Black Holes: Edge of Infinity

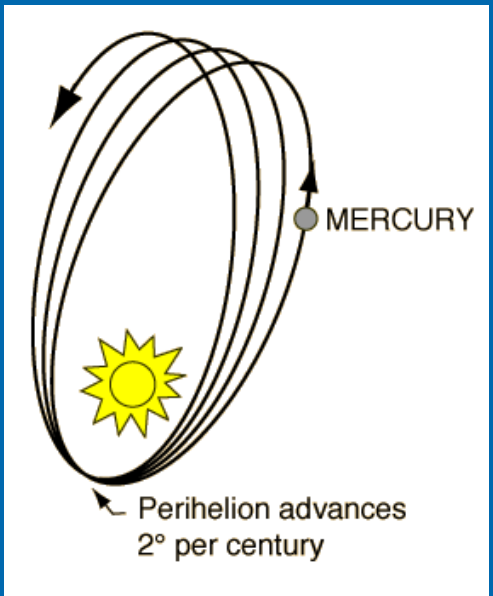
Jonathan McKinney

Outline

- Newton: Why he had to be wrong
- Einstein: Why he was right
  - Principles of relativity
  - Concept of space-time
- Black Holes:
  - What are they?
  - How do we find them?
  - Where do they come from?
  - What is their impact on evolution of Universe?
  - How do we use them to test Einstein's GR?
- Black Holes not so black:
  - Actually brightest systems in universe
  - Ergosphere: drags space-time allowing energy out
  - Disks and Jets
  - Simulations and Movies

Problems with Newton's Gravity

- Why inertial mass gravitates?
  - F=ma vs. F=GMm/r^2
- Instantaneous force incompatible with special relativity
- Post-Newtonian required:
  - Perihelion shift of Mercury
  - Small effects add up secularly
- 1783: Mitchell: Escape velocity:



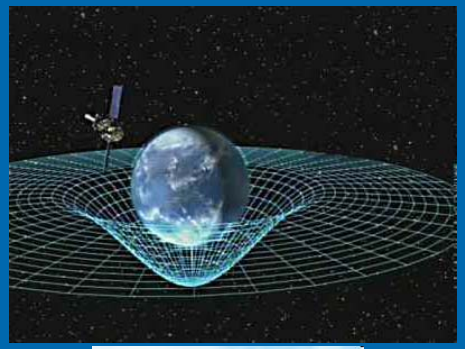
$$mv^2/2 = GMm/r \;\rightarrow\; v = \sqrt{2GM/r}$$

$$v = c \;\rightarrow\; r = 2GM/c^2$$

Black Hole Theorists

- 1916: Einstein: Equivalence, speed of light constant
- 1916: Schwarzschild: Exact static spherical solution
- 1931: Chandrasekhar: >1.4$M_\odot$ white dwarf -> neutron star
- 1935: Eddington: >>1.4$M_\odot$, then black hole must form
- 1963: Kerr: rotating black hole solution
- 1970's: Wheeler: Better "PR" for black holes
- 1969: Penrose process: Spin energy extractable
- 1970: Bardeen: Last stable orbit and efficient emission
- 1973: Shakura & Sunyaev: Quasars powered by BHs
- 1974: Hawking: BH is a black body
- 1977+: Blandford: Jets powered by BH/Disks
- 80's: Simulations of Disks
- 90's: Simulations of BHs, novae, supernovae
- 00's: Simulations of Formation of BHs, disks, jets

Einstein's Gravity



- Freely-falling objects move straight in curved space-time
- Principles of Equivalence:
  - **Weak**: Test body motion independent of composition
  - **Einstein**: Non-gravitational experimental result same in any inertial frame
  - **Strong**: Applies to all experiments for entire Universe
  - Dimensionless numbers constant

- No known violations of relativity:
  - Perihelion **shift** of Mercury (+43arcsec/century)
  - Sun **bends** light 2X more than Newton (1.75arcsec)
  - Gravitational **redshift** (from equivalence principle)
  - Shapiro light **delay**: Mars by Sun (+250μs / 45mins)
  - Relativistic geodetic **precession** (7arcsec/year)
  - Gravitational wave decay of binary pulsars
  - Constancy of Newton's G (lunar ranging): 1 in 10^{12}/year
  - Gravitomagnetism (Venus-Sun: -0.0003 arcsec/century)

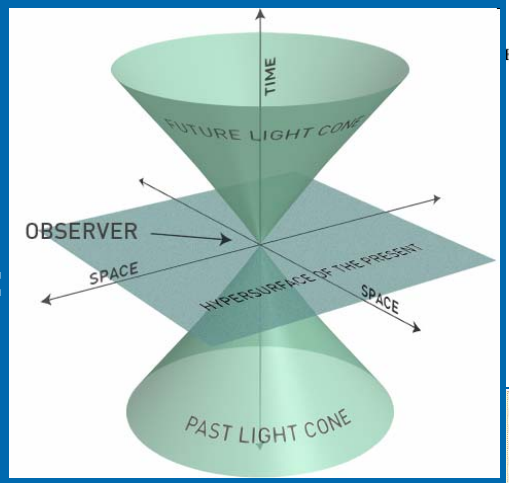
Space-Time

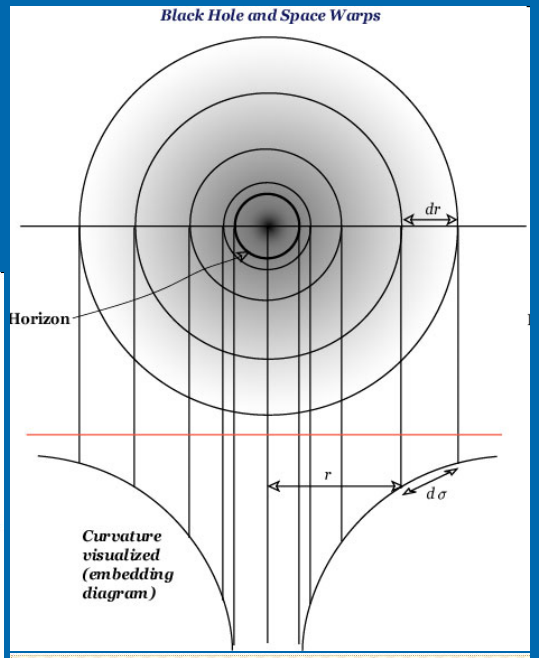
- **Metric**: Space-time is a smoothly warping manifold

$$d(\mathbf{x}, \mathbf{y}) = \sqrt{(x_1 - y_1)^2 + \ldots + (x_n - y_n)^2}$$

$$d s^2 = g_{\alpha\beta}\, d x^\alpha\, d x^\beta$$
$$= (d x^1)^2 + (d x^2)^2 + (d x^3)^2$$

$$(g)_{\alpha\beta} \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

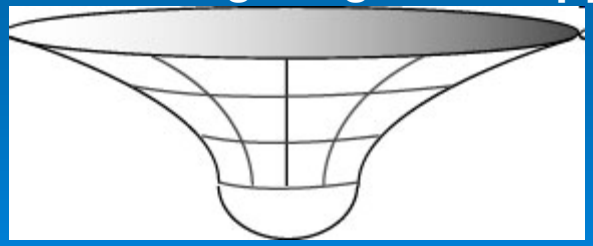


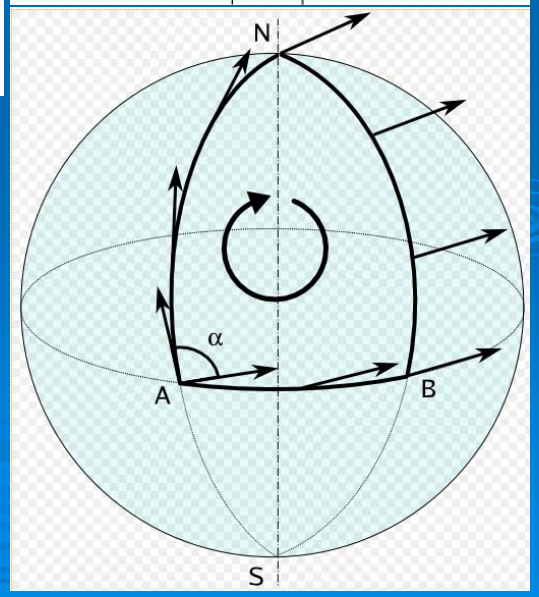


- **Minkowski metric/Light Cone:**

$$\eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

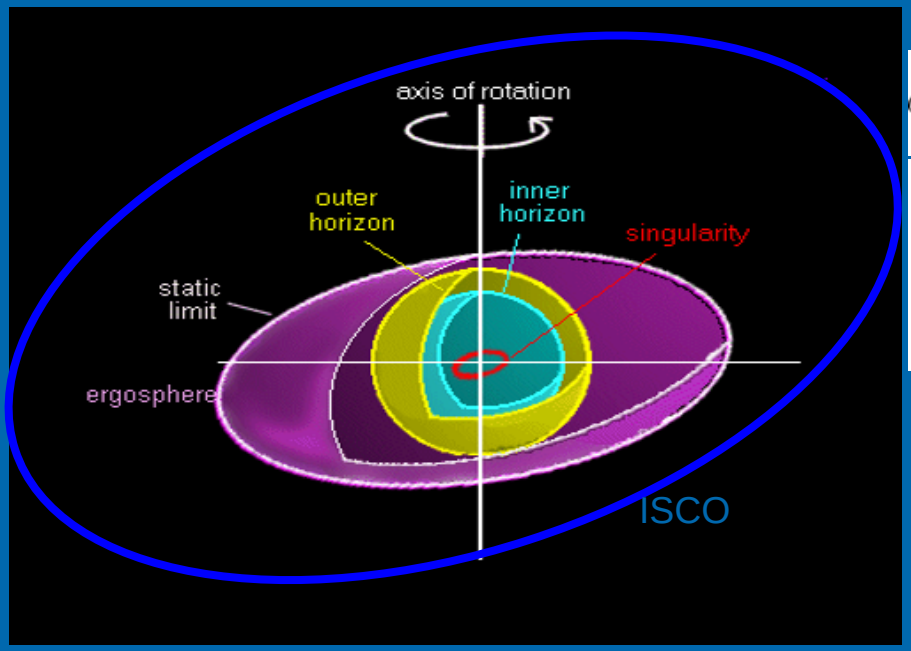
- **Embedding Diagram: Mapping of full geometry**





- **Parallel Transport**:
  - Describes motion of a vector on manifold.
  - Many such transports can be used to build up metric

Black Holes



Schwarzschild Metric (BH w/ only Mass):

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 & 0 & 0 & 0 \\ 0 & \alpha^{-2} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix} \qquad \alpha = \left(1 - \frac{2M}{r}\right)^{1/2}$$

Kerr Metric (BH w/ Spin):

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \omega^2 \varpi^2 & 0 & 0 & -\omega\varpi^2 \\ 0 & \rho^2/\Delta & 0 & 0 \\ 0 & 0 & \rho^2 & 0 \\ -\omega\varpi^2 & 0 & 0 & \varpi^2 \end{pmatrix}$$

Horizon: $r_H = M \pm \sqrt{M^2 - a^2}$

Static Limit: $r_S = M \pm \sqrt{M^2 - a^2 \cos^2\theta}$

Wheeler's No-Hair Theorem:

Mass: M, Spin: J, Charge: Q

Inner-most stable circular orbit (ISCO): Inside no circular orbits ($3r_H$ for a=0 , $1r_H$ for a=1)

Photon Sphere: Inside, objects cannot orbit at all, at (~$3/2r_H$ for a=0)

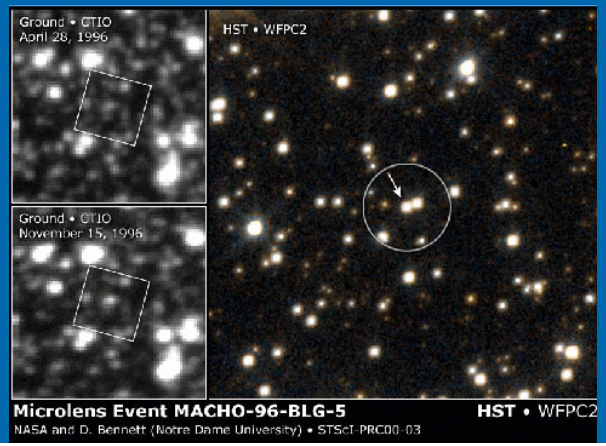
Static Limit: Inside, objects cannot be static (varies from $1r_H$ to $2r_H$ for a=M)

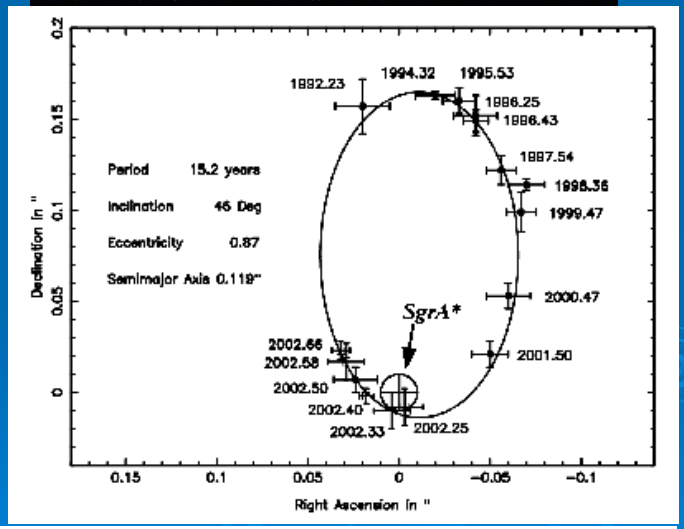
Horizon or Schwarzschild radius: Inside r_H, objects must fall

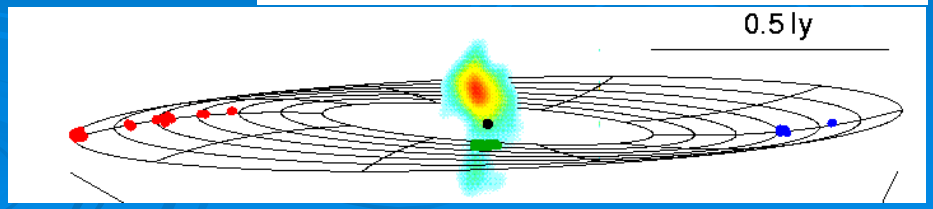
Singularity: Near, physics breaks down (need quantum gravity), reached in finite time

Detecting Black Holes

- Power and Efficiency
  - Rule out fusion
- Compactness:
  - Rule out other extreme objects such as boson stars
  - Gravitational microlensing
- Kepler's Laws:
  - Binary orbit with star
  - Many stellar orbits
  - Stellar velocity dispersion
  - Gas motion around BH
  - Maser emission
- Empirical Relations
  - Mass-Luminosity
  - Mass-Velocity
- Gravitational Waves

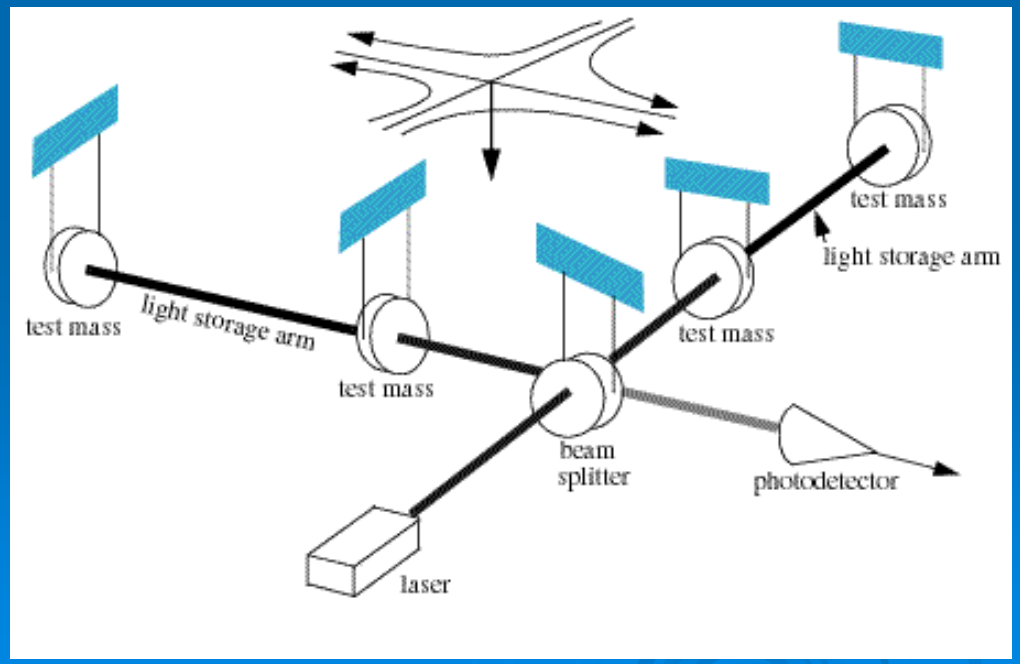


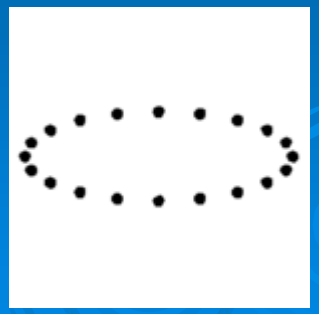




Gravitational Waves

- Generated by quadrapolar motion
- No direct detection yet!
- LIGO/advanced LIGO/LISA

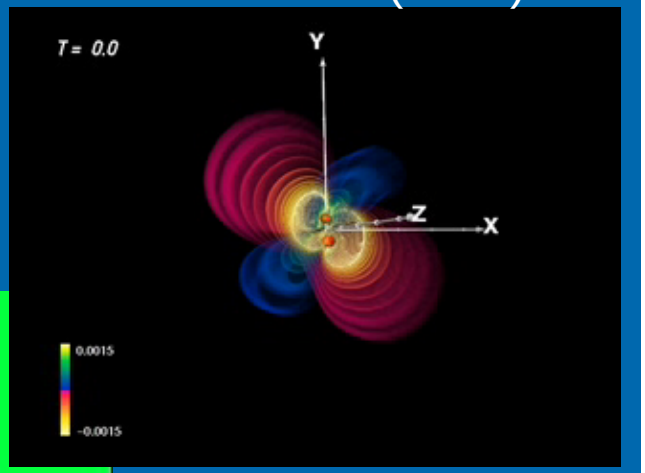




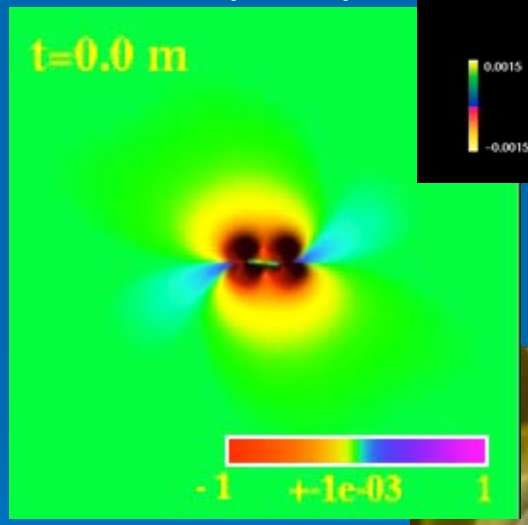
Black Hole Collision Simulations

- Purpose:
  - Waveforms for LIGO/LISA
  - Kicks

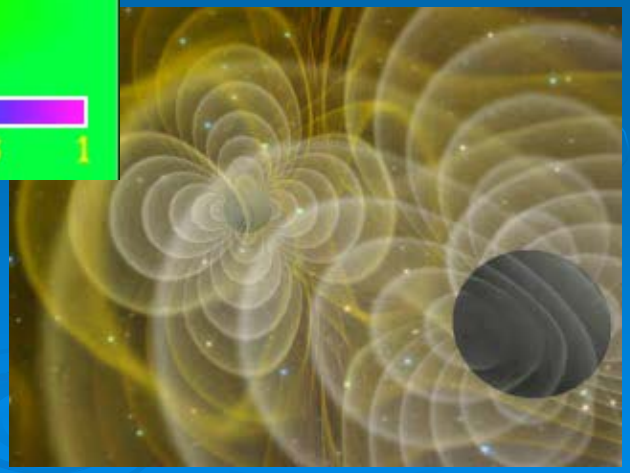
Alcubierre (1999)



Pretorius (2005)



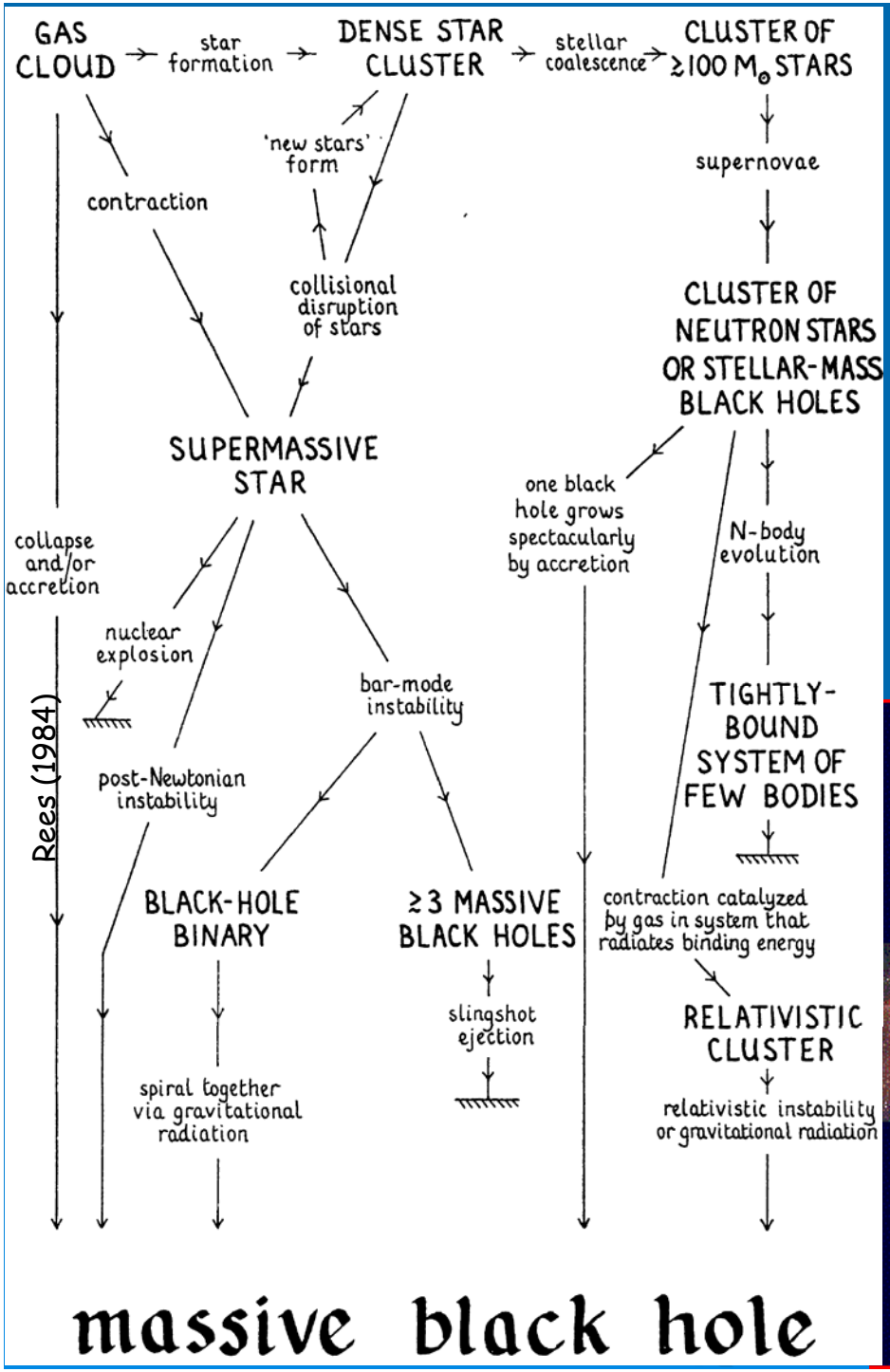
- Simulate BH/NSs
  - Singularity/horizon evolution difficult
- Simulate colliding BH-BH
  - Slow progress in 90's
  - Breakthroughs in 2006 full merger, ring down, waveforms
- Kicks:
  - 175km/s for a=0
  - 2500km/s for anti-aligned spins

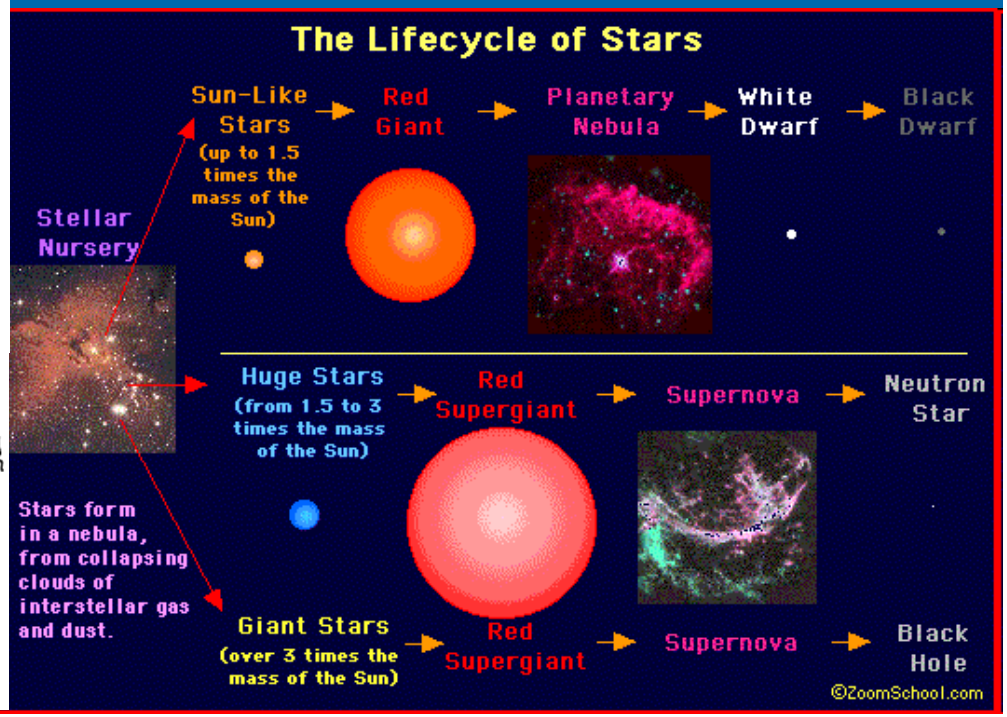


Baker (2006) / Campanelli (2006)

Black Hole Origins

- Death of massive star
- Collision of compact stars
- Single massive gaseous uniform collapse
- Many events (accretion or mergers)
- Primordially in early Universe





Supernovae and Gamma-Ray Bursts

- Supernova: Death of Massive Star to NS/BH
- 2/century in our galaxy
- BH Jet creates GRB lasting seconds seen about 2/day

Hypernova and GRB

NS-NS collision and GRB

NS-BH collision and GRB

Cosmology/Matter Evolution

- Friedmann-Robertson-Walker solution
  - Homogenous and Isotropic
  - Dark Energy is Cosmological Constant
  - Λ-CDM (cold dark matter)

$$-c^2 \mathrm{d}\tau^2 = -c^2 \mathrm{d}t^2 + a(t)^2 \left(\frac{\mathrm{d}r^2}{1 - kr^2} + r^2 \mathrm{d}\theta^2 + r^2 \sin^2\theta \, \mathrm{d}\phi^2 \right)$$

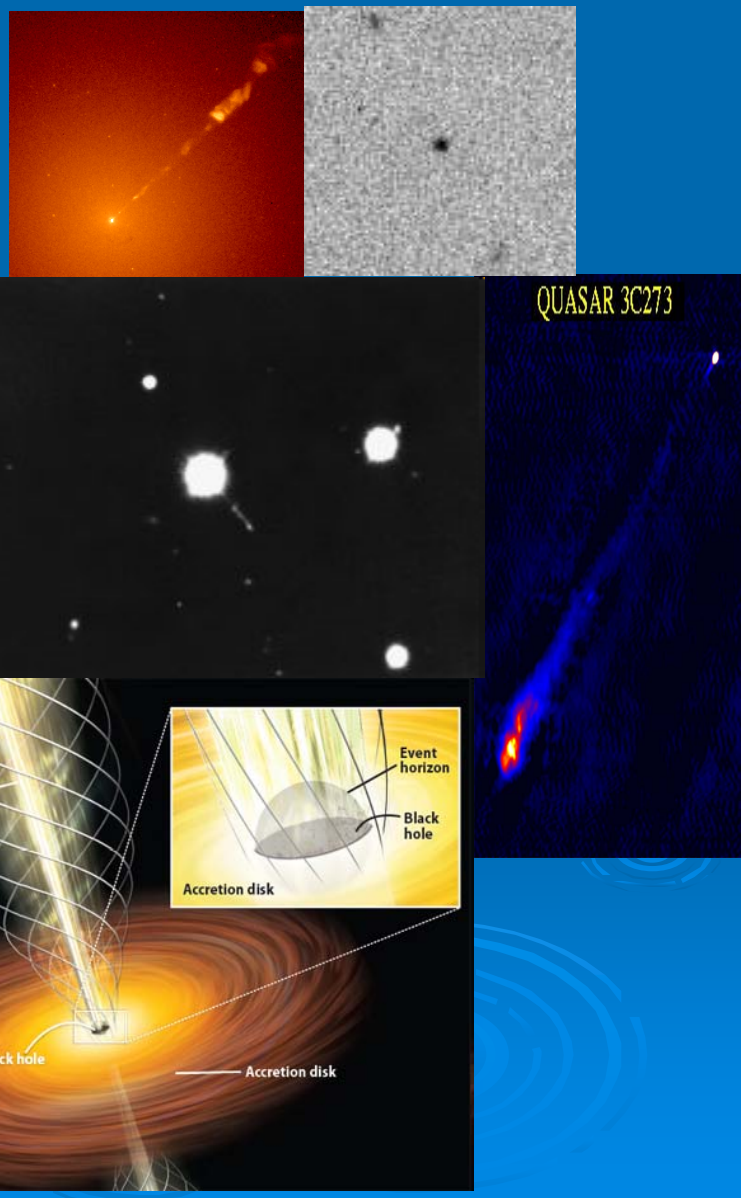
$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3}\rho$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \Lambda c^2 = -\frac{8\pi G}{c^2}p.$$

- First Stars and Quasars with black holes
  - Feedback from stars and black holes
  - Correlation between stellar velocity dispersion and black hole mass
  - Implies historical link or co-evolution

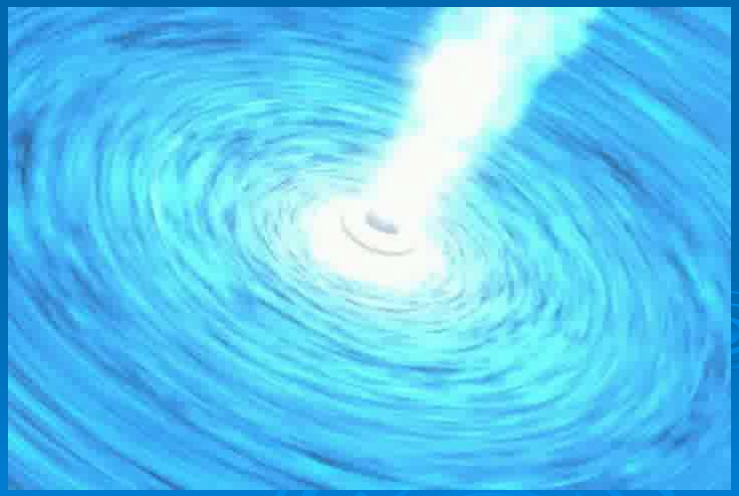
Quasars and Jets

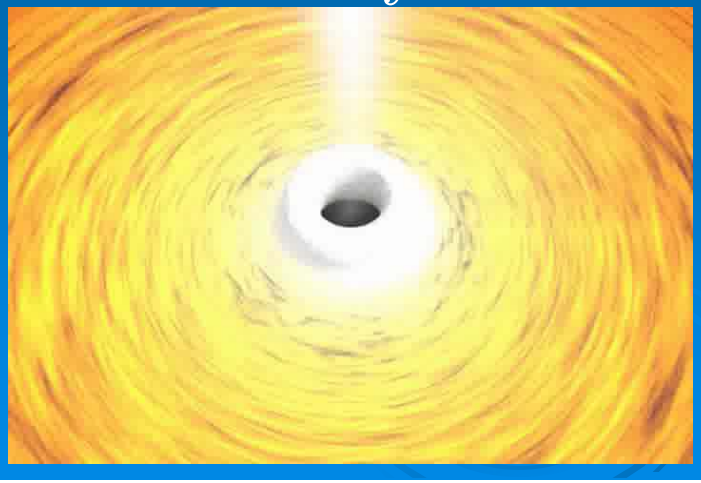
- First Jet:
  - 1917: Heber Curtis: M87
- Quasars: Quasi-Stellar radio source
  - 1950s: Stars with odd lines
  - 1960s: 3C273: Optical Counterpart
    High redshift: z=0.158 749Mpc
- Problems:
  - Deep gravity or large distance?
  - Implied > Fusion Efficiency
- BH with Accretion Disk Solution:
  - 1970s: Gravity to Radiation in Disks
  - 1980s: Unified models of diverse appearance of quasars
- 1990s: >12,000 observed quasars
- 2007: Highest redshift: z=6.43 10Gpc

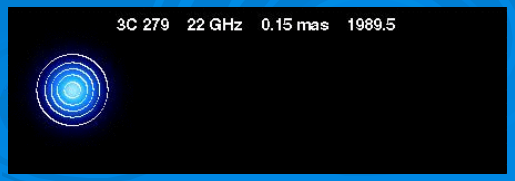


Active Galactic Nuclei

- Million-Billion solar mass BHs
- Nearly every galaxy has BH
- BH surrounded by disk
- Stars can form in outer disk
- 10% are “active” with jets
- Jets observed mostly in radio

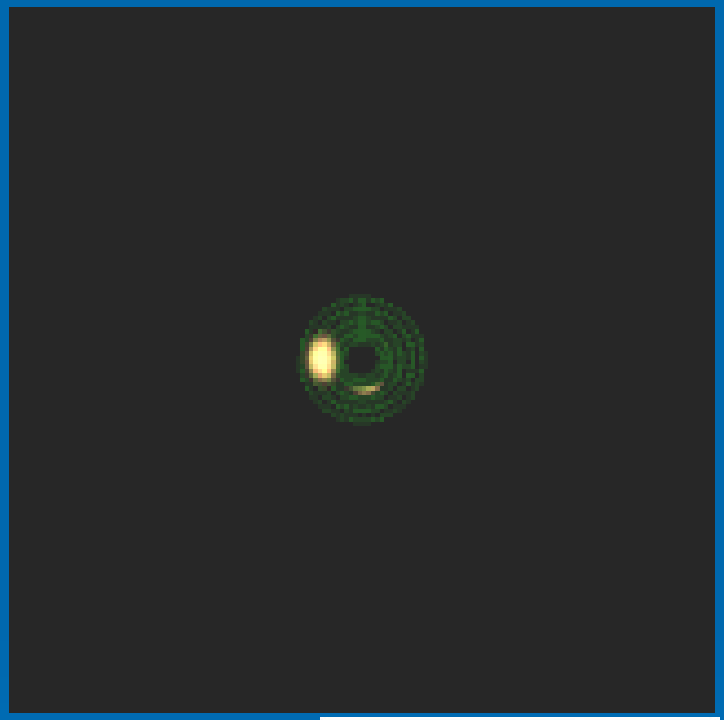




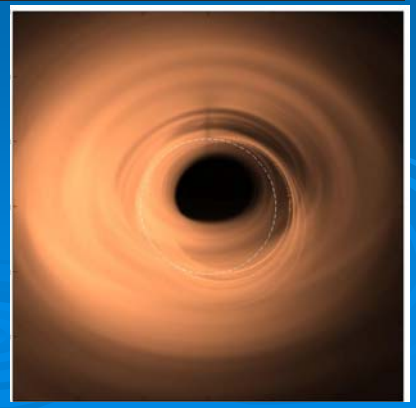


Sagittarius A*: In our Galaxy

- Center of galaxy
  - 2.6 million solar mass BH
  - Radio source
  - Accretion flow
- Near BH trace hot spots
- Back-lit BH
  - Silhouette: ~5M
  - 2.5X larger than horizon
- Used to measure:
  - mass, spin
  - Compare to Kerr space-time

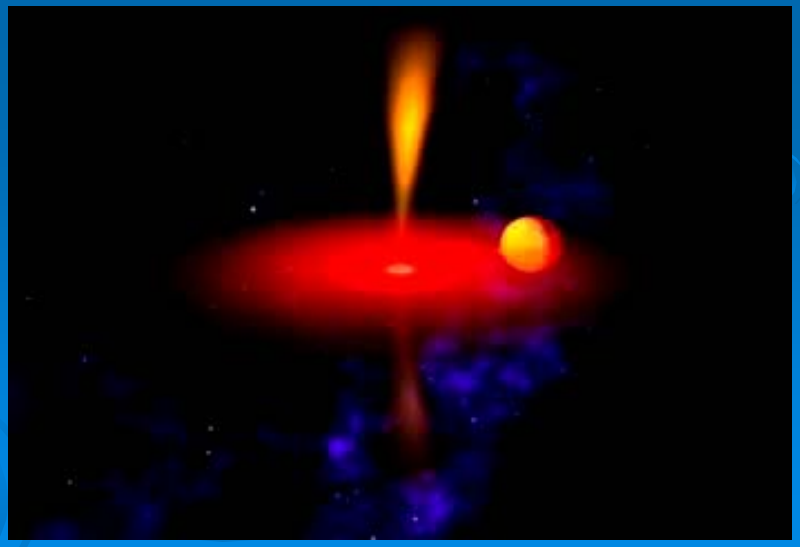


Broderick



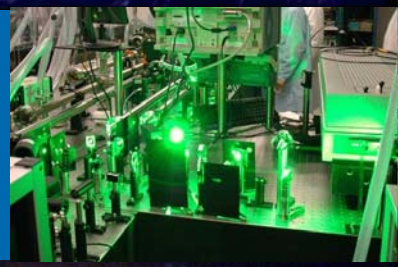
X-Ray Binaries

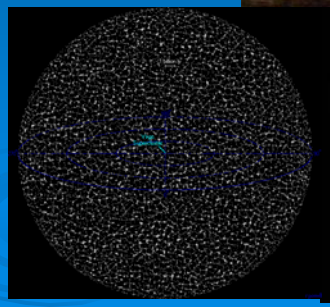
- Roughly 250 X-ray Binaries
- 3-25 Solar Mass BH
- Disk comes from companion star
- Disk emits in X-rays
- 25 BH X-ray Binaries
- 10 Produce Relativistic Jets
- 1 (GRS1915+105) may contain nearly maximally spinning BH



Black Hole Power (Energy/Time)

- 100 Watts: 1 Light Bulb
- 10^4 Watts: Each US Citizen
- 10^9 Watts (GigaWatt): Largest Power Station
- 10^{13} Watts: Human Total or 1 Hurricane
- 10^{15} Watts (PetaWatt): Most Powerful Laser
- 10^{24} Watts: Russian Tsar Bomba(50Megatons)
- 10^{27} Watts: Sun (10^{17} hits Earth)
- 10^{32} Watts (10^8 Tsar): X-Ray Binaries
- 10^{37} Watts: 1 Galaxy
- 10^{37} Watts: Active Galactic Nucleus
- 10^{45} Watts: Supernova
- 10^{45} Watts: Gamma-Ray Burst
- 10^{48} Watts: All stars in obs. universe
  - (10^{11} galaxies/10^{23} stars)




(Spinning) Black Holes Efficiency

Efficiency:

- Defined: [Usable Energy out] / [Energy in]
- Total free energy in rapidly spinning BH: 30% of its mass

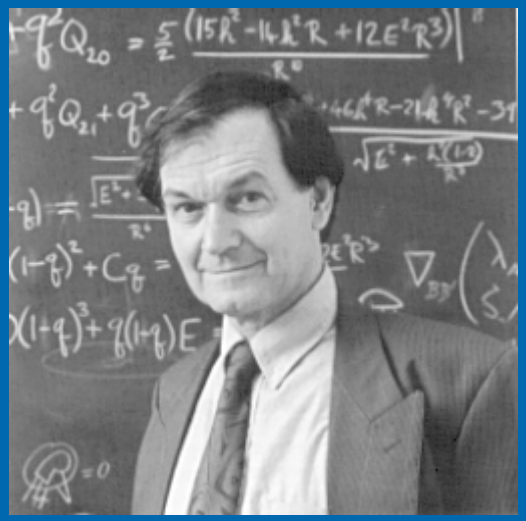
Observed Efficiency of BHs:

- Quasars = old active galaxies with active galactic nucleus
- Quasars w/ BHs: ~20% efficient

Efficiency of Mechanisms to Extract Free Energy ($E=mc^2$):

- **Antimatter**-matter: 100% of mass
- Nuclear **Fusion**: 0.07%
- **Gravitational Accretion Friction**: 5% (no spin) to 42% (max spin)
- Penrose **Particle Explosion** Process: up to 20%
- Blandford-Znajek **ElectroMagnetic** Process:
  - Typically10% but can be >100% !

Penrose Process

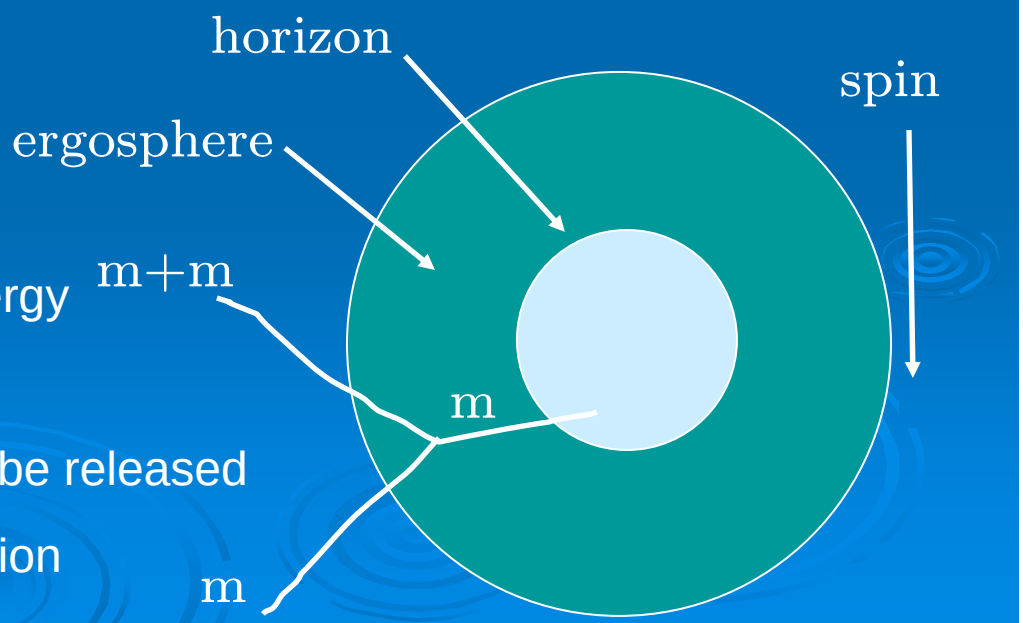


Assumptions:

- Rotating BH
- Particle explodes into 2 parts inside ergosphere
- One particle goes backwards through rotating BH
- Second particle explodes off to infinity before reaching horizon

Find:

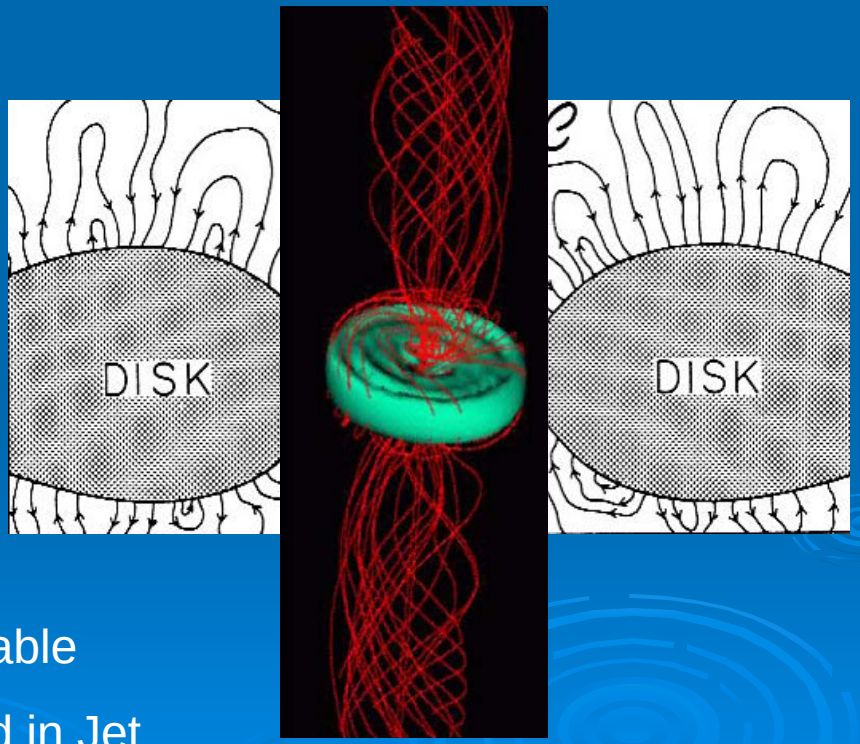
- One particle absorbs BH spin energy
- BH spins down
- 20% of particle mass-energy can be released
- No obvious astrophysical application



Blandford & Znajek (1977)

Assumptions:

- Rotating BH
- Electricity & Magnetism
- Hot Plasma in disk



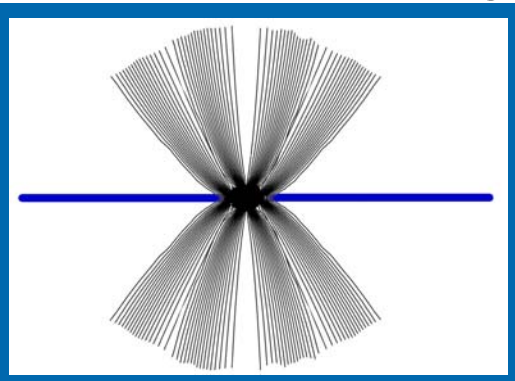
Find:

- BH Emits Energy!
- 30% of total BH mass-energy releasable
- 10% of mass-energy can be released in Jet
- Jet power scales with BH mass

Energy Extraction

- Dimensional Analysis:

$$\text{Power} \sim [B^2][\tfrac{4\pi}{3}L^3][T^{-1}]$$



$$L \equiv GM/c^2$$
$$T \equiv L/c$$
$$j \equiv J/M^2$$
$$r_+ \equiv L(1+\sqrt{1-j^2})$$
$$\Omega_H \equiv jc/2r_+$$

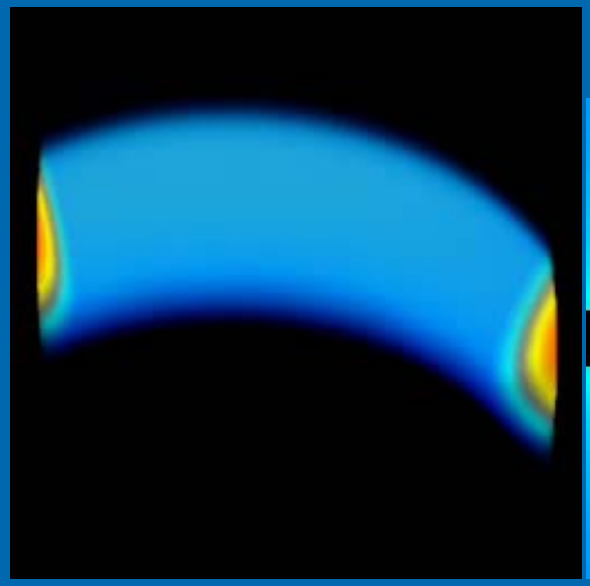
- Correct Answer:

$$\text{Power} \sim [1\%][j^2][B^2][\tfrac{4\pi}{3}L^3][T^{-1}]$$

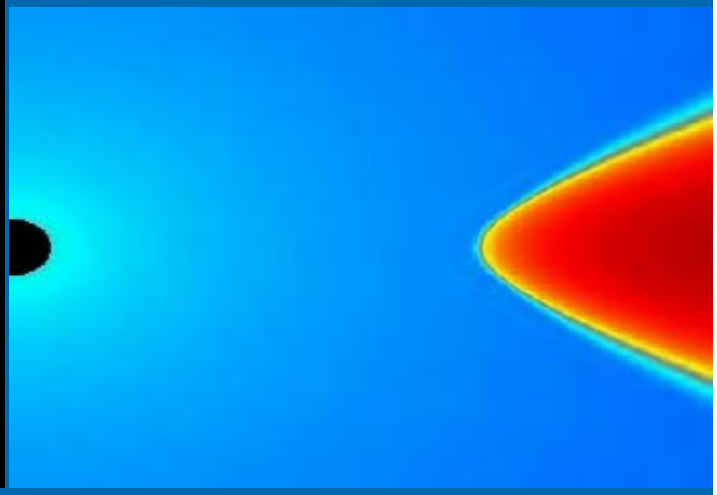
General Relativistic Plasma Computer Simulations

Simulations:

- Rotating BH
- Hot Electromagnetic Plasma in Disk



Hawley



McKinney

Purpose:

- What is the structure of disks around black holes?
- How do jets form?
- Simulation show jets naturally collimated and accelerate to high Lorentz factors $\Gamma = 1/\sqrt{1-(v/c)^2}$

Summary about Black Holes

- Black Holes are more efficient than Fusion
  - Spinning BH: up to 30% stored as spin energy
  - Gravity Accretion: 5 (0 spin) to 42% (max spin)
  - Magnetic field + BH: Typical: 10% , Maybe >100% during long periods
- Creating and Colliding black holes:
  - GRBs: NS-NS, NS-BH collisions and star death
  - Produce kicks up to 2500km/s
  - Gravitational waveforms for LIGO/LISA
- Black Holes brightest objects in sky:
  - Accretion converts Gravity to Radiation
  - BH threaded by magnetic field makes Jets/Outflows