{r} \]

\[ f(r) = \ln(1 + r/r_s) - \frac{r/r_s}{1 + r/r_s} \]

\[ r_{v_{\text{max}}} = 2.163r_s \]
halo concentrations

Bullock et al 2001

$\frac{c_{\text{vir}}}{(1+z)}$

68% intrinsic scatter in halo population

Neto et al 2007
concentration vs. formation time

\[ c_{\text{vir}} = c_1 \frac{a_{\text{obs}}}{a_c} \]

for all masses and redshifts

scatter at a given mass and redshift caused by scatter in mass accretion histories

→ correlated with galaxy type?

RW et al 02
\[ a_c(M,z) \]

simply inverting this plot gives you \( c(M,z) + \text{scatter} \). The same model for formation time based on \( M^* \) applies.

This means that we can understand how concentrations depend on the power spectrum.

\[
c_{\text{vir}} = K \frac{a}{a_c} \sim 9 (M/M^*)^{-0.13}
\]

\[ c_{200} = 5.26 \left( \frac{M_{200}}{10^{14} h^{-1} M_\odot} \right)^{-0.10}, \]

for relaxed haloes, and

\[ c_{200} = 4.67 \left( \frac{M_{200}}{10^{14} h^{-1} M_\odot} \right)^{-0.11}, \]

for the complete halo sample.
- low mass halos form early, when the universe was denser
- reducing mass fluctuations on galaxy scales (low $\sigma_8$, tilt) reduces concentrations
the inner slope
(“cuspy halo crisis”)
a diversity of inner slopes?
do baryons matter for profiles?
adiabatic contraction

correction

cluster of galaxies
gas cooling: hierarchical structure formation
gas cooling: hierarchical structure formation

Gnedin et al 2004
effect of baryons on concentrations

Rudd et al 2007
triaxial halos

Jing & Suto 2002
triaxial halos

- wide distribution of shape parameters, but all halos fairly triaxial and prolate
halo shapes

- halos get rounder with time & further out in radius
- low mass halos are the most spherical
- early forming halos more spherical

\[ s = 0.54 \frac{M_{\text{vir}}}{M_*}^{-0.05} \]

Allgood et al 2006

shape = shortest axis/longest axis
dark matter substructure

- Simulation by A. Kravtsov
- Massive host halo
- Galactic subhalo

Simulation by VIRGO consortium
Simulation by B. Allgood

Simulation by A. Kravtsov
Substructure

- The best evidence for a hierarchical structure formation
- The distribution and properties of substructure contains information about the entire hierarchy and history of merging galaxies
- This includes information about the properties and nature of dark matter
- Substructures in large halos are likely the host for galaxies

Stewart, Bullock, Wechsler in prep
substructure studies
only a decade old

"... by no stretch of the imagination does one form "galaxies" in the current cosmological simulations: the physical model and dynamic range are inadequate to follow any but the crudest details..."

Full Box - 7 Mpc
Central Region - 700 kpc

Fig. 1.—Two plots which characterize the dark matter distribution in the simulation. Length scales here and throughout are given in physical units at $z = 1$. On the left is the entire simulation volume (a 7 Mpc cube) showing the central group, a second group above and right of center, and various filamentary structures. For clarity, only one-fourth of the particles are plotted. The right-hand side details the central region, $1/10$ the box length on a side, and shows all of the particles.

first serious substructure studies in 1998:
Warm Dark Matter

mass variance

halo concentration

Zentner & Bullock 2002; 2003
resolving substructure in simulations
CDM substructure may extend over 18-20 orders of magnitude in mass

one of the first objects to form, at z~60.
smooth halo with a cuspy density profile.
earth ($10^{-6}$) mass substructures, size of the solar system.
$10^{15}$ of these inside the galactic halo

Diemand et al 2006

model: neutralino
100 GeV DM
power spectrum cut off at ~0.6 Mpc

axion DM: no cut off here ($10^{-13}$)
(approximately)

self-similar substructure

Moore et al. 1999
Abundance of subhalos in a given halo

is determined by competition between accretion of new subhalos and disruption of old subhalos

disruption = loss of identity via merging with other halos or significant mass loss due to tidal stripping

Formation of a galaxy-sized halo in LCDM, $M_{\text{vir}}=3\times10^{12}h^{-1} \text{Msun}$; $R_{\text{vir}}=293h^{-1} \text{kpc}$;
what processes affect substructure?


Orbit decays by dynamical friction: central merger.

Tidal forces act to disrupt the accreted halo.

Survival
dynamical friction

- galaxies can lose orbital energy due to a gravitational drag force

\[ F_{DF} = \frac{4\pi ln(\Lambda)G^2 M_{\text{sat}}^2 \rho(r)}{V_{\text{orb}}^2} \left[ \text{erf}(x) - \frac{2X}{\sqrt{\pi}} \exp(-X^2) \right] \]

Chandrasekar 1943
Tidal stripping of subhalos: three examples

Kravtsov, Gnedin & Klypin 2004

VM = \text{max of circ velocity curve} = (GM(<r)/r)^{1/2}

\text{distance to the host}

\text{tidal force}

Vmax and grav. bound mass
subhalo abundance:  
~ linear with halo mass

\[
\langle N(M) \rangle_{\text{sat}} = \frac{M}{M_1} \exp \left( - \frac{M_{\text{cut}}}{M} \right)
\]

\[
\log (M_{\text{cut}}) = 0.76 \log (M_1) + 2.3
\]

Kravtsov et al. 2004

Tinker et al. 2005; Conroy et al. 2006
satellite number correlates with formation time and halo concentration

Zentner et al 2005; Wechsler et al 2006
abundance depends on power spectrum

Average # of subhalos above a given circular velocity

\[ \alpha_s, N_s \]


Tasitsiomi et al. 2007

host halo mass

\[ M_r \leq -19 \]

\[ L_{80_{0.9}}, L_{80_{0.75}}, L_{120_{0.9}} \]
The amount of primordial power on scales relevant for galaxy formation is not well constrained \( \lambda < \text{Mpc} \) and scale-invariance is almost always assumed.

Suppression of small-scale power from dark matter produced through decays (Feng et al. 2003; Cembranos et al. 2005).

Suppression of small-scale power via models of non-thermal, sterile neutrino dark matter (Abazajian 2005; Asaka et al. 2005).

Warm Dark Matter has two effects:

**Suppression of Linear Power Spectrum** due to free-streaming on scales with \( k \lambda_{\text{DM}} \gg 1 \).

For WDM models \( \lambda_{FS} \approx 3 \ h\text{Mpc}^{-1}(m/\text{keV})^{-4/3} \)

High densities limited by finite "Phase-Packing" because initial phase-space density is relatively low, \( Q \equiv \rho/(v^2)^{3/2} = 10^{-4} \ (\text{Msun}/\text{pc}^3/\text{km}^3\text{s}^{-3}) = 10^{-24} \ Q \chi \)

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The amount of primordial power on scales relevant for galaxy formation is not well constrained \( \lambda < \text{Mpc} \) and scale-invariance is almost always assumed.
fraction of mass in substructure depends on DM properties.
tidal streams in M31

Figure 1. Panel a: Map of RGB count density, from Irwin et al. (2005). The edges of the NE and W shelves that are the focus of this paper are marked with red dotted lines. The H313 field of Reines et al. (2000) and four fields from Ibata et al. (2004) discussed below are marked with crosses. Panel b: Sobel-filtered version of Panel a, which detects sharp edges in the count map. To create this map, we fill in sharp features such as plot annotations and noise spikes with the surrounding pixel values, smooth the map, and apply the Sobel operator. We then fit a smooth curve to the shelf edges, which again are shown by dotted lines.
observational signatures of DM substructure

• substructure mass function in clusters from lensing
• multiple-image strong lenses of quasars
• abundance of dwarf satellite galaxies
• dynamical effects on galactic disks
• (future) annihilation signal from DM subhalos in the local group
• galaxy abundance in groups & clusters
Substructure Summary

- substructures are ubiquitous in CDM
- mass function of substructure is roughly self-similar.
- the number of substructures is a trade-off between accretion (the halo merging rate) and destruction (dynamical friction; tidal stripping)
- abundance & properties may constrain small scale power spectrum (inflation; DM physics), but much work is still needed.
- lensing & stellar structure may be ways of probing dark and disrupted substructures.
gas in density peaks shock heats to virial temperature, and then radiates and cools. expect to have a galaxy at the center of every density peak (halo) that is massive enough to allow cooling ($\sim 10^4$ K).
galaxy properties are tightly correlated with halo properties.

Tully-Fisher relation
halo occupation of galactic halos

\[ \alpha^2 \equiv \frac{\langle N(N-1) \rangle}{\langle N \rangle^2} \]

\[ N_{\text{sub}} \sim M \]

- a physically motivated way of characterizing non-linear bias of galaxies/subhalos
- no smoothing scale; naturally incorporates stochasticity, naturally brings about scale dependence

**Average number of galactic (sub)halos**

**Host halo mass**

Kravtsov, Berlind, Wechsler, et al 2004
assume every galaxy lives in a subhalo. how do they cluster?

SDSS, z=0  
conroy, wechsler & kravtsov 2006

data: zehavi et al 2004

1-halo term

2-halo term

characteristic size of halo

dark matter

bright galaxies

dimmer galaxies

DEEP, z=1  
data: coil et al 2006
The Halo Model

- Basic idea:
  - assume that stuff (e.g.: mass, galaxies, quasars, gas) lives in dark matter halos.
  - use knowledge of dark matter halo properties + relation of stuff to halos to determine the clustering properties of the stuff (e.g., non-linear power spectrum, galaxy clustering, etc...)
  - or use clustering to constrain the relation
the halo occupation approach

- assume galaxies live in halos
- assume $P(N|M)$ is just a function of $M$

The relation between the clustering of Dark Matter and any class of galaxies (luminosity, type, etc.) is fully defined by the Halo Occupation Distribution (HOD):

- The probability distribution $P(N|M)$ that a halo of mass $M$ contains $N$ galaxies of that class.
- The relation between the spatial distributions of galaxies and DM within halos.
- The relation between the velocity distributions of galaxies and DM within halos.
Galaxy Formation

Gas cooling, Star formation, Feedback, Mergers, etc.

Cosmological Model

$\Omega, P(k), \text{etc.} + \text{Gravity}$

Dark Halo Population

$n(M), \rho(r|M), \xi(r|M), v(r|M)$

Halo Occupation Distribution

$P(N|M)$
Spatial distribution within halos
Velocity distribution within halos

Galaxy clustering

slide credit: Berlind
How do we compute clustering statistics?

**Correlation function**

**Small scales:** All pairs come from same halo.

1-halo term

\[
1 + \xi^1_g(r) = \left(2\pi r^2 n_g^2\right)^{-1} \int_0^\infty dM \frac{dn}{dM} \frac{\langle N(N-1) \rangle}{2} M^{\lambda(r|M)}
\]

**Large scales:** Pairs come from separate halos.

\[
\xi_g(r) = b_g^2 \xi_m(r)
\]

\[
b_g = n_g^{-1} \int_0^\infty dM \frac{dn}{dM} \langle N \rangle M b_h(M)
\]

Berlind & Weinberg (2002)
The HOD contains information about galaxy formation physics

- Baryon/DM fraction
- Gas cooling
- Star formation efficiency
- Dynamical friction
- Tidal disruption
- DM halo merger statistics

![Graph showing the relationship between mass and other physical parameters.](image)
can also use the halo model to calculate the Non-Linear Power Spectrum

- Non-linear power spectrum is composed of dark matter halos that are clustered according to the halo bias + the clustering due to the halo density profile

\[
P_{nl}(k, z) = I_2^2(k, z) P(k, z) + I_1(k, z)
\]

where

\[
I_2(k, z) = \int d\ln M \left( \frac{M}{\rho_m(z = 0)} \right) \frac{dn}{d\ln M} b(M) y(k, M)
\]

\[
I_1(k, z) = \int d\ln M \left( \frac{M}{\rho_m(z = 0)} \right)^2 \frac{dn}{d\ln M} y^2(k, M)
\]

and \( y \) is the Fourier transform of the halo profile with \( y(0, M) = 1 \)

\[
y(k, M) = \frac{1}{M} \int_{0}^{r_h} dr 4\pi r^2 \rho(r, M) \frac{\sin(kr)}{kr}
\]
Summary