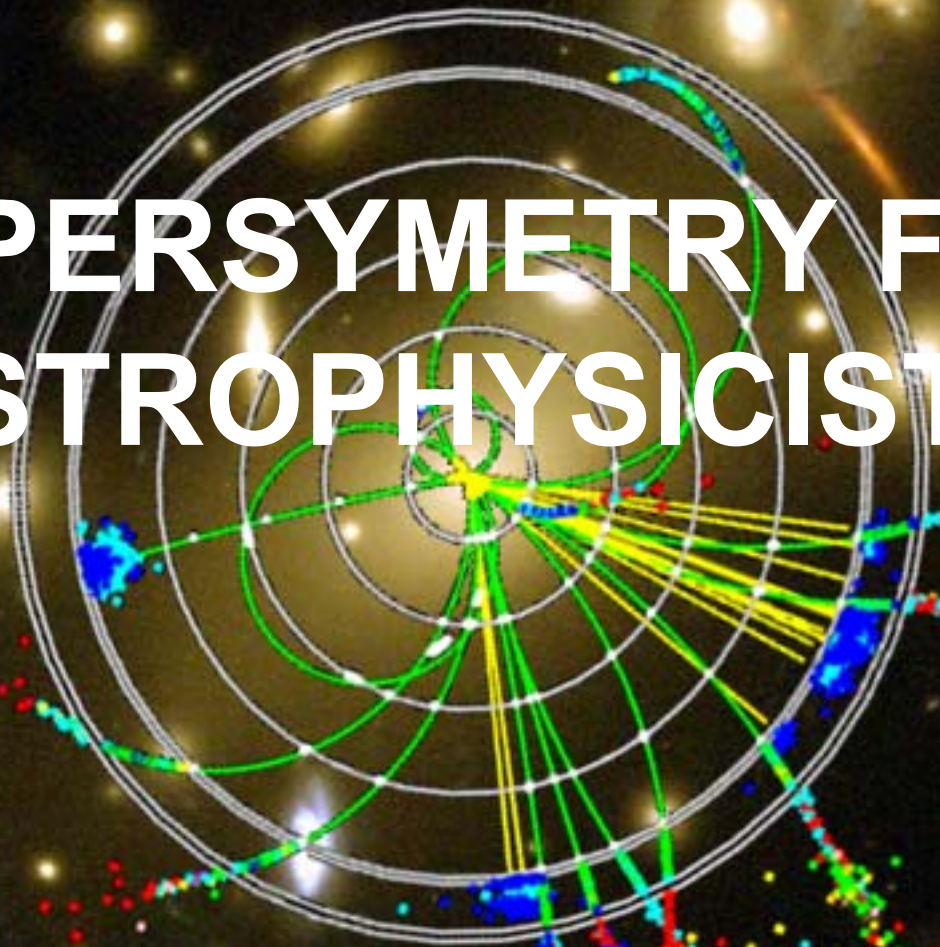


Dark Matter: From the Cosmos to the Laboratory

SUPERSYMMETRY FOR ASTROPHYSICISTS

A complex diagram of particle tracks and orbits, likely representing supersymmetry or dark matter, overlaid on a starry background. The diagram features several concentric white circles representing orbits. A central yellow point is the origin of numerous colored lines (green, yellow, blue, red) that radiate outwards and then curve back towards the center, forming intricate paths. The background is a dark space filled with numerous bright, multi-colored stars and galaxies.

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SLAC Summer Institute

POLLING DATA

I'm giving summer school lectures titled, "Supersymmetry for Astrophysicists." What should I talk about?

- Astrophysicist #1: "Beats me. I couldn't care less about supersymmetry. Maybe you can get out of it somehow."
- Astrophysicist #2: "Dark matter, of course. Isn't that the only motivation for supersymmetry?"

OUTLINE

LECTURE 1: SUSY ESSENTIALS

Standard Model; SUSY Motivations; LSP Stability and Candidates

LECTURE 2: NEUTRALINOS

Properties; Production; Direct Detection; Indirect Detection; Collider Signals

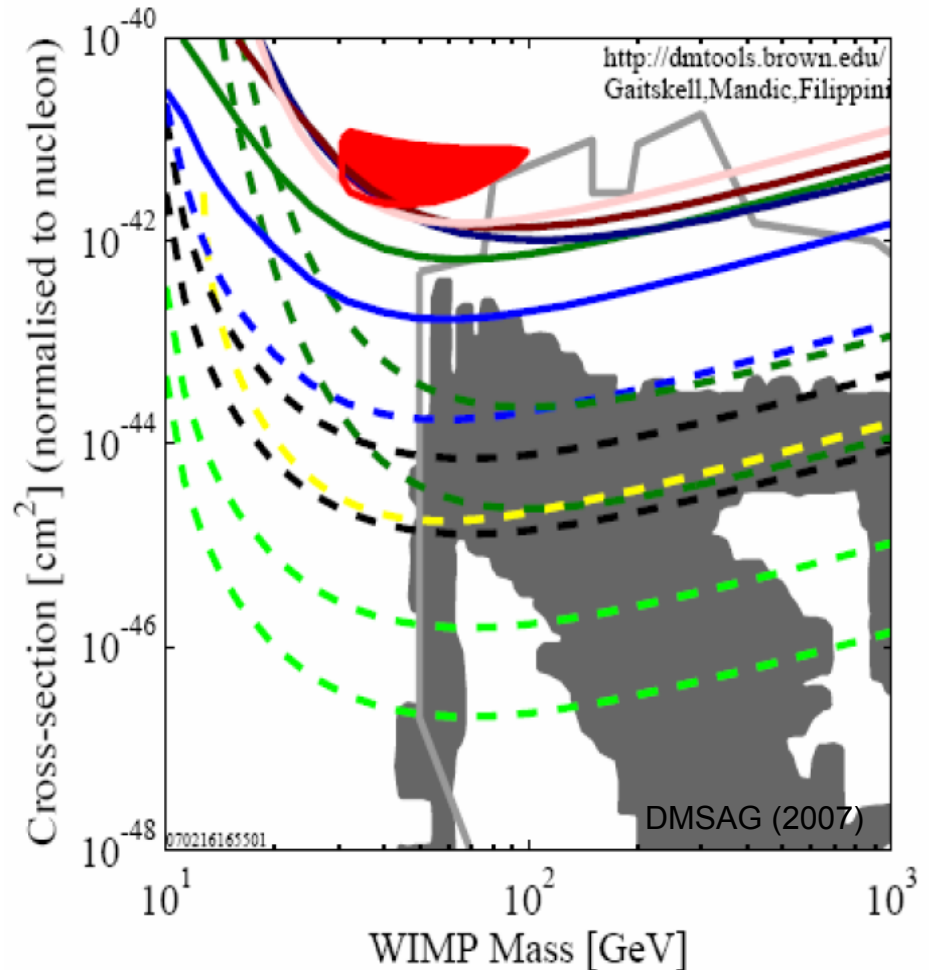
LECTURE 3: GRAVITINOS

Properties; Production; Astrophysical Detection; Collider Signals

SUSY ESSENTIALS

First discuss motivations for supersymmetry. Why?

- Supersymmetry is the best motivated framework for new particle physics
- Generic properties vs. special models (What do these shaded regions mean?)
- Direct implications for astrophysics



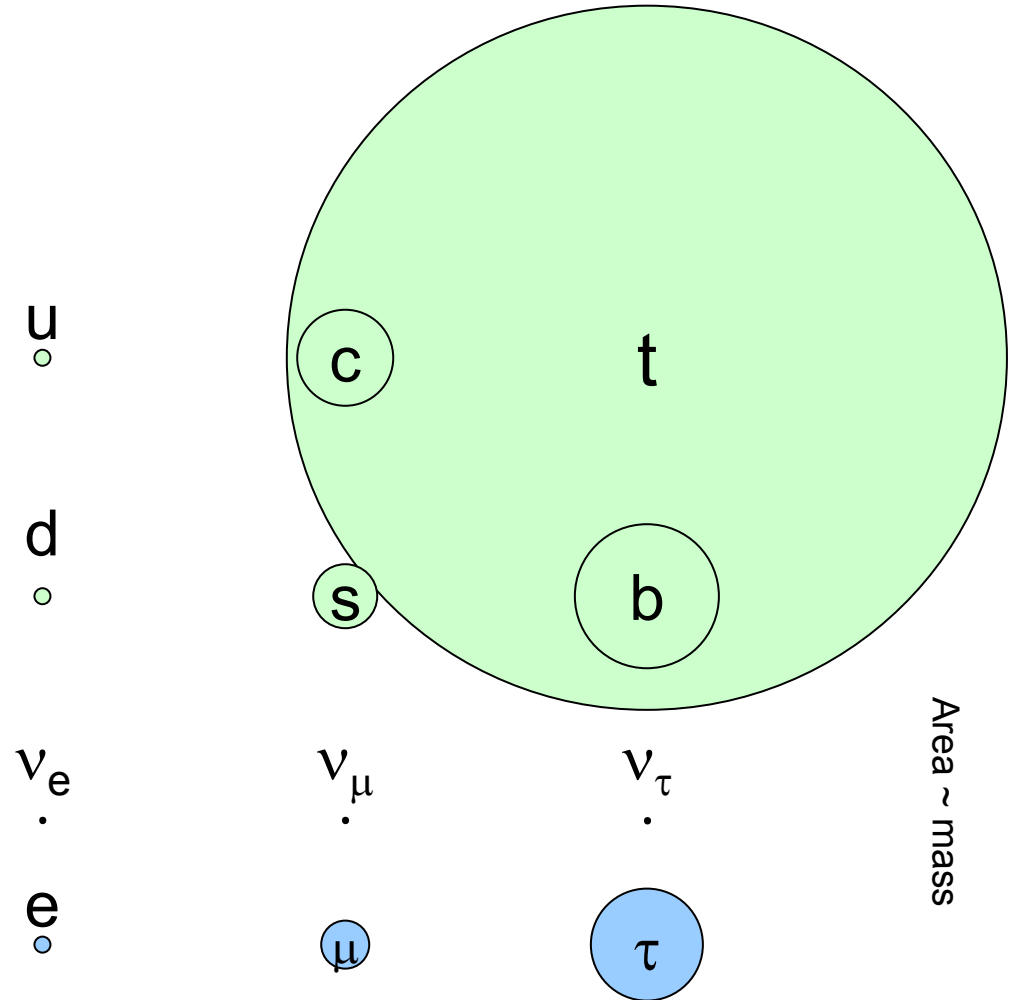
STANDARD MODEL

- Matter Particles
 - Quarks and leptons
 - Spin $\frac{1}{2}$ fermions
- Force Particles
 - Photon (EM)
 - W , Z (weak)
 - Gluons (strong)
 - Spin 1 bosons
- Higgs Particle
 - Undiscovered
 - Spin 0 boson

Quarks	u up	c charm	t top	g gluon	Force Carriers	
	d down	s strange	b bottom			γ photon
Leptons	ν_e e neutrino	ν_μ μ neutrino	ν_τ τ neutrino	W W boson		
	e electron	μ muon	τ tau	Z Z boson		
3 \rightarrow	I	II	III	\leftarrow Generations		

Matter Particles

- Most of the unexplained parameters of the SM are here
- Interactions determined by unusual quantum numbers
- Masses span at least 11 orders of magnitude
 - Neutrinos \sim eV
 - Electron: 511 keV
 - Top quark: 171 GeV
- The top quark is heavy!



Force Particles

- Couplings $\alpha \equiv g^2/(4\pi)$ at m_Z

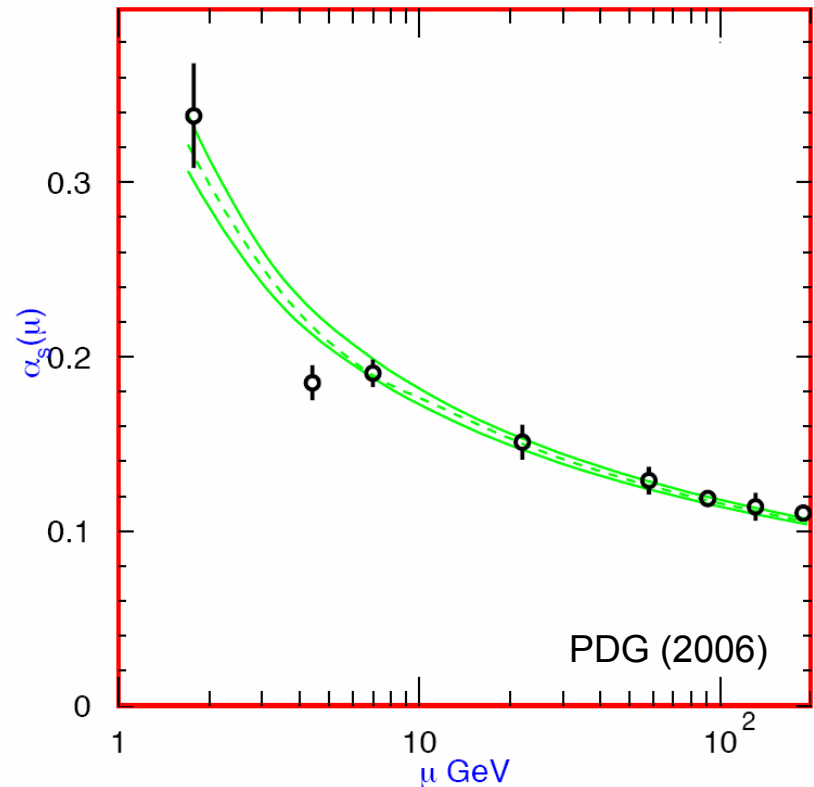
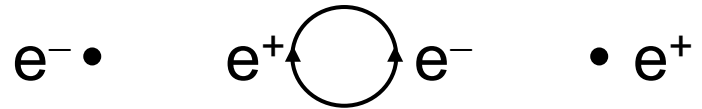
- $\alpha_{EM} = 0.007818 \pm 0.000001$
- $\alpha_{weak} = 0.03381 \pm 0.00002$
- $\alpha_s = 0.118 \pm 0.002$

- At observable energies,

$$\alpha_{EM} < \alpha_{weak} < \alpha_s$$

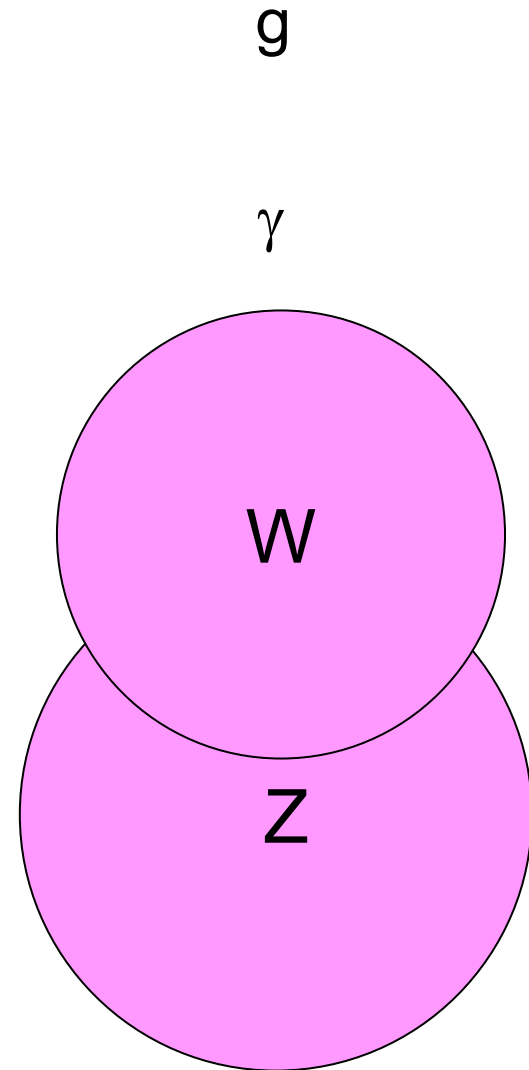
- Precisely measured

- Scale-dependent – the quantum vacuum has dielectric properties



Force Particles

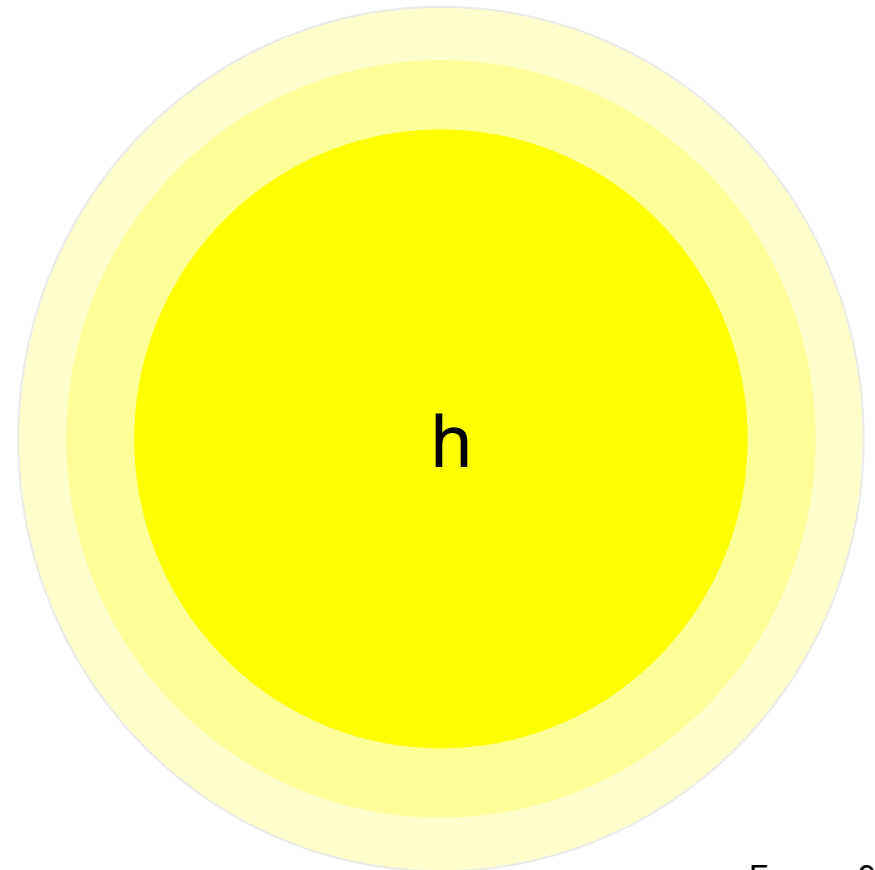
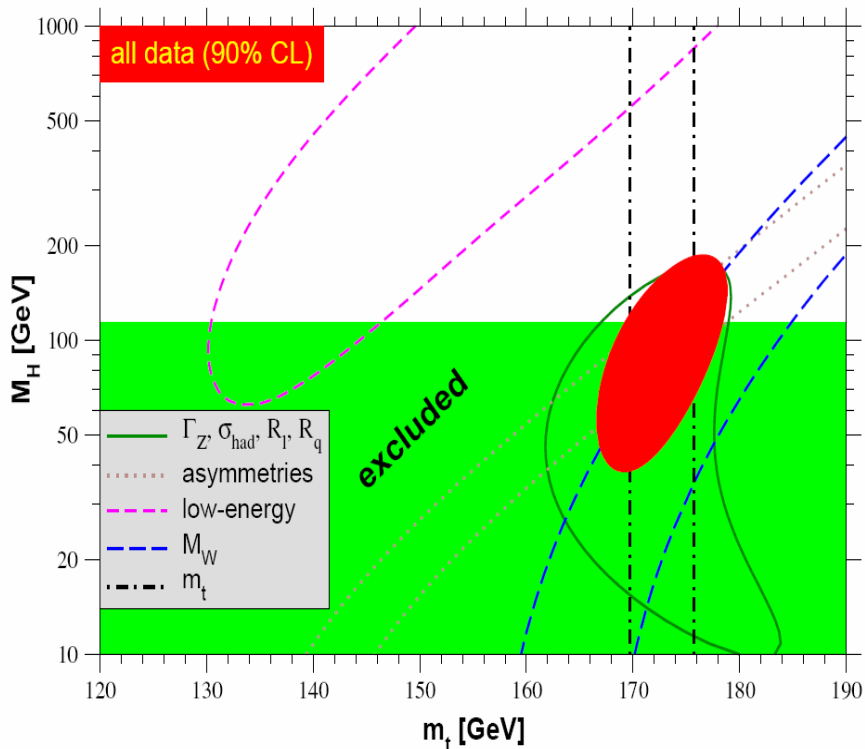
- Masses
 - $m_\gamma = 0$: U(1) conserved
 - $m_g = 0$: SU(3) conserved
 - $m_W = 80$ GeV: SU(2) broken
 - $m_Z = 91$ GeV: SU(2) broken
- SU(2) is broken, the others aren't



Higgs Particle

- Mass

- Direct searches: $m_h > 115$ GeV
- Indirect constraints from precision data: $40 \text{ GeV} < m_h < 200 \text{ GeV}$



NATURALNESS

- We know 3 fundamental constants
 - Special relativity: speed of light c
 - Quantum mechanics: Planck's constant h
 - General relativity: Newton's constant G
- From these we can form the Planck mass

$$M_{\text{Pl}} = \sqrt{\frac{hc}{G}} \approx 10^{19} \text{ GeV}$$

- Why are $m_h, m_W, m_Z, \dots \ll M_{\text{Pl}}$?

Gauge Hierarchy Problem

$$m_h^2 = (m_h^2)_0 - \frac{1}{16\pi^2} \lambda^2 \Lambda^2$$

In the SM, m_h is naturally $\sim \Lambda$, the largest energy scale

$m_h \sim 100 \text{ GeV}$, $\Lambda \sim 10^{19} \text{ GeV} \rightarrow$ cancellation of 1 part in 10^{34}

SUPERSYMMETRY

SYMMETRIES OF NATURE	Exact	Broken
Gauge	$U(1)_{EM}, SU(3)_C$	$SU(2) \times U(1)_Y$
Global	B, L	L_e, L_μ, L_τ
Spacetime	Rotations, Boosts, Translations	SUSY

Supersymmetry is a qualitatively new class of symmetry

Superpartners

- Translations: particle P at $x \rightarrow$ particle P at x'
- SUSY: particle P at $x \rightarrow$ particle \tilde{P} at x , where
 - P and \tilde{P} differ in spin by $\frac{1}{2}$: fermions \leftrightarrow bosons
 - P and \tilde{P} are identical in all other ways (mass, couplings)
- New particles
 - Superpartners of matter particles: Spin 0 bosons, add “s” (selectron, sneutrinos, squark, ...)
 - Superpartners of force particles: Spin $\frac{1}{2}$ fermions, add “ino” (photino, Wino, ...)
 - Superpartners of Higgs particles: Spin $\frac{1}{2}$ fermions, “Higgsinos”

SUSY AND NATURALNESS

The diagram shows the renormalization of the Higgs mass squared. On the left, a vertical dashed line with a solid black dot represents the tree-level mass m_h^2 . This is equal to the sum of three terms:

- Classical:** A vertical dashed line with a cross (X) at the top, representing the tree-level mass $(m_h^2)_0$.
- Quantum (Loop):** A solid circle loop with a vertical dashed line entering from the top and exiting from the bottom. The top vertex is labeled λ , the bottom vertex is labeled λ , and the two internal vertices are labeled f . This represents a fermion loop correction.
- Quantum (Loop):** A dashed circle loop with a vertical dashed line entering from the top and exiting from the bottom. The top vertex is labeled \tilde{f} , the bottom vertex is labeled λ^2 , and the two internal vertices are labeled \tilde{f} . This represents a superpartner loop correction.

Below the diagrams, the corresponding mathematical equation is shown:

$$m_h^2 = (m_h^2)_0 - \underbrace{\frac{1}{16\pi^2}\lambda^2\Lambda^2 + \frac{1}{16\pi^2}\lambda^2\Lambda^2}_{\text{Quadratic divergences}} + \frac{1}{16\pi^2}\lambda^2(m_{\tilde{f}}^2 - m_f^2)\ln(\Lambda/m_h)$$

Dependence on Λ is softened to a logarithm

SUSY solves the gauge hierarchy problem, even if broken, provided superpartner masses are ~ 100 GeV

Higgs Doubling

- SUSY requires 2 Higgs doublets to cancel anomalies and to give mass to both up- and down-type particles
- E.g., anomaly cancelation requires $\Sigma Y^3 = 0$, where Y is hypercharge and the sum is over fermions. This holds in the SM
- SUSY adds an extra fermion with $Y = -1$:

$$\begin{pmatrix} h^0 \\ h^- \end{pmatrix} \equiv \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$$

- To cancel the anomaly we add another Higgs doublet with $Y = +1$:

$$\begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$$

SUSY PARAMETERS

SUSY breaking introduces many unknown parameters. These are

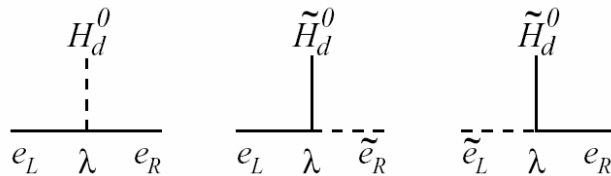
- Masses for sleptons and squarks: $m_{f ij}^2$
- Masses for gauginos: M_1, M_2, M_3
- Trilinear scalar couplings (similar to Yukawa couplings): A_{ij}^f
- Mass for the 2 Higgsinos: $\mu \tilde{H}_u \tilde{H}_d$
- Masses for the 2 neutral Higgs bosons: $B H_u H_d + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2$
- The 2 neutral Higgs bosons both contribute to electroweak symmetry breaking:

$$v^2 = (174 \text{ GeV})^2 \rightarrow v_u^2 + v_d^2 = (174 \text{ GeV})^2$$

The extra degree of freedom is called $\tan\beta = v_u/v_d$

TAKING STOCK

- SUSY is a single symmetry, which implies many new particles
- Many new parameters, but
 - Dimensionless couplings are fixed (no “hard” breaking)

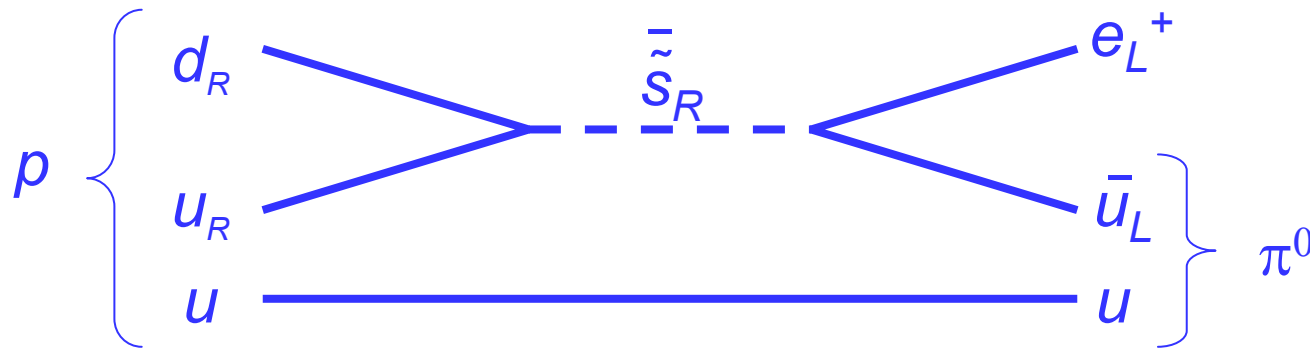


- Dimensionful parameters are allowed (soft breaking), but should be ~ 100 GeV
- Even the dimensionful parameters cannot be arbitrary

Analogy	Soap Bubble	SM
Large Parameter	Length L Height H	M_{Pl}
Small Parameter	L - H	m_h
Symmetry explanation	Rotational invariance	SUSY
Symmetry breaking	Gravity	M_{SUSY}
Natural if	Gravity weak	M_{SUSY} small

R-PARITY AND STABLE SUPERPARTNERS

- One problem: proton decay



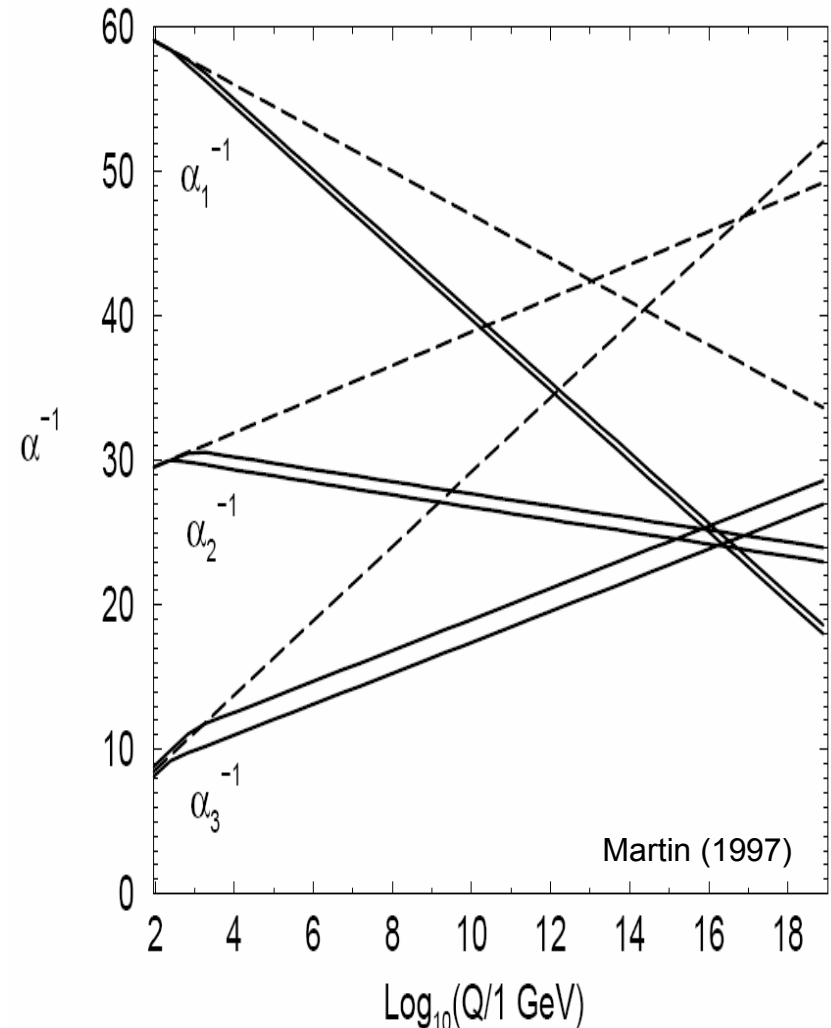
- Forbid this with R-parity conservation: $R_p = (-1)^{3(B-L)+2S}$
 - SM particles have $R_p = 1$, SUSY particles have $R_p = -1$
 - Requires 2 superpartners in each interaction
- Consequence: the lightest SUSY particle (LSP) is stable and cosmologically significant. What is the LSP?

Neutral SUSY Particles

	U(1)	SU(2)	Up-type	Down-type			
Spin	M_1	M_2	μ	μ	$m_{\tilde{\nu}}$	$m_{3/2}$	
2						G graviton	
3/2		Neutralinos: $\{\chi \equiv \chi_1, \chi_2, \chi_3, \chi_4\}$					\tilde{G} gravitino
1	B	W^0	↑				
1/2	\tilde{B} Bino	\tilde{W}^0 Wino	\tilde{H}_u Higgsino	\tilde{H}_d Higgsino	ν		
0			H_u	H_d	$\tilde{\nu}$ sneutrino		

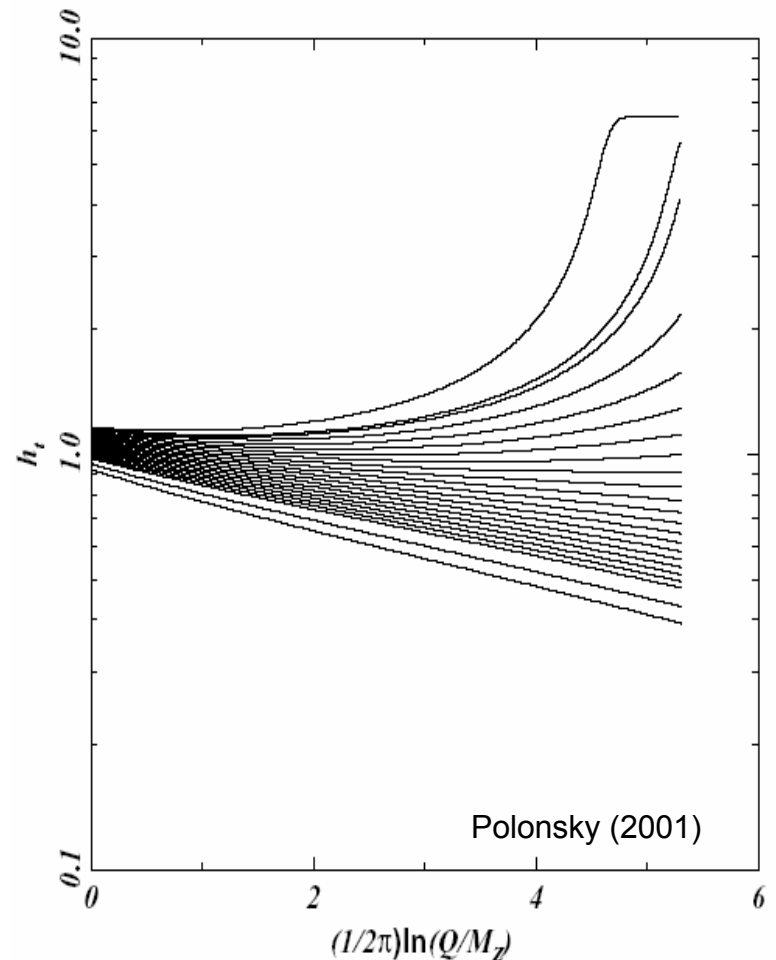
FORCE UNIFICATION

- Can the 3 forces be unified, e.g., $SU(3) \times SU(2) \times U(1) \rightarrow SO(10)$?
- Superpartners modify the scale dependence of couplings
- With TeV superpartners, 3 couplings meet at a point!
 - No free parameters
 - % level “coincidence”
 - Coupling at unification: $\alpha^{-1} > 1$
 - Scale of unification
 - $Q > 10^{16}$ GeV (proton decay)
 - $Q < 10^{19}$ GeV (quantum gravity)
- SUSY explains $\alpha_{EM} < \alpha_{weak} < \alpha_s$
- Gaugino mass unification implies $M_1:M_2:M_3 \approx \alpha_1:\alpha_2:\alpha_3 \approx 1:2:7$, the Bino is the lightest gaugino



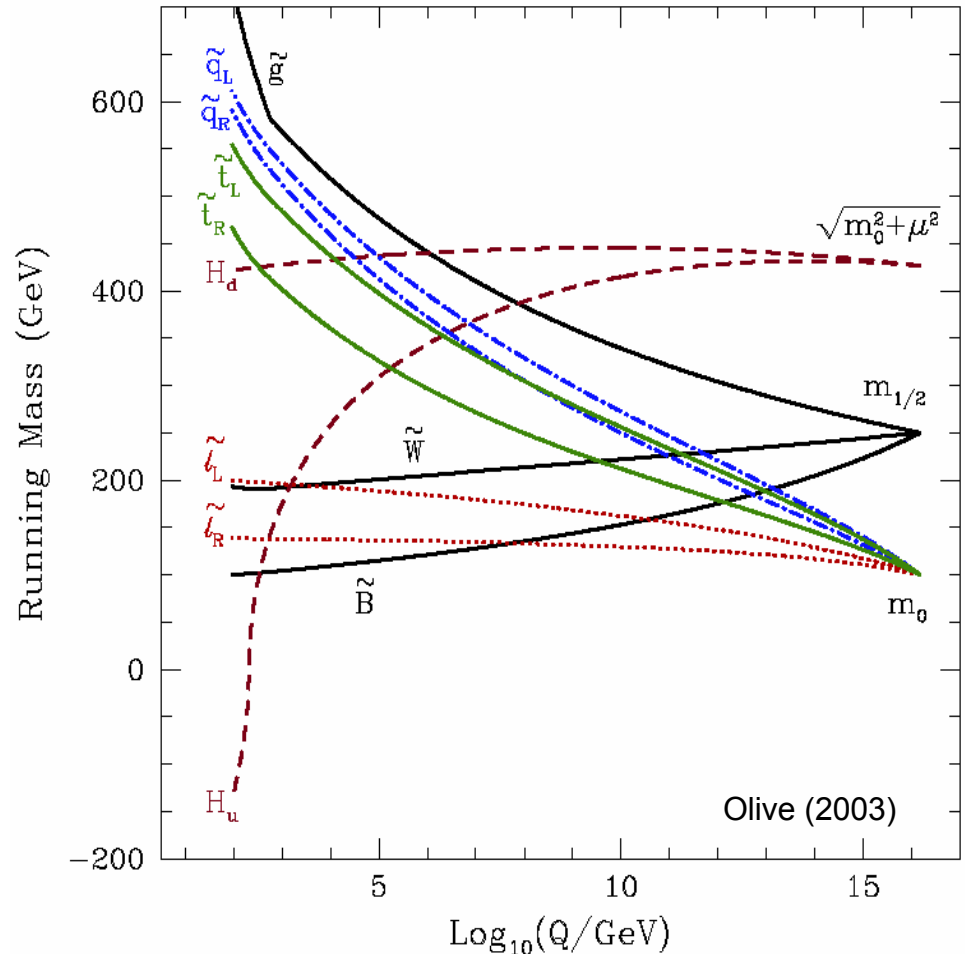
TOP QUARK MASS

- Force unification suggests we can extrapolate to very high energy scales
- All parameters (masses, couplings) have scale dependence
- The top quark Yukawa coupling has a quasi-fixed point near its measured value
- SUSY “explains” heavy top



SCALAR MASSES

- How do scalar masses change with scale?
- Gauge couplings increase masses; Yukawa couplings decrease masses
- H_u has large top quark Yukawa, but no compensating strong interaction
- H_u is the lightest scalar. In fact, it's typically tachyonic!



ELECTROWEAK SYMMETRY BREAKING

- The Higgs boson potential is

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (BH_u^0 H_d^0 + \text{c.c.}) + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2$$

- Minimizing this, one finds (for moderate/large $\tan\beta$)

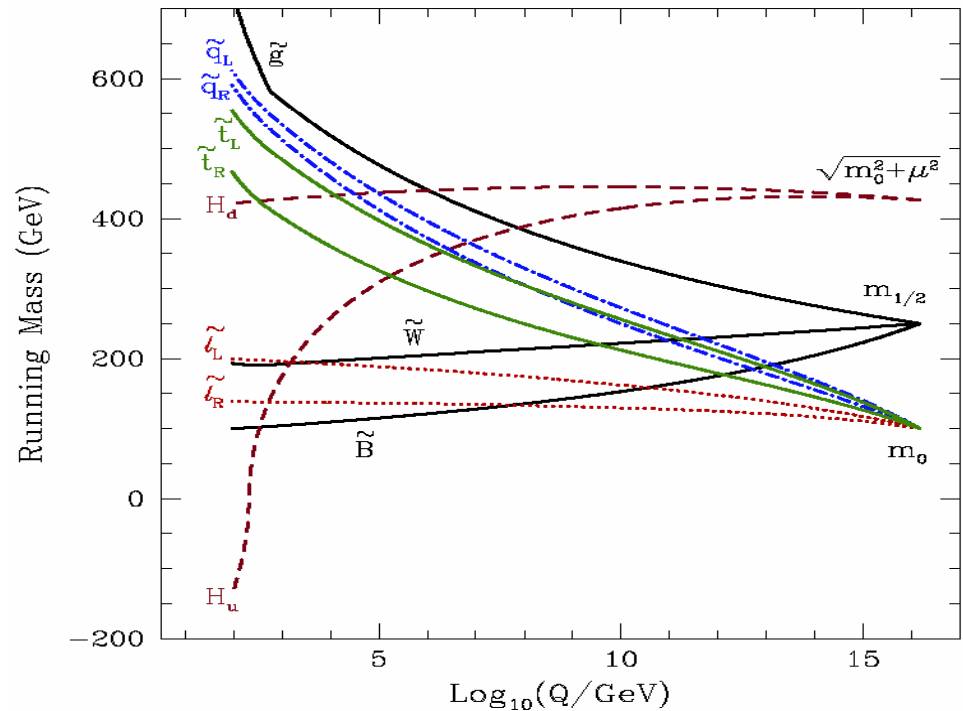
$$\frac{1}{2}m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - |\mu|^2 \approx -m_{H_u}^2 - |\mu|^2$$

- EWSB requires $m_{H_u}^2 < 0$

SUSY explains why SU(2) is broken and SU(3) and U(1) aren't

SNEUTRINOS AND HIGGSINOS

- Lightest physical scalars are typically the right-handed sleptons
- Sneutrinos
 - have SU(2) interactions, and so are typically heavier
 - Disfavored as LSPs by direct searches
- EWSB also fixes Higgsino mass μ



$$\frac{1}{2}m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - |\mu|^2$$

LECTURE 1 SUMMARY

- The Standard Model is incomplete
- SUSY provides elegant solutions
 - Naturalness
 - Force unification
 - Electroweak symmetry breaking
- Proton decay \rightarrow R-parity, stable LSP
- Natural LSPs: neutralino (Bino/Higgsino), gravitino