Two Lectures on Neutrino Theory/Phenomenology

André de Gouvêa

Northwestern University

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Revolutions on the Horizon: A Decade of New Experiments
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Tentative Outline for Today and Tomorrow

1. What Are Neutrinos? – Very Brief!;
2. Neutrino Puzzles – The Discovery of Neutrino Masses;
3. Mass-Induced Neutrino Oscillations;
4. What We Know We Don't Know – Next-Generation ν Oscillations;
5. What We Know We Don't Know – Majorana versus Dirac Neutrinos;
6. Neutrino Masses As Physics Beyond the Standard Model;
7. Ideas for Tiny Neutrino Masses, and Some Consequences;

Some References (Biased):

- M.C. Gonzalez-Garcia, M. Maltoni, 0704.1800 [hep-ph];
- A. Strumia and F. Vissani, hep-ph/0606054;
- R. Mohapatra *et al.*, hep-ph/0510213;
- AdG, hep-ph/0503086;
- AdG, hep-ph/0411274.

Neutrino History:

"Are There Really Neutrinos? – An Evidential History," Allan Franklin, Perseus Books, 2001.

Recent Neutrino Textbook:

"Fundamentals of neutrino physics and astrophysics," Carlo Giunti and Chung Wook Kim, Oxford University Press, 2007.

1– Neutrino Timeline, abridged:

1. 1930: Postulated by Pauli to (a) resolve the problem of continuous β-ray spectra, and (b) reconcile nuclear model with spin-statistics theorem.

2. 1934: Fermi theory of Weak Interactions – current-current interaction

$$\mathcal{H} \sim G_F \left(\bar{p}\Gamma n\right)\left(\bar{e}\Gamma \nu_e\right), \qquad \text{where } \Gamma = \{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$$

   Way to "see" neutrinos: $\bar{\nu}_e + p \to e^+ + n$. Prediction for the cross-section – too small to ever be observed...

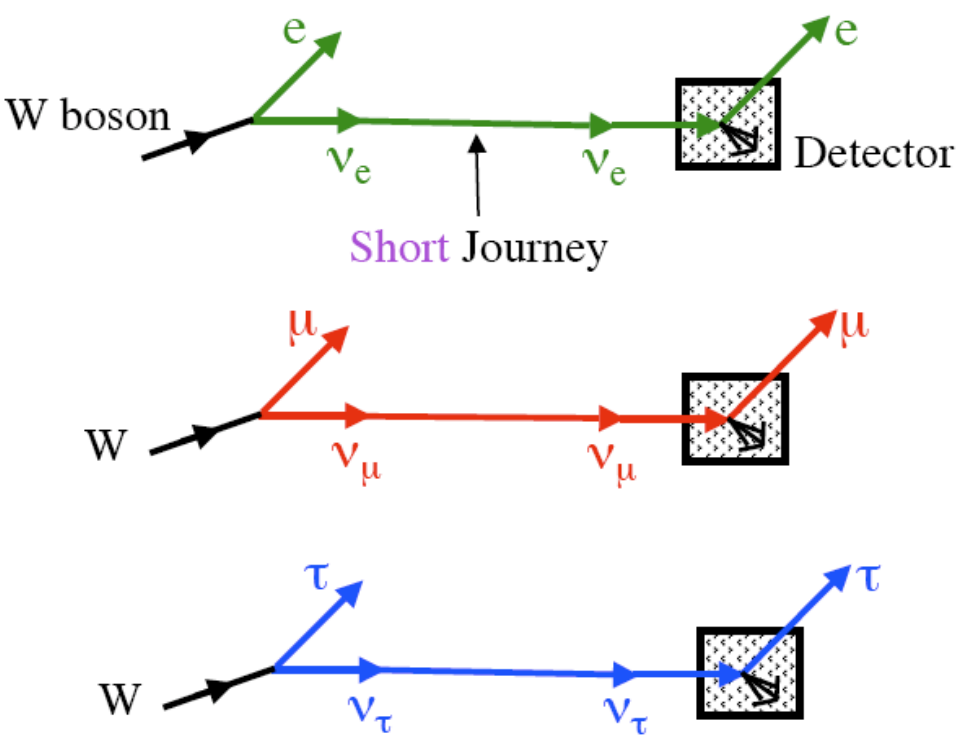
3. 1956: "Discovery" of the neutrino (Reines and Cowan) in the Savannah River Nuclear Reactor site. $\bar{\nu}_e + p \to e^+ + n$.

4. 1962: The second neutrino: $\nu_\mu \neq \nu_e$ (Lederman, Steinberger, Schwartz at BNL). First neutrino beam.

$$p + Z \to \pi^+ X \to \mu^+ \nu_\mu \quad \Rightarrow \quad \begin{array}{l} \nu_\mu + Z \to \mu^- + Y \ (\text{“always”}) \\ \nu_\mu + Z \to e^- + Y \ (\text{“never”}) \end{array}$$

5. 2001: ν_τ directly observed (DONUT experiment at FNAL). Same strategy: $\nu_\tau + Z \to \tau^- + Y$. ($\tau$-leptons discovered in the 1970's).

Until recently,† this is how we pictured neutrinos:



- come in three flavors (see figure);
- interact only via weak interactions ($W^\pm, Z^0$);
- have ZERO mass – helicity good quantum number;
- ν_L field describes 2 degrees of freedom:
  - left-handed state ν,
  - right-handed state $\bar{\nu}$ (CPT conjugate);
- neutrinos carry lepton number:
  - $L(\nu) = +1$,
  - $L(\bar{\nu}) = -1$.

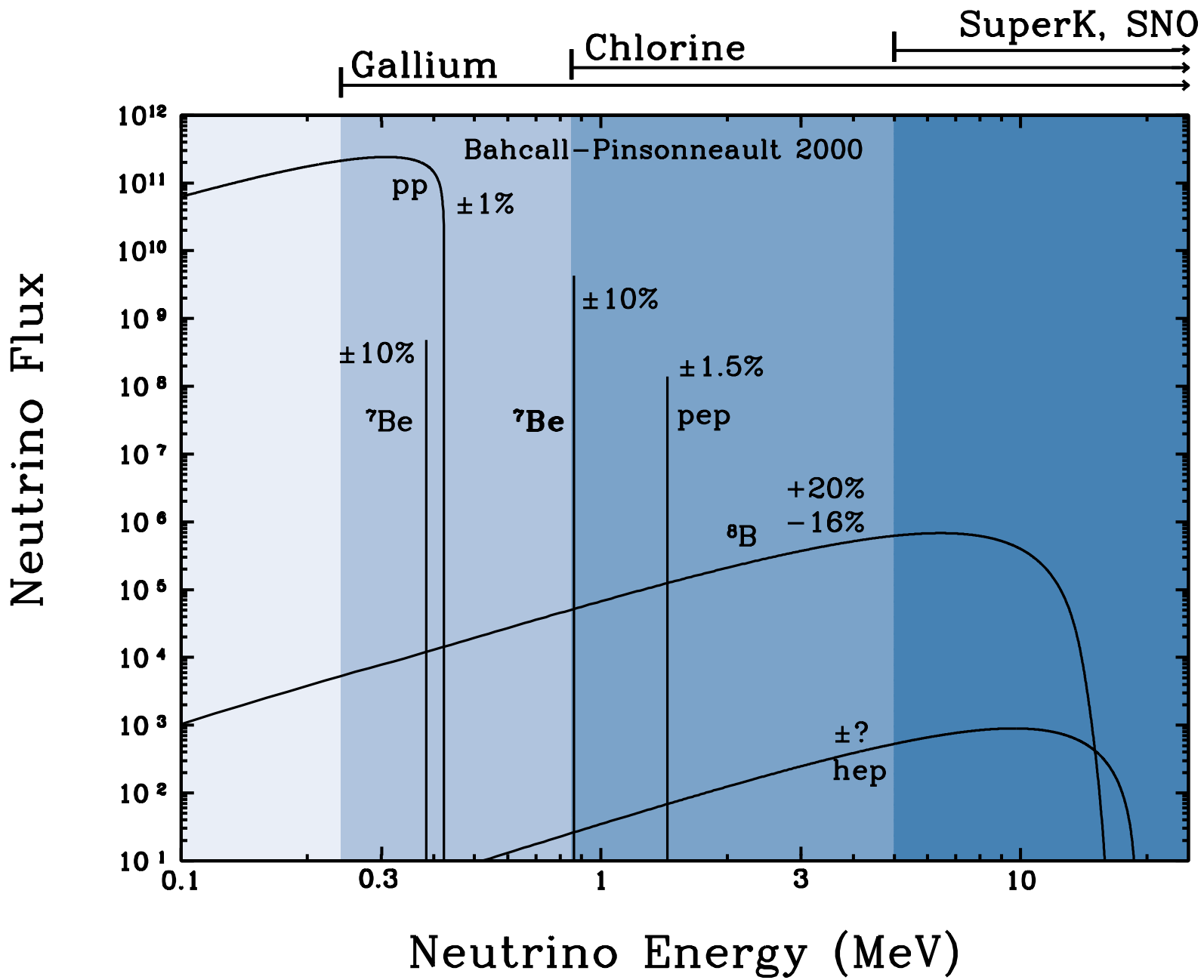
†things changed qualitatively after SuperKamiokande results in 1998 - over one decade ago!

2– Neutrino Puzzles

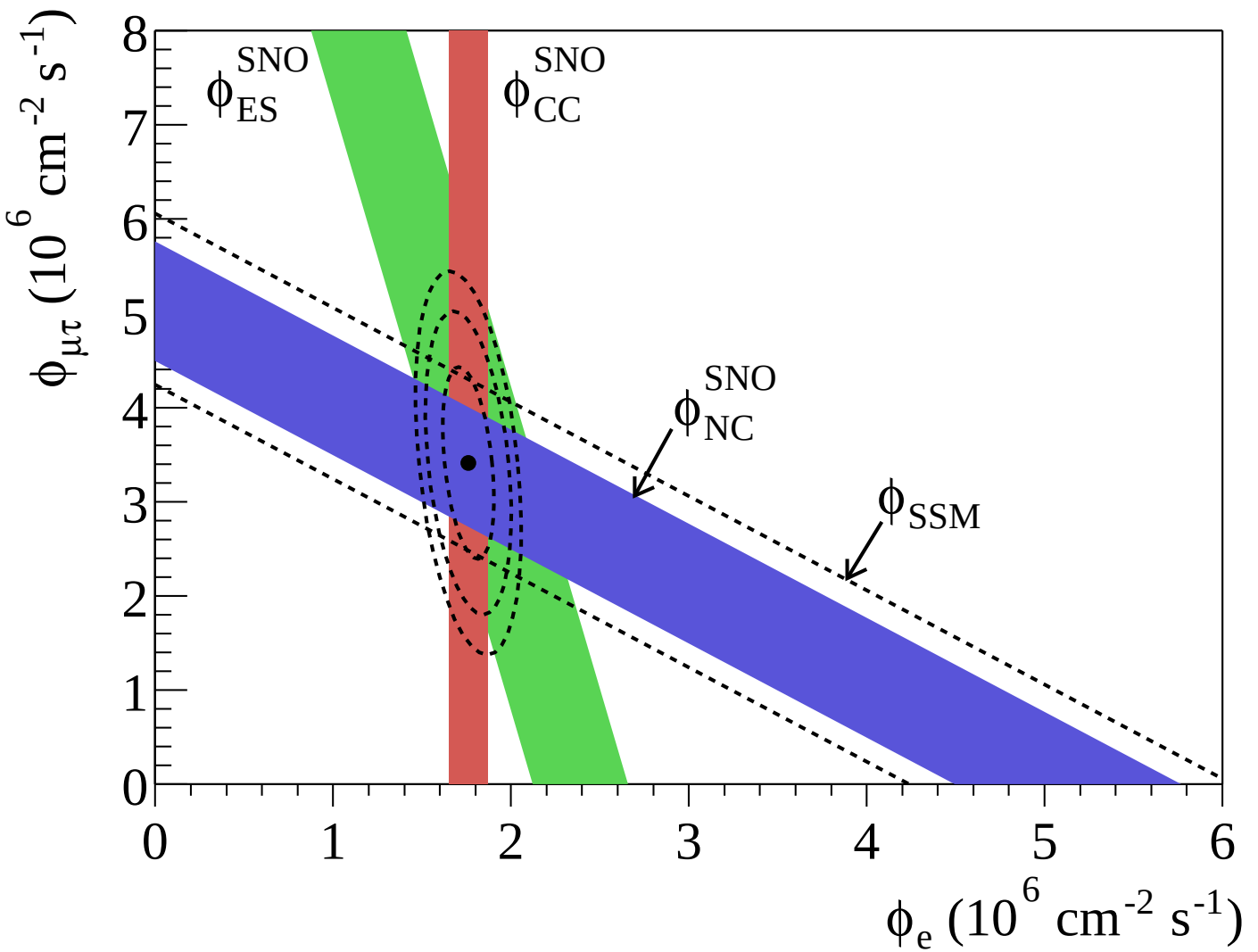
Long baseline neutrino experiments have revealed that neutrinos change flavor after propagating a finite distance, violating the definitions in the previous slide. The rate of change depends on the neutrino energy E_ν and the baseline L.

- $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ — atmospheric experiments ["indisputable"];
- $\nu_e \to \nu_{\mu,\tau}$ — solar experiments ["indisputable"];
- $\bar{\nu}_e \to \bar{\nu}_{\rm other}$ — reactor neutrinos ["indisputable"];
- $\nu_\mu \to \nu_{\rm other}$ from accelerator experiments ["indisputable"].

$\Rightarrow$ More Details: See Mike Shaevitz, Aug. 12, 13, 14


SuperK, SNO
Chlorine
Gallium
Bahcall−Pinsonneault 2000
pp
±1%
±10%
^{7}Be
±10%
^{7}Be
±1.5%
pep
+20%
−16%
^{8}B
±?
hep
Neutrino Flux
Neutrino Energy (MeV)
10^{12}
10^{11}
10^{10}
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
0.1
0.3
1
3
10

The SNO Experiment: conclusive evidence for flavor change



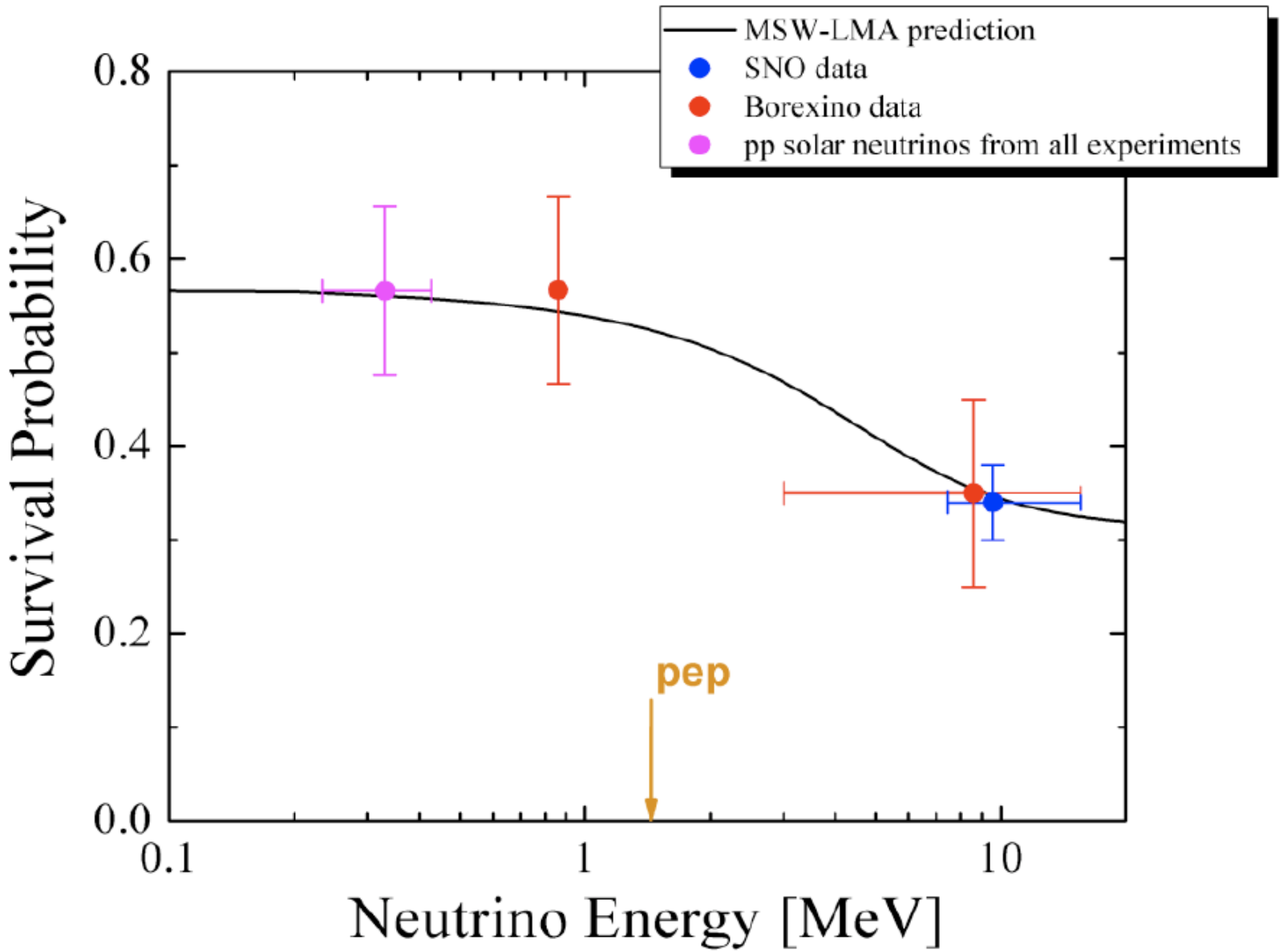
SNO Measures:

$[CC]\ \nu_e + {}^2H \rightarrow p + p + e^-$

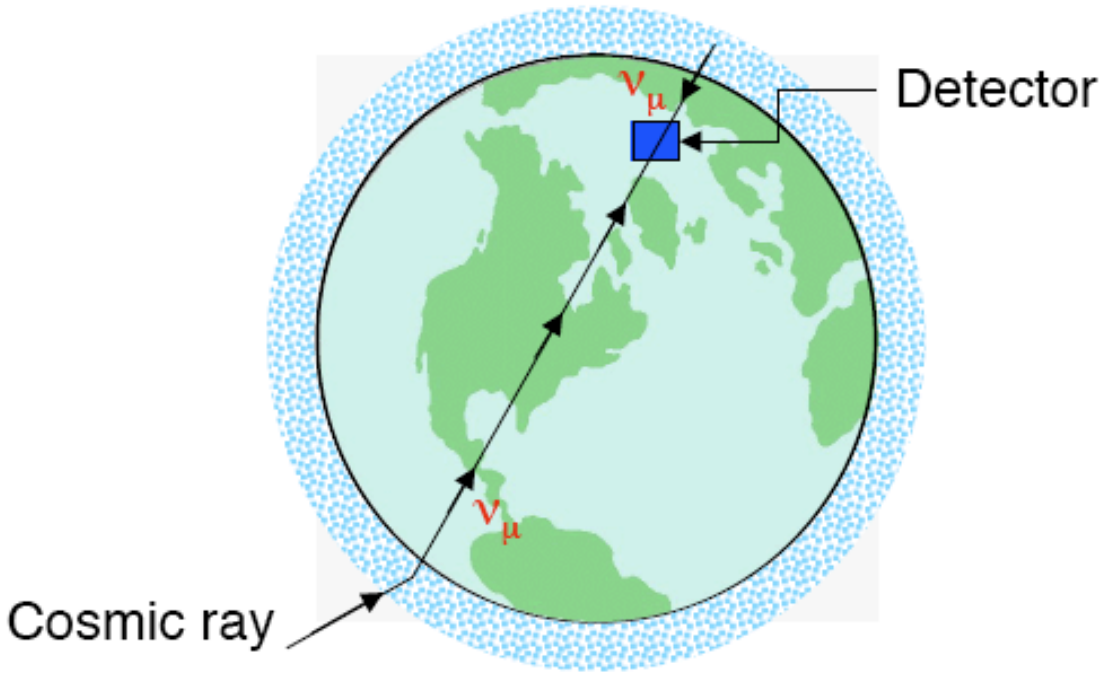
$[ES]\ \nu + e^- \rightarrow \nu + e^-$

$[NC]\ \nu + {}^2H \rightarrow p + n + \nu$

different reactions sensitive to different neutrino flavors.


MSW-LMA prediction
SNO data
Borexino data
pp solar neutrinos from all experiments
Survival Probability
0.8
0.6
0.4
0.2
0.0
0.1
1
10
Neutrino Energy [MeV]
pep

Atmospheric Neutrinos



Isotropy of the $\gtrsim$ 2 GeV cosmic rays + Gauss' Law + No ν_μ disappearance

$$\Longrightarrow \quad \frac{\phi_{\nu_\mu}(\mathrm{Up})}{\phi_{\nu_\mu}(\mathrm{Down})} = 1 \ .$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV

$$\frac{\phi_{\nu_\mu}(\mathrm{Up})}{\phi_{\nu_\mu}(\mathrm{Down})} = 0.54 \pm 0.04 \ .$$

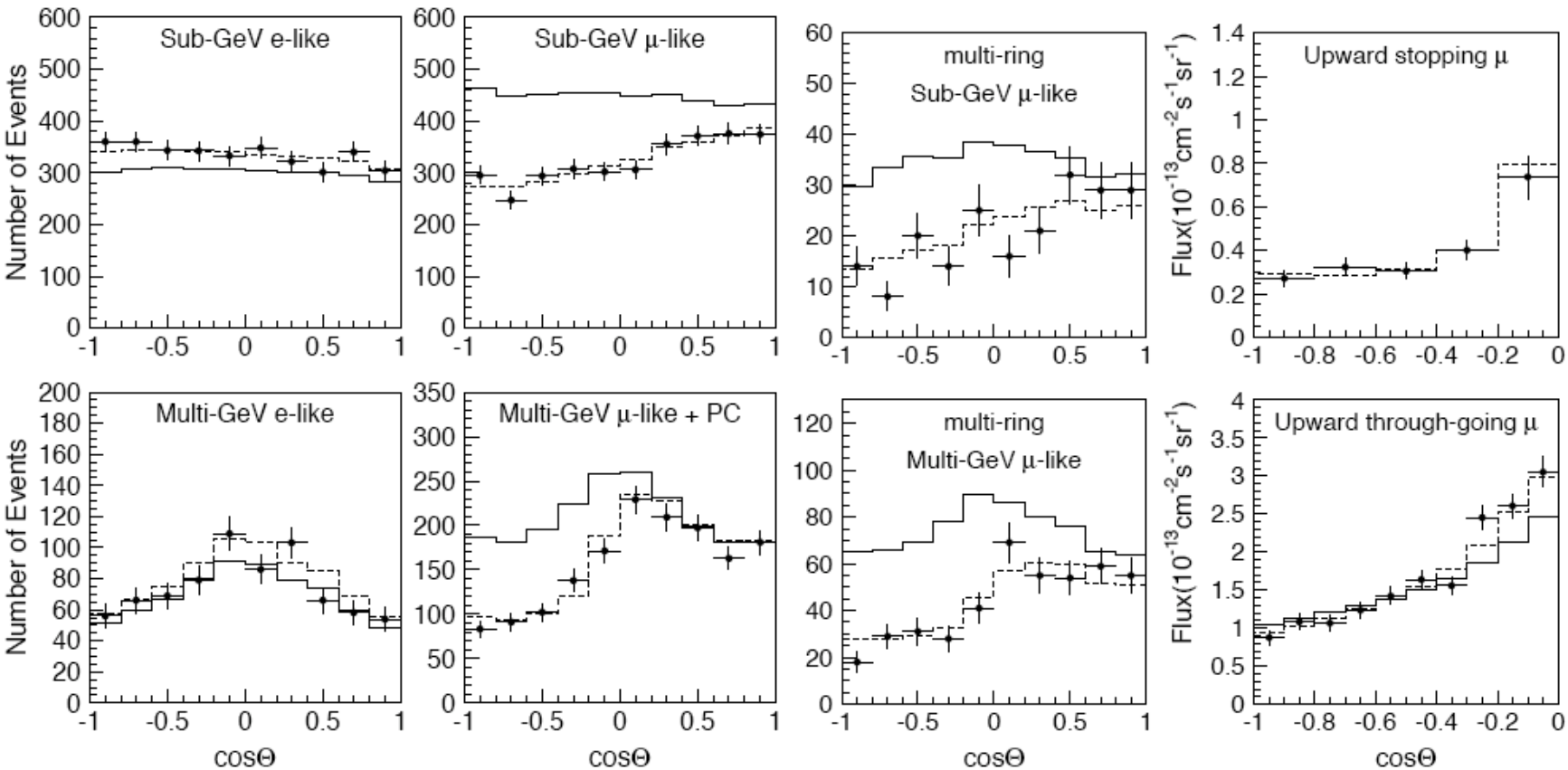


Figure 4. Zenith angle distribution for fully-contained single-ring e-like and μ-like events, multi-ring μ-like events, partially contained events and upward-going muons. The points show the data and the solid lines show the Monte Carlo events without neutrino oscillation. The dashed lines show the best-fit expectations for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. From M. Ishitsuka [Super-Kamiokande Collaboration], hep-ex/0406076.

3- Mass-Induced Neutrino Flavor Oscillations

Neutrino Flavor change can arise out of several different mechanisms. The simplest one is to appreciate that, once **neutrinos have mass**, **leptons can mix**. If neutrinos have mass, there are two different ways to define the different neutrino states.

(1) Neutrinos with a well defined mass:

$$\nu_1, \nu_2, \nu_3, \ldots \qquad \text{with masses} \;\; m_1, m_2, m_3, \ldots$$

(2) Neutrinos with a well defined flavor:

$$\nu_e, \nu_\mu, \nu_\tau$$

These are related by a unitary transformation:

$$\nu_\alpha = U_{\alpha i}\nu_i \qquad \alpha = e, \mu, \tau, \quad i = 1, 2, 3$$

U is a unitary mixing matrix.

The Propagation of Massive Neutrinos – The "Hand-Waving" Version

Neutrino mass eigenstates are eigenstates of the free-particle Hamiltonian:

$$|\nu_i\rangle = e^{-iE_i t}|\nu_i\rangle, \qquad E_i^2 - |\vec{p}_i|^2 = m_i^2$$

The neutrino flavor eigenstates are linear combinations of ν_i's, say:

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle. \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle. \end{aligned}$$

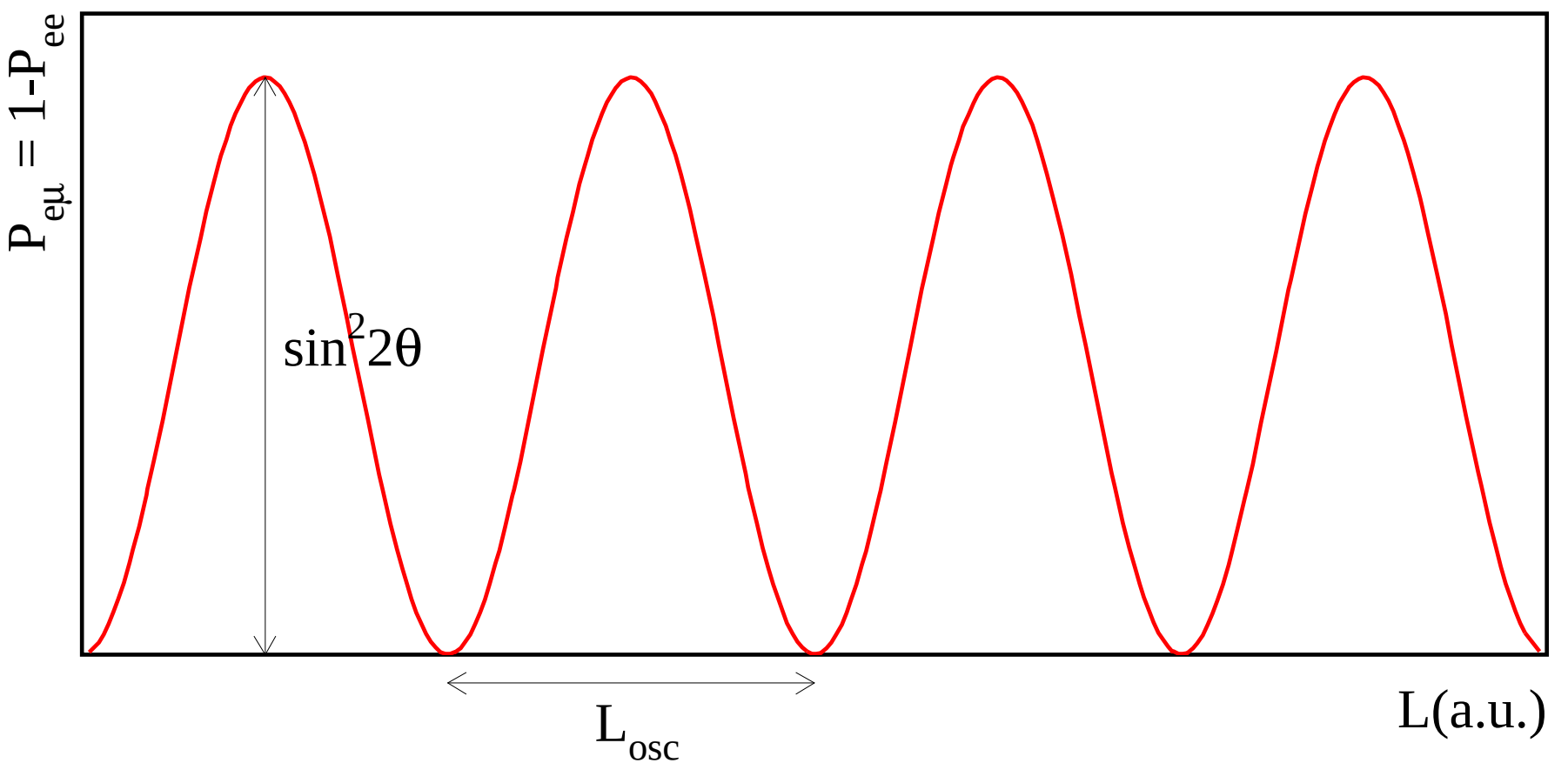
If this is the case, a state produced as a ν_e evolves in vacuum into

$$|\nu(t,\vec{x})\rangle = \cos\theta e^{-ip_1 x}|\nu_1\rangle + \sin\theta e^{-ip_2 x}|\nu_2\rangle.$$

It is trivial to compute $P_{e\mu}(L) \equiv |\langle\nu_\mu|\nu(t, z = L)\rangle|^2$. It is just like a two-level system from basic undergraduate quantum mechanics! In the ultrarelativistic limit (always a good bet), $t \simeq L$, $E_i - p_{z,i} \simeq (m_i^2)/2E_i$, and

$$\boxed{P_{e\mu}(L) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)}$$

oscillation parameters: $\begin{cases} \pi \frac{L}{L_{\rm osc}} \equiv \frac{\Delta m^2 L}{4E} = 1.267 \left(\frac{L}{\rm km}\right) \left(\frac{\Delta m^2}{\rm eV^2}\right) \left(\frac{\rm GeV}{E}\right) \\ \text{amplitude } \sin^2 2\theta \end{cases}$



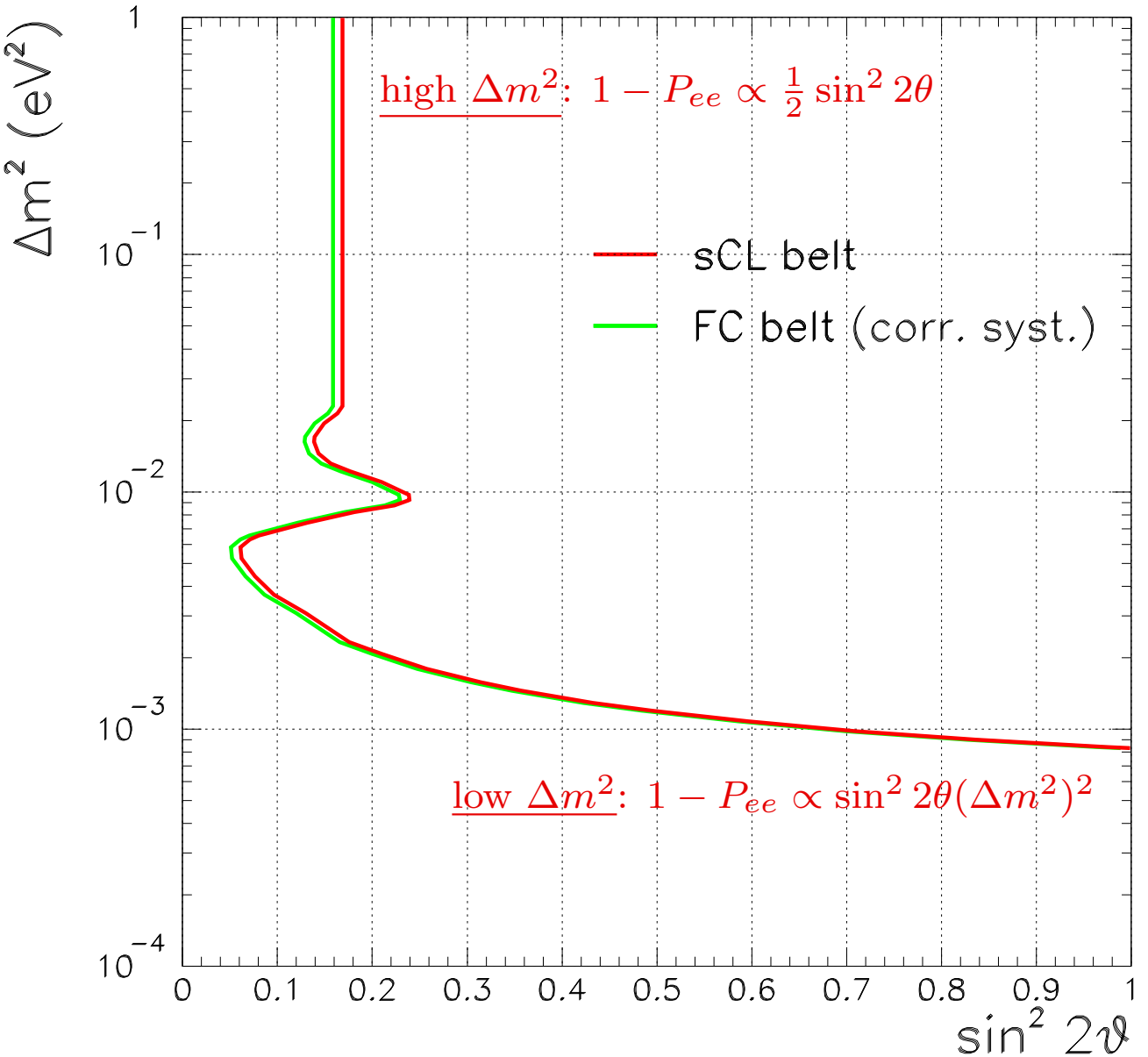
There is a long (and oftentimes confused and confusing) history behind this derivation and several others. A recent nice (very comprehensive) discussion can be found in

E.K. Akhmedov, A. Yu. Smirnov, 0905.1903 [hep-ph]

In a nutshell, neutrino oscillations as described above occur whenever

- Neutrino Production and Detection are Coherent $\rightarrow$ cannot "tell" ν_1 from ν_2 from ν_3 but "see" ν_e or ν_μ or ν_τ.
- Decoherence effects due to wave-packet separation are negligible $\rightarrow$ baseline not too long that different "velocity" components of the neutrino wave-packet have time to physically separate.
- The energy released in production and detection is large compared to the neutrino mass $\rightarrow$ so we can assign all of the effect to the neutrino propagation, independent from the production process. Also assures ultra-relativistic approximation good.

CHOOZ experiment



$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

result: $1 - P_{ee} < 0.05$

$$P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Works great for $\sin^2 2\theta \sim 1$ and $\Delta m^2 \sim 10^{-3}$ eV^2

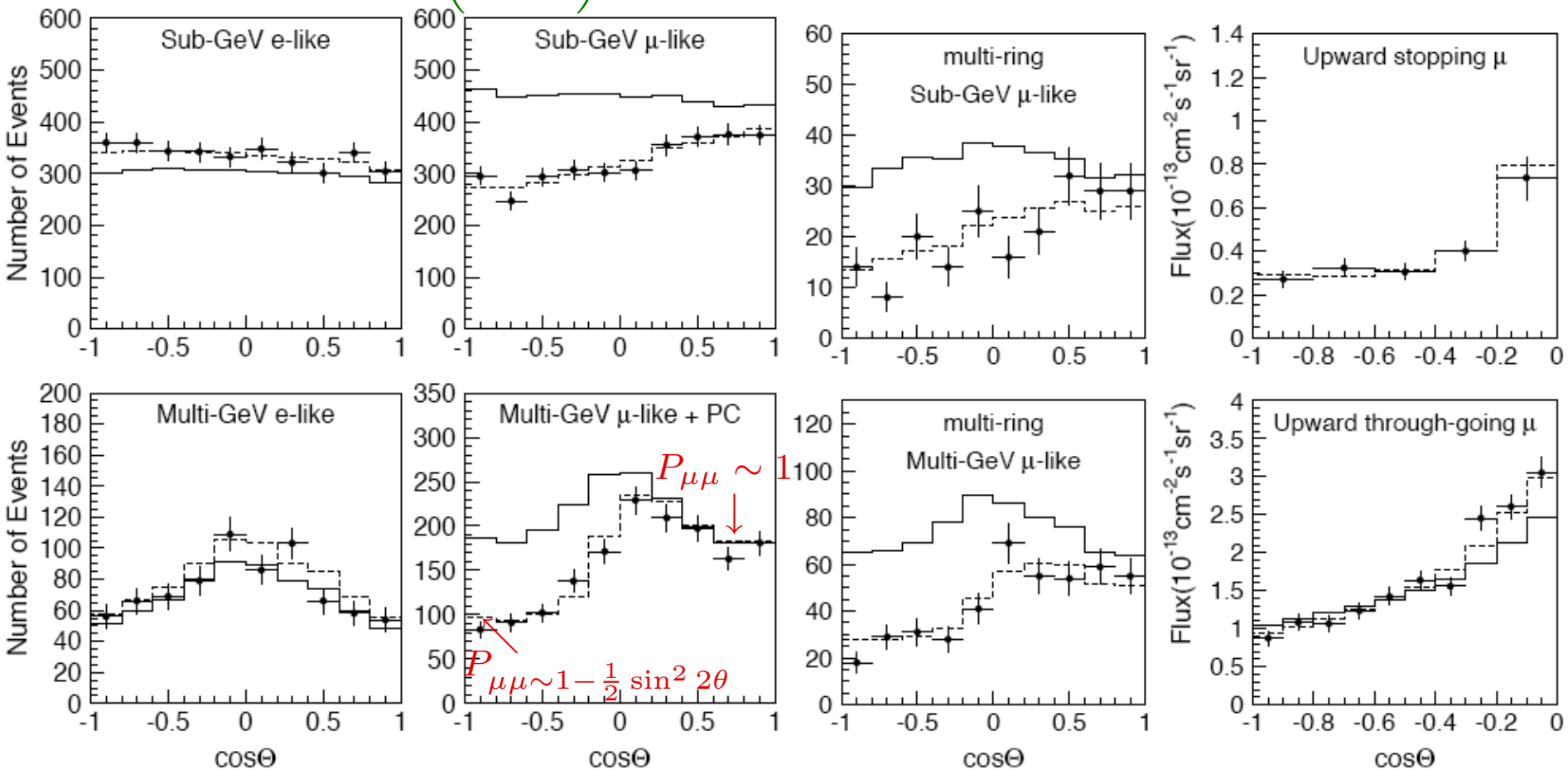


Figure 4. Zenith angle distribution for fully-contained single-ring e-like and μ-like events, multi-ring μ-like events, partially contained events and upward-going muons. The points show the data and the solid lines show the Monte Carlo events without neutrino oscillation. The dashed lines show the best-fit expectations for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. From M. Ishitsuka [Super-Kamiokande Collaboration], hep-ex/0406076.

Matter Effects

The neutrino propagation equation, in the ultra-relativistic approximation, can be re-expressed in the form of a Shrödinger-like equation. In the mass basis:

$$i\frac{\mathrm{d}}{\mathrm{d}L}|\nu_i\rangle = \frac{m_i^2}{2E}|\nu_i\rangle,$$

up to a term proportional to the identity. In the weak/flavor basis

$$i\frac{\mathrm{d}}{\mathrm{d}L}|\nu_\beta\rangle = U_{\beta i}\frac{m_i^2}{2E}U^\dagger_{i\alpha}|\nu_\alpha\rangle.$$

In the 2×2 case,

$$i\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \frac{\Delta m^2}{2E}\begin{pmatrix} \sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix},$$

(again, up to additional terms proportional to the 2×2 identity matrix).

Fermi Lagrangian, after a Fiertz rearrangement of the charged-current terms:

$$\mathrm{L} \supset \bar{\nu}_{eL} i\partial_\mu \gamma^\mu \nu_{eL} - 2\sqrt{2} G_F \left(\bar{\nu}_{eL}\gamma^\mu \nu_{eL}\right)\left(\bar{e}_L \gamma_\mu e_L\right) + \ldots$$

Equation of motion for one electron neutrino state in the presence of a non-relativistic electron background, in the rest frame of the electrons:

$$\langle \bar{e}_L \gamma_\mu e_L \rangle = \delta_{\mu 0} \frac{N_e}{2}$$

where $N_e \equiv e^\dagger e$ is the average electron number density (at rest, hence $\delta_{\mu 0}$ term). Factor of 1/2 from the "left-handed" half.

Dirac equation for a one neutrino state inside a cold electron "gas" is (ignore mass)

$$(i\partial^\mu \gamma_\mu - \sqrt{2} G_F N_e \gamma_0)|\nu_e\rangle = 0.$$

In the ultrarelativistic limit, (plus $\sqrt{2} G_F N_e \ll E$), dispersion relation is

$$E \simeq |\vec{p}| \pm \sqrt{2} G_F N_e, \qquad + \text{ for } \nu, \quad - \text{ for } \bar{\nu}$$

$$i\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \left[\frac{\Delta m^2}{2E}\begin{pmatrix} \sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta \end{pmatrix} + \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}\right]\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix},$$

$A = \pm\sqrt{2}G_F N_e$ ($+$ for neutrinos, $-$ for antineutrinos).

Note: Similar effect from neutral current interactions common to all (active) neutrino species $\rightarrow$ proportional to the identity.

In general, this is hard to solve, as A is a function of L: two-level non-relativistc quantum mechanical system in the presence of time dependent potential.

In some cases, however, the solution is rather simple.

Constant A: good approximation for neutrinos propagating through matter inside the Earth [exception: neutrinos that see Earth's internal structure (the crust, the mantle, the outer core, the inner core)]

$$i\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} A & \Delta/2\sin 2\theta \\ \Delta/2\sin 2\theta & \Delta\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}, \qquad \Delta \equiv \Delta m^2/2E.$$

$$P_{e\mu} = \sin^2 2\theta_M \sin^2\left(\frac{\Delta_M L}{2}\right),$$

where

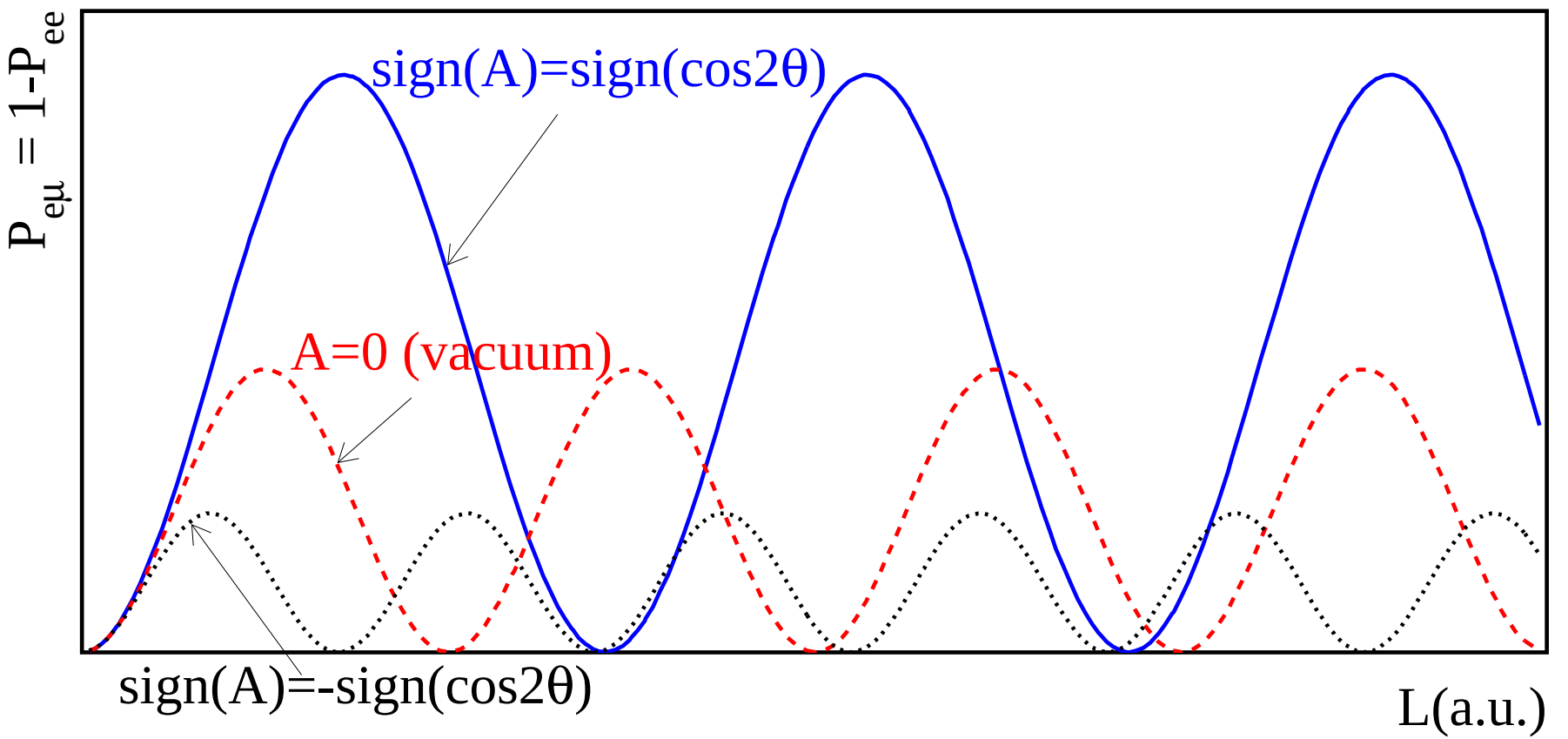
$$\begin{aligned} \Delta_M &= \sqrt{(A-\Delta\cos 2\theta)^2 + \Delta^2\sin^2 2\theta}, \\ \Delta_M \sin 2\theta_M &= \Delta\sin 2\theta, \\ \Delta_M \cos 2\theta_M &= A - \Delta\cos 2\theta. \end{aligned}$$

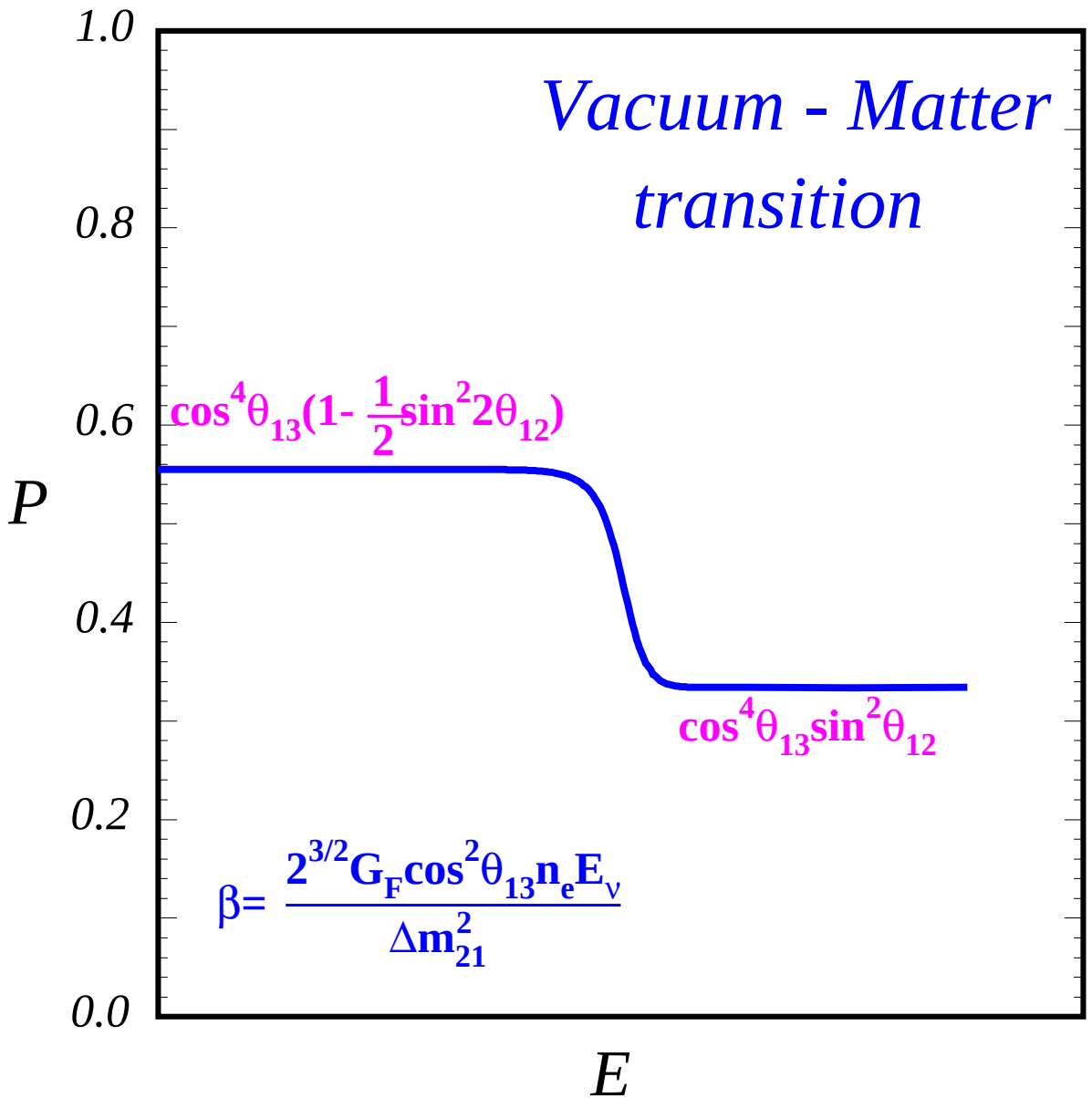
The presence of matter affects neutrino and antineutrino oscillation differently.

Nothing wrong with this: CPT-theorem relates the propagation of neutrinos in an electron background to the propagation of antineutrinos in a positron background.

Enlarged parameter space in the presence of matter effects.

For example, can tell whether $\cos 2\theta$ is positive or negative.





Solar νs

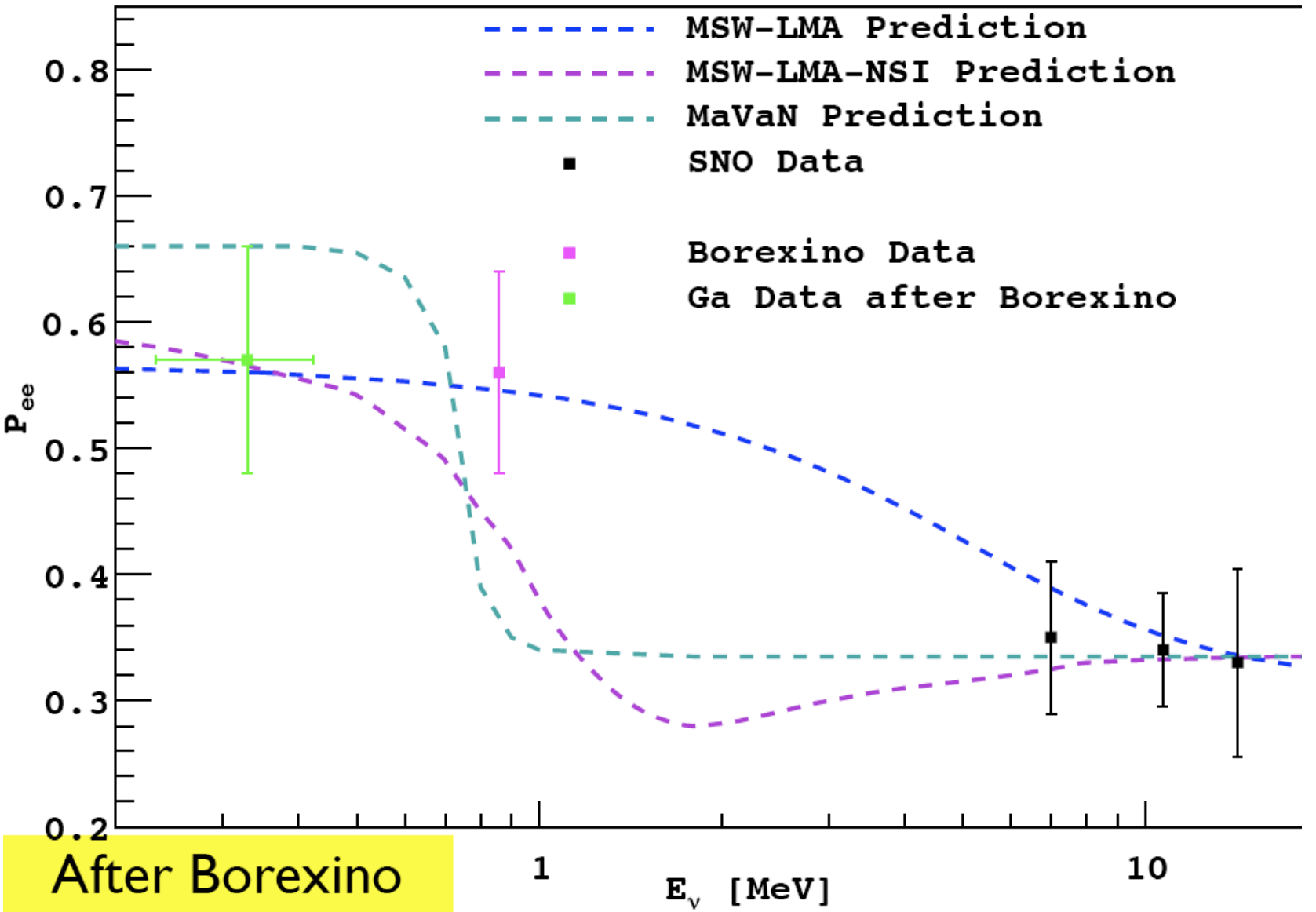
more complicated...

- $P_{ee} \sim 0.3$ (^{8}B neutrinos)
- $P_{ee} \sim 0.6$ (^{7}Be, pp neutrinos)

$\Rightarrow \quad \sin^2\theta \sim 0.3$

$\Rightarrow \quad \Delta m^2 \sim 10^{-(5 \text{ to } 4)}\ \text{eV}^2$

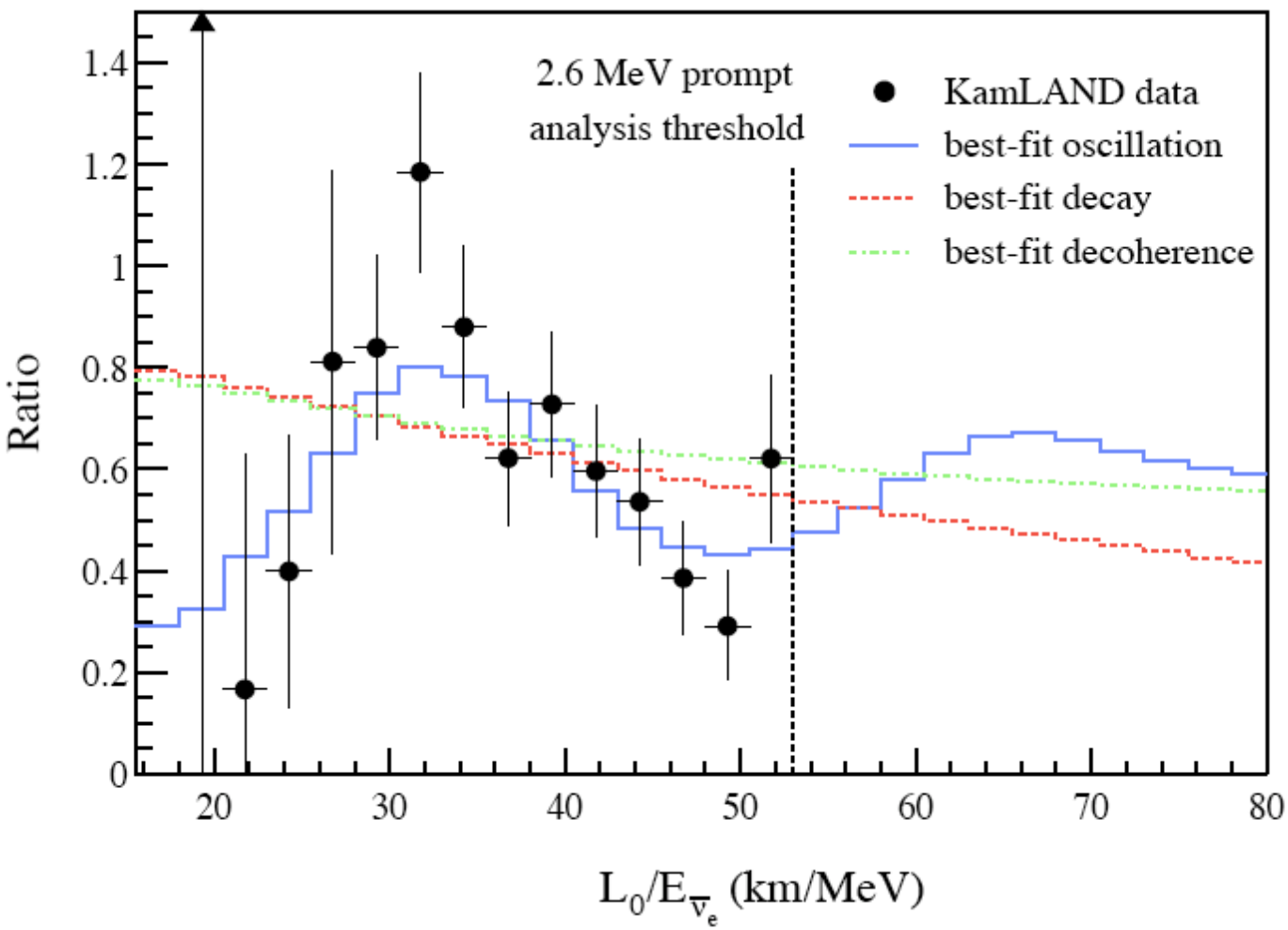
Solar Neutrino Survival Probability



After Borexino

Solar oscillations confirmed by Reactor experiment: KamLAND!!!

$$\text{phase} = 1.27 \left(\frac{\Delta m^2}{5\times10^{-5}\ \text{eV}^2}\right)\left(\frac{5\ \text{MeV}}{E}\right)\left(\frac{L}{100\ \text{km}}\right)$$



$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

oscillatory behavior!

Summarizing:

Both the solar and atmospheric puzzles can be properly explained in terms of two-flavor neutrino oscilations:

- **solar**: $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-4}$ eV^2, $\sin^2\theta \sim 0.3$.
- **atmospheric**: $\nu_\mu \leftrightarrow \nu_\tau$: $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin^2\theta \sim 0.5$ ("maximal mixing").

[Maltoni and Schwetz, arXiv: 0812.3161]

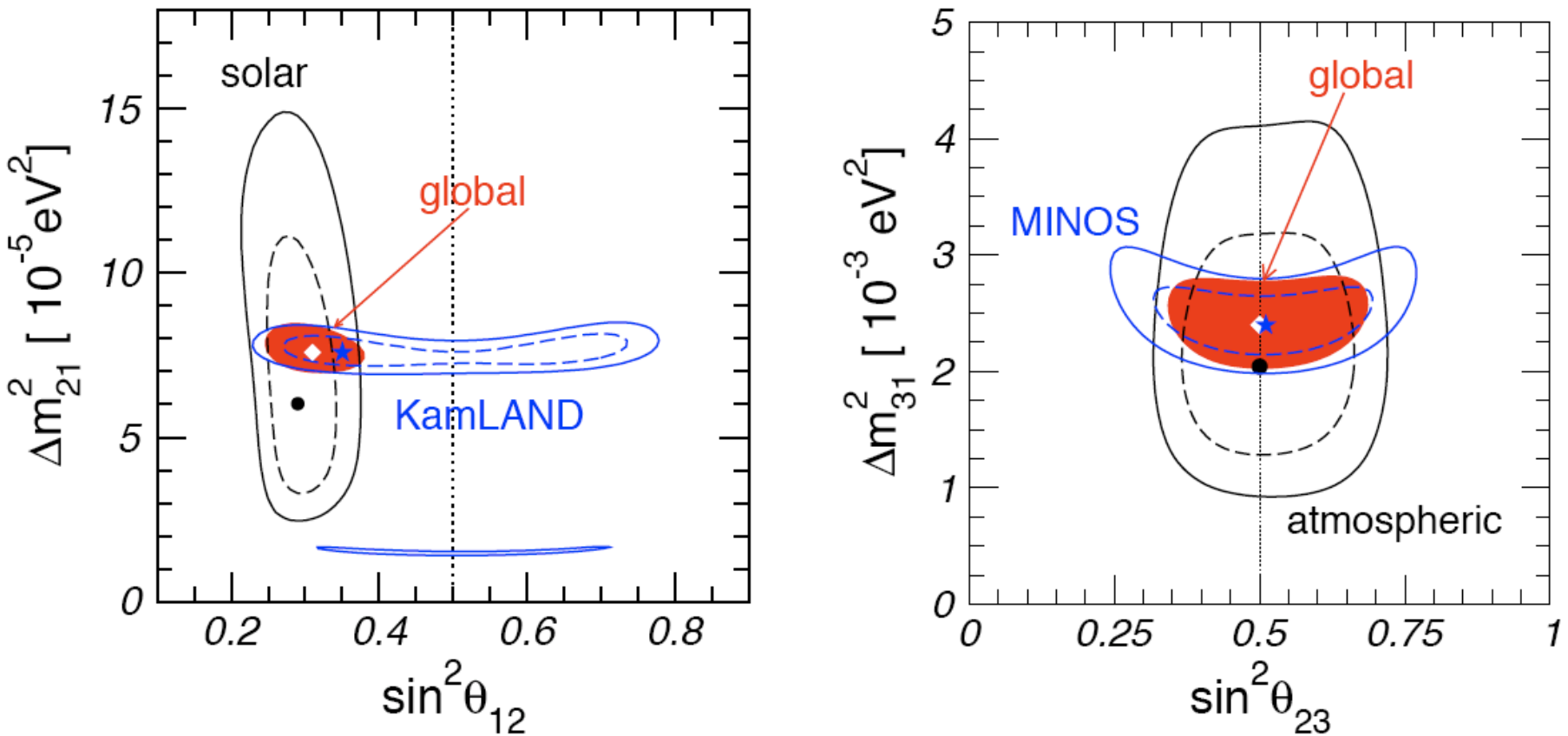


Figure 1: Determination of the leading "solar" and "atmospheric" oscillation parameters [1]. We show allowed regions at 90% and 99.73% CL (2 dof) for solar and KamLAND (left), and atmospheric and MINOS (right), as well as the 99.73% CL regions for the respective combined analyses.

Putting it all together – 3 flavor mixing:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Definition of neutrino mass eigenstates (who are ν_1, ν_2, ν_3?):

- $m_1^2 < m_2^2$ — $\Delta m_{13}^2 < 0$ – Inverted Mass Hierarchy
- $m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2|$ — $\Delta m_{13}^2 > 0$ – Normal Mass Hierarchy

$$\tan^2\theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2\theta_{23} \equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}; \quad U_{e3} \equiv \sin\theta_{13} e^{-i\delta}$$

[For a detailed discussion see AdG, Jenkins, PRD78, 053003 (2008)]

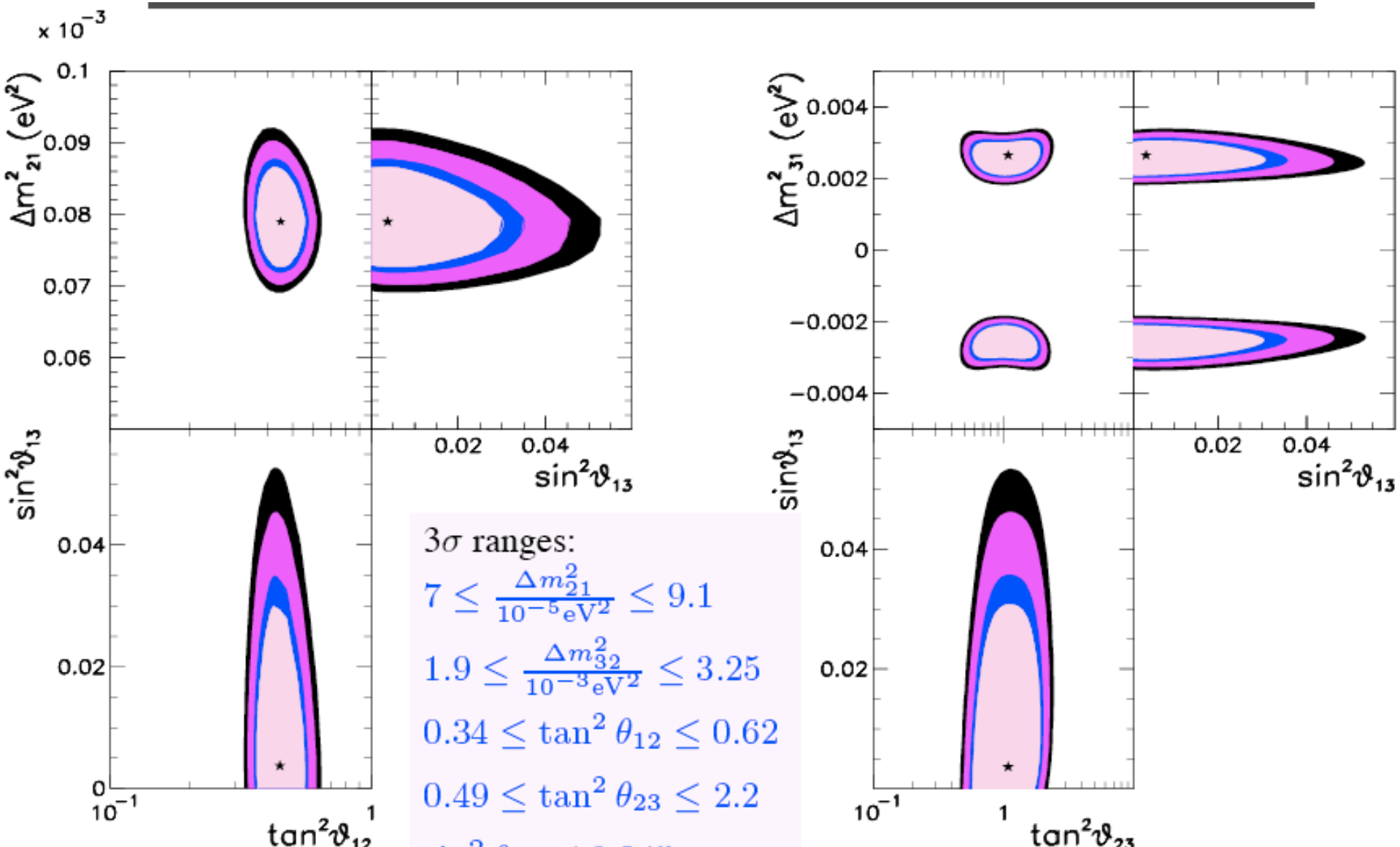
It Turns Out That ...

- Two Mass-Squared Differences Are Hierarchical, $\Delta m^2_{12} \ll |\Delta m^2_{13}|$;
- One of the Mixing Angles Is Small, $\sin^2\theta_{13} < 0.04$.

$\Rightarrow$ Two Puzzles Decouple, and Two-Flavor Interpretation Captures Almost All the Physics:

- Atmospheric Neutrinos Determine $|\Delta m^2_{13}|$ and θ_{23};
- Solar Neutrinos Determine Δm^2_{12} and θ_{12}.

(small θ_{13} guarantees that $|\Delta m^2_{13}|$ effects governing electron neutrinos are small, while $\Delta m^2_{12} \ll |\Delta m^2_{13}|$ guarantees that Δm^2_{12} effects are small at atmospheric and accelerator experiments).



3σ ranges:

$$7 \leq \frac{\Delta m^2_{21}}{10^{-5}\mathrm{eV}^2} \leq 9.1$$

$$1.9 \leq \frac{\Delta m^2_{32}}{10^{-3}\mathrm{eV}^2} \leq 3.25$$

$$0.34 \leq \tan^2\theta_{12} \leq 0.62$$

$$0.49 \leq \tan^2\theta_{23} \leq 2.2$$

$$\sin^2\theta_{13} \leq 0.045$$

$$-\pi \leq \delta \leq \pi$$

[Gonzalez-Garcia, PASI 2006]

Three Flavor Mixing Hypothesis Fits All Data Really Well.

⇒ Good Measurements of Oscillation Observables

parameter	Ref. [1] best fit$\pm 1\sigma$	Ref. [1] 3σ interval	Ref. [2] (MINOS updated) best fit$\pm 1\sigma$	Ref. [2] (MINOS updated) 3σ interval
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	$7.65^{+0.23}_{-0.20}$	7.05–8.34	$7.67^{+0.22}_{-0.21}$	7.07–8.34
Δm^2_{31} $[10^{-3}\mathrm{eV}^2]$	$\pm 2.40^{+0.12}_{-0.11}$	$\pm$(2.07–2.75)	-2.39 ± 0.12 $+2.49 \pm 0.12$	$-$(2.02–2.79) $+$(2.13–2.88)
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.25–0.37	$0.321^{+0.023}_{-0.022}$	0.26–0.40
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.36–0.67	$0.47^{+0.07}_{-0.06}$	0.33–0.64
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	≤ 0.056	0.003 ± 0.015	≤ 0.049

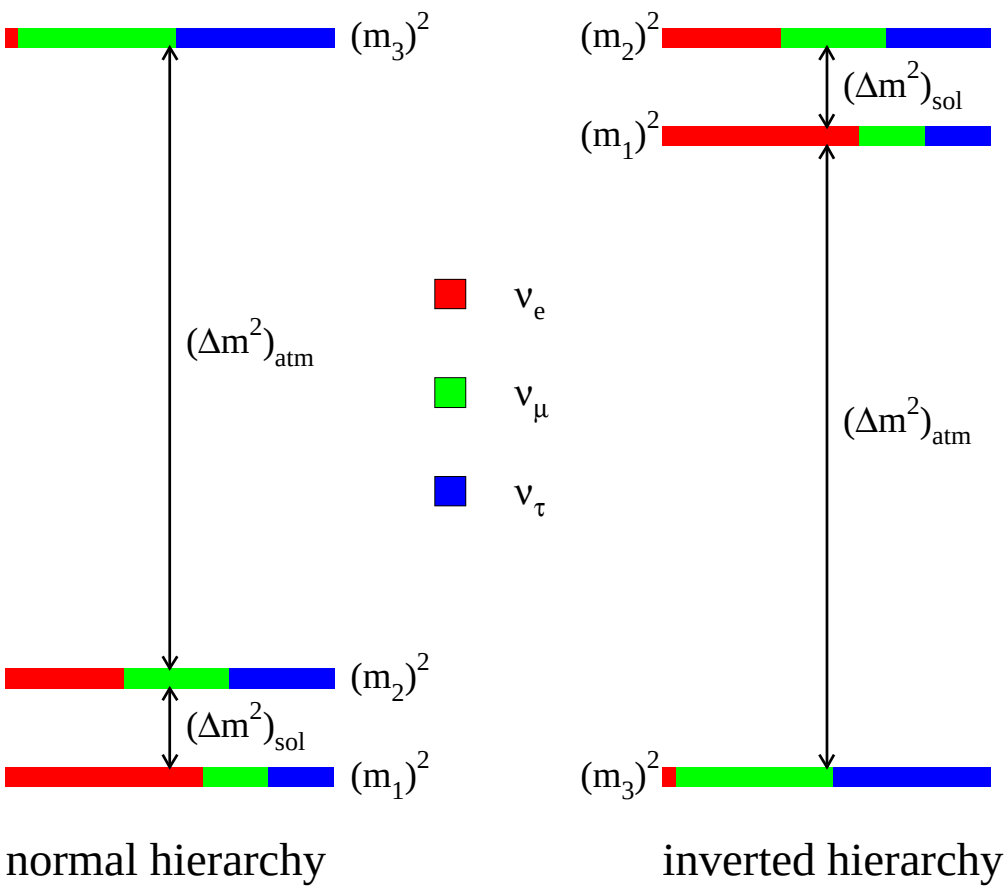
Table 1: Determination of three–flavour neutrino oscillation parameters from 2008 global data [1, 2].

[1] Schwetz, Tortola and Valle, arXiv:0808.2016

[2] Gonzalez-Garcia and Maltoni, arXiv:0704.1800

[Maltoni and Schwetz, arXiv: 0812.3161]

4– What We Know We Don't Know (i)



- What is the ν_e component of ν_3? ($\theta_{13} \neq 0$?)
- Is CP-invariance violated in neutrino oscillations? ($\delta \neq 0, \pi$?)
- Is ν_3 mostly ν_μ or ν_τ? ($\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, or $\theta_{23} = \pi/4$?)
- What is the neutrino mass hierarchy? ($\Delta m^2_{13} > 0$?)

$\Rightarrow$ All of the above can "only" be addressed with new neutrino oscillation experiments

Ultimate Goal: Not Measure Parameters but Test the Formalism (Over-Constrain Parameter Space)

Hunting For θ_{13} (or U_{e3})

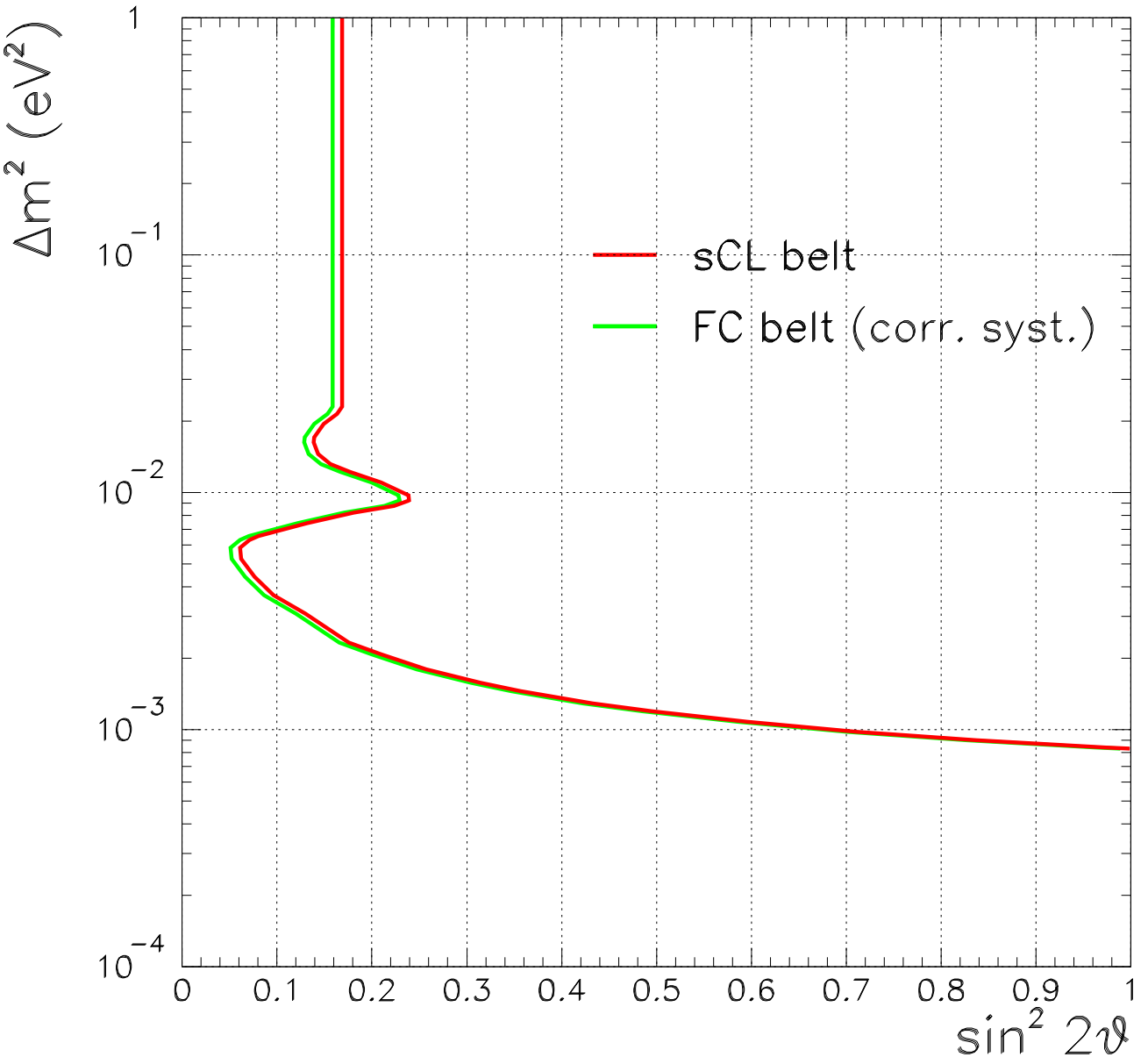
The best way to hunt for θ_{13} is to look for oscillation effects involving electron (anti)neutrinos, governed by the atmospheric oscillation frequency, Δm^2_{13} (other possibility, precision measurement of ν_μ disappearance...).

One way to understand this is to notice that if $\theta_{13} \equiv 0$, the ν_e state only participates in processes involving Δm^2_{12}.

Example:

$$P_{ee} \simeq 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m^2_{13} L}{4E} \right) + O \left(\frac{\Delta m^2_{12}}{\Delta m^2_{13}} \right)^2$$

Reactor Neutrino Searches for θ_{13}



- $L \sim 1$ km
- $E_\nu \sim 5$ MeV

next-generation: aim at improving CHOOZ bound by an order of magnitude.

[see Mike Shaevitz]

$\nu_\mu \leftrightarrow \nu_e$ at Long-Baseline Experiments

REQUIREMENTS: ν_μ beam, detector capable of seeing electron appearance. This is the case of "Superbeam Experiments" like T2K (2009) and NOνA (2012). Mike Shaevitz will discus these in detail.

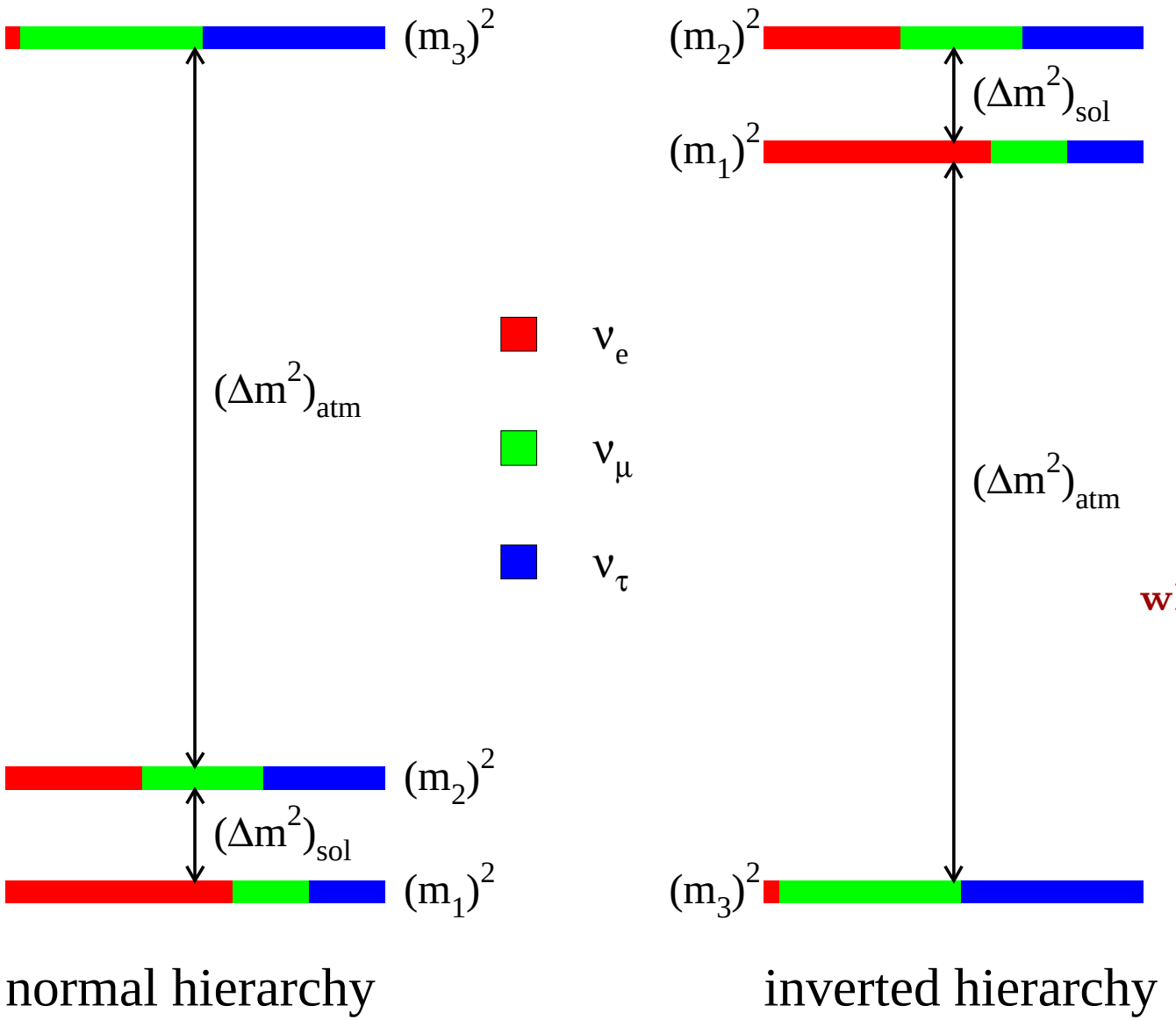
or

ν_e beam and detector capable of detecting muons (usually including sign). This would be the case of "Neutrino Factories" ($\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$) and "Beta Beams" ($Z \to (Z \pm 1) e^\mp \nu_e$).

In vaccum

$$P_{\mu e} = \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{13} L}{4E}\right) + \text{ "subleading"}.$$

• Sensitivity to $\sin^2\theta_{13}$. More precisely, $\sin^2\theta_{23}\sin^2 2\theta_{13}$. This leads to one potential degeneracy.



The Neutrino Mass Hierarchy

which is the right picture?

Why Don't We Know the Neutrino Mass Hierarchy?

Most of the information we have regarding θ_{23} and Δm^2_{13} comes from atmospheric neutrino experiments (SuperK). Roughly speaking, they measure

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m^2_{13} L}{4E} \right) + \text{ subleading.}$$

It is easy to see from the expression above that the leading term is simply not sensitive to the sign of Δm^2_{13}.

On the other hand, because $|U_{e3}|^2 < 0.05$ and $\frac{\Delta m^2_{12}}{\Delta m^2_{13}} < 0.06$ are both small, we are yet to observe the subleading effects.

Determining the Mass Hierarchy via Oscillations – the large U_{e3} route

Again, necessary to probe $\nu_\mu \to \nu_e$ oscillations (or vice-versa) governed by Δm^2_{13}. This is the oscillation channel that (almost) all next-generation, accelerator-based experiments are concentrating on, including the next generation experiments T2K and NOνA.

In vaccum

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{13}L}{4E}\right) + \text{ “subleading”},$$

so that, again, this is insensitive to the sign of Δm^2_{13} at leading order. However, in this case, matter effects may come to the rescue.

As I discussed already, neutrino oscillations get modified when these propagate in the presence of matter. Matter effects are sensitive to the neutrino mass ordering (in a way that I will describe shortly) and different for neutrinos and antineutrinos.

If $\Delta_{12} \equiv \frac{\Delta m^2_{12}}{2E}$ terms are ignored, the $\nu_\mu \to \nu_e$ oscillation probability is described, in constant matter density, by

$$P_{\mu e} \simeq P_{e\mu} \simeq \sin^2\theta_{23}\sin^2 2\theta_{13}^{\rm eff}\sin^2\left(\frac{\Delta_{13}^{\rm eff}L}{2}\right),$$

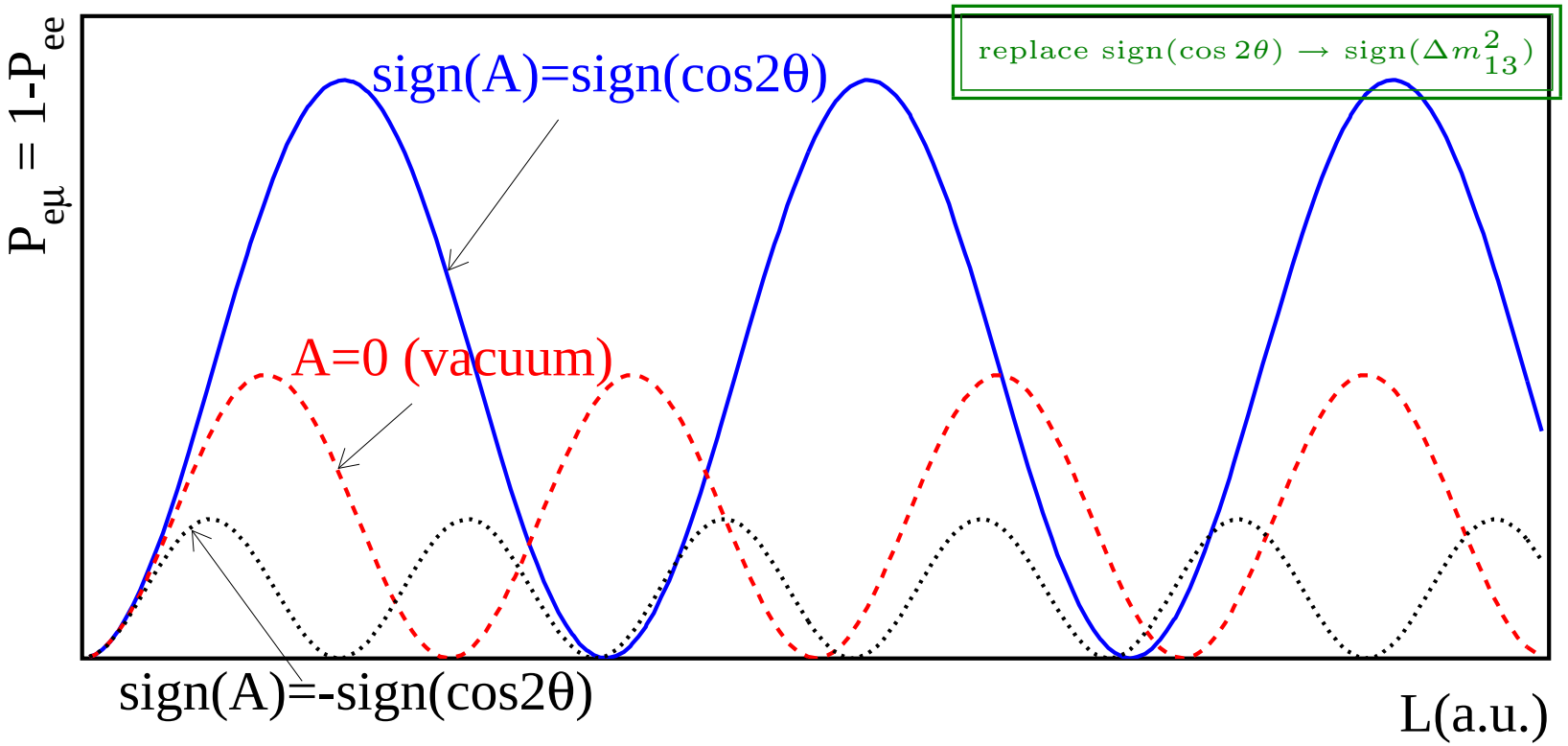
$$\sin^2 2\theta_{13}^{\rm eff} = \frac{\Delta_{13}^2\sin^2 2\theta_{13}}{(\Delta_{13}^{\rm eff})^2},$$

$$\Delta_{13}^{\rm eff} = \sqrt{(\Delta_{13}\cos 2\theta_{13} - A)^2 + \Delta_{13}^2\sin^2 2\theta_{13}},$$

$$\Delta_{13} = \frac{\Delta m^2_{13}}{2E},$$

$A \equiv \pm\sqrt{2}G_F N_e$ is the matter potential. It is positive for neutrinos and negative for antineutrinos.

$P_{\mu e}$ depends on the relative sign between Δ_{13} and A. It is different for the two different mass hierarchies, and different for neutrinos and antineutrinos.



Requirements:

- $\sin^2 2\theta_{13}$ large enough – otherwise there is nothing to see!
- $|\Delta_{13}| \sim |A|$ – matter potential must be significant but not overwhelming.
- $\Delta_{13}^{\rm eff} L$ large enough – matter effects are absent near the origin.

The "Holy Graill" of Neutrino Oscillations – CP Violation

In the old Standard Model, there is only one[a] source of CP-invariance violation:

$\Rightarrow$ The complex phase in V_{CKM}, the quark mixing matrix.

Indeed, as far as we have been able to test, all CP-invariance violating phenomena agree with the CKM paradigm:

- ϵ_K;
- ϵ'_K;
- $\sin 2\beta$;
- etc.

Recent experimental developments, however, provide strong reason to believe that this is not the case: neutrinos have mass, and leptons mix!

[a] modulo the QCD θ-parameter, which will be "willed away" henceforth.

CP-invariance Violation in Neutrino Oscillations

The most promising approach to studying CP-violation in the leptonic sector seems to be to compare $P(\nu_\mu \to \nu_e)$ versus $P(\bar{\nu}_\mu \to \bar{\nu}_e)$.

The amplitude for $\nu_\mu \to \nu_e$ transitions can be written as

$$A_{\mu e} = U_{e2}^* U_{\mu 2} \left(e^{i\Delta_{12}} - 1\right) + U_{e3}^* U_{\mu 3} \left(e^{i\Delta_{13}} - 1\right)$$

where $\Delta_{1i} = \frac{\Delta m_{1i}^2 L}{2E}$, $i = 2, 3$.

The amplitude for the CP-conjugate process can be written as

$$\bar{A}_{\mu e} = U_{e2} U_{\mu 2}^* \left(e^{i\Delta_{12}} - 1\right) + U_{e3} U_{\mu 3}^* \left(e^{i\Delta_{13}} - 1\right).$$

[remember: according to unitarty, $U_{e1} U_{\mu 1}^* = -U_{e2} U_{\mu 2}^* - U_{e3} U_{\mu 3}^*$]

In general, $|A|^2 \neq |\bar{A}|^2$ (CP-invariance violated) as long as:

- Nontrivial "Weak" Phases: $\arg(U_{ei}^* U_{\mu i}) \to \delta \neq 0, \pi$;
- Nontrivial "Strong" Phases: Δ_{12}, $\Delta_{13} \to L \neq 0$;
- Because of Unitarity, we need all $|U_{\alpha i}| \neq 0 \to$ three generations.

All of these can be satisfied, with a little luck: given that two of the three mixing angles are known to be large, **we need $|U_{e3}| \neq 0$.**

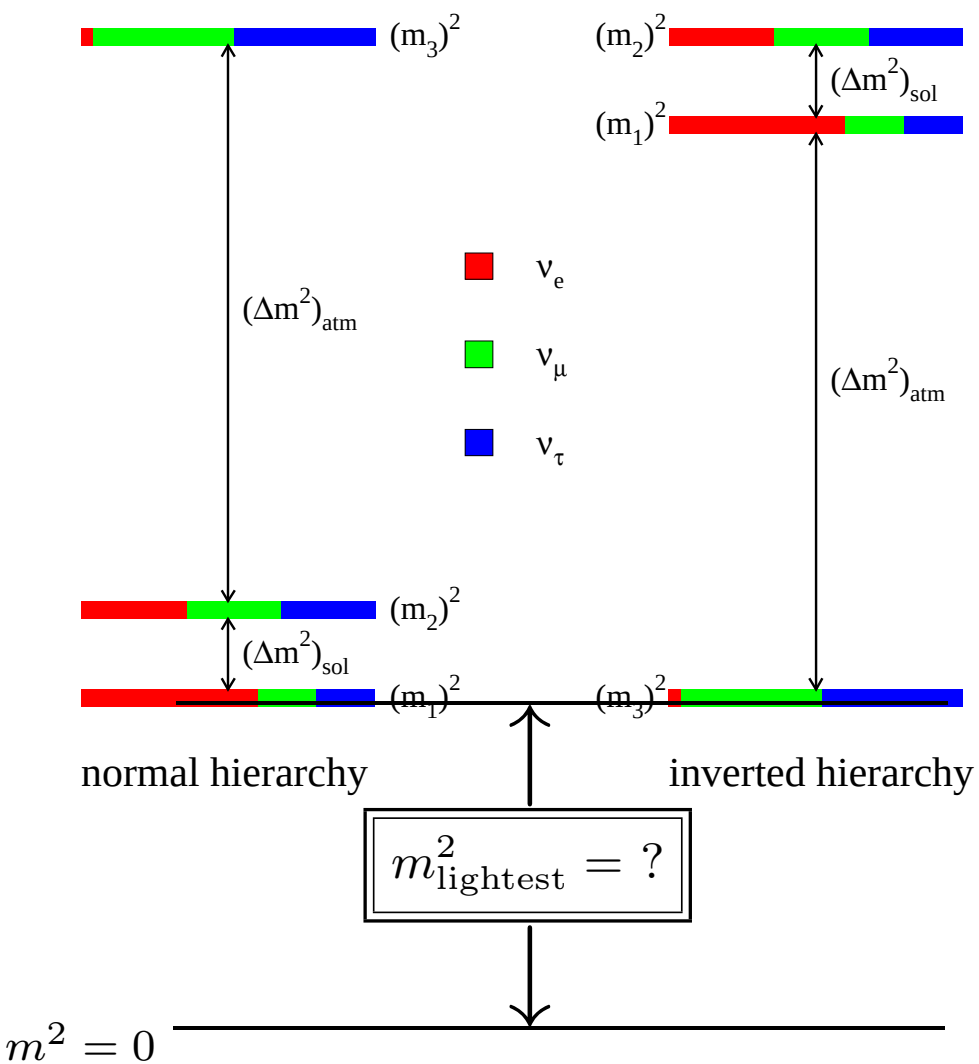
The goal of next-generation neutrino experiments is to determine the magnitude of $|U_{e3}|$. We need to know this in order to understand how to study CP-invariance violation in neutrino oscillations!

In the real world, life is much more complicated. The lack of knowledge concerning the mass hierarchy, θ_{13}, θ_{23} leads to several degeneracies.

Note that, in order to see CP-invariance violation, we **need** the "subleading" terms!

In order to ultimately measure a new source of CP-invariance violation, we will need to combine different measurements:
– oscillation of muon neutrinos and antineutrinos,
– oscillations at accelerator and reactor experiments,
– experiments with different baselines,
– etc.

4- What We Know We Don't Know (ii): How Light is the Lightest Neutrino?



So far, we've only been able to measure neutrino mass-squared differences.

The lightest neutrino mass is only poorly constrained: $m^2_{\rm lightest} < 1$ eV^2

qualitatively different scenarios allowed:

- $m^2_{\rm lightest} \equiv 0$;
- $m^2_{\rm lightest} \ll \Delta m^2_{12,13}$;
- $m^2_{\rm lightest} \gg \Delta m^2_{12,13}$.

Need information outside of neutrino oscillations.

The most direct probe of the lightest neutrino mass – precision measurements of β-decay

Observation of the effect of non-zero neutrino masses **kinematically**.

When a neutrino is produced, some of the energy exchanged in the process should be spent by the non-zero neutrino mass.

Typical effects are very, very small – we've never seen them! The most sensitive observable is the electron energy spectrum from tritium decay.

$$^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^- + \bar{\nu}$$

Why tritium? Small Q value, reasonable abundances. Required sensitivity proportional to m^2/Q^2.

In practice, this decay is sensitive to an effective "electron neutrino mass":

$$m_{\nu_e}^2 \equiv \sum_i |U_{ei}|^2 m_i^2$$

Experiments measure the **shape** of the end-point of the spectrum, not the value of the end point. This is done by counting events as a function of a low-energy cut-off. note: LOTS of Statistics Needed!

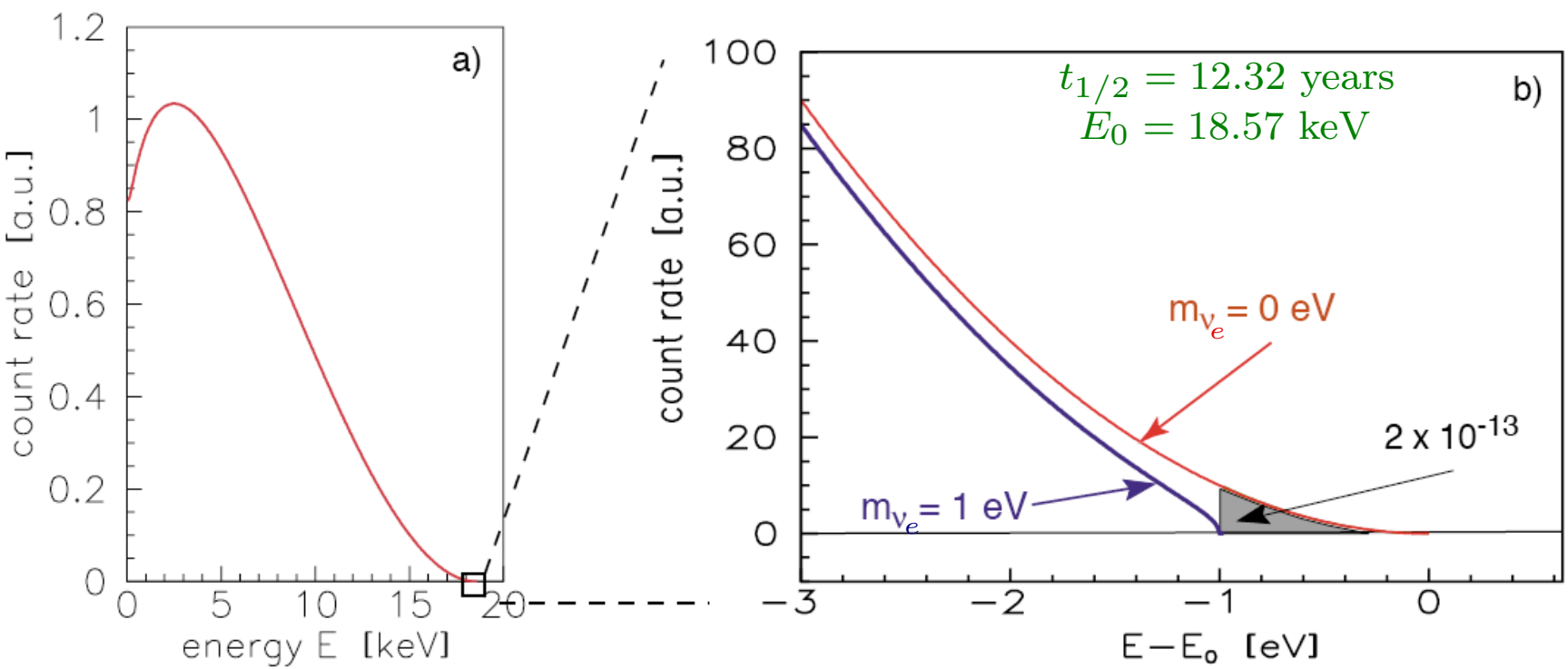


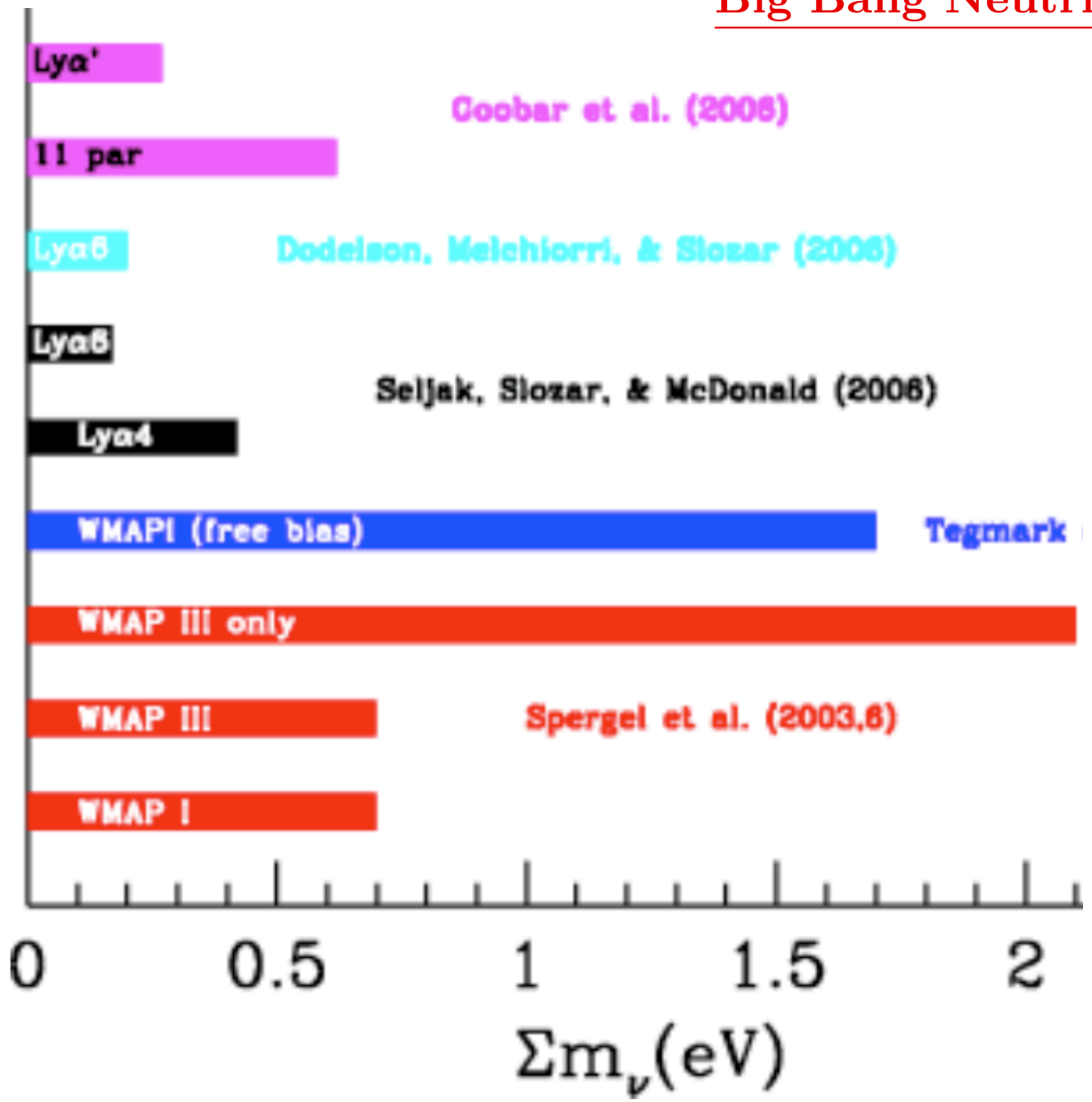
Figure 2: The electron energy spectrum of tritium β decay: (a) complete and (b) narrow region around endpoint E_0. The β spectrum is shown for neutrino masses of 0 and 1 eV.

NEXT GENERATION: The Karlsruhe Tritium Neutrino (KATRIN) Experiment:

(not your grandmother's table top experiment!)

sensitivity $m^2_{\nu_e} > (0.2\ \text{eV})^2$

Big Bang Neutrinos are Warm Dark Matter



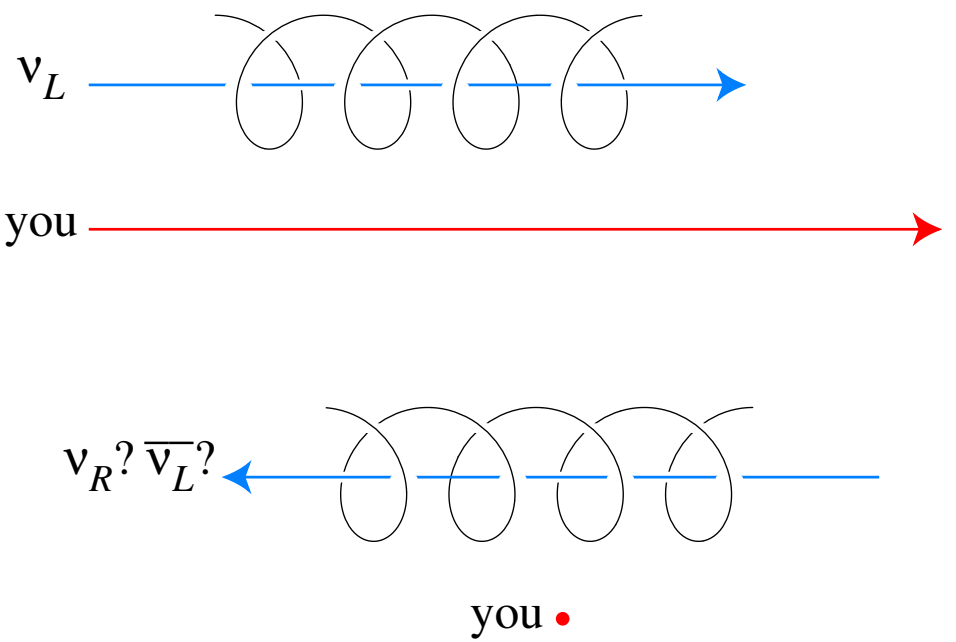
- Constrained by the Large Scale Structure of the Universe.

Constraints depend on

- Data set analysed;
- "Bias" on other parameters;
- ...

Bounds can be evaded with non-standard cosmology. Will we learn about neutrinos from cosmology or about cosmology from neutrinos?

5– What We Know We Don't Know (iii) – Are Neutrinos Majorana Fermions?



A massive charged fermion (s=1/2) is described by 4 degrees of freedom:

$$(e_L^- \leftarrow \text{CPT} \rightarrow e_R^+)$$
$$\updownarrow \text{Lorentz}$$
$$(e_R^- \leftarrow \text{CPT} \rightarrow e_L^+)$$

A massive neutral fermion (s=1/2) is described by 4 or 2 degrees of freedom:

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\updownarrow \text{Lorentz} \qquad \text{“DIRAC”}$$
$$(\nu_R \leftarrow \text{CPT} \rightarrow \bar{\nu}_L)$$

How many degrees of freedom are required to describe massive neutrinos?

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\text{“MAJORANA”} \qquad \updownarrow \text{Lorentz}$$
$$(\bar{\nu}_R \leftarrow \text{CPT} \rightarrow \nu_L)$$

Why Don't We Know the Answer (Yet)?

If neutrino masses were indeed zero, this is a nonquestion: there is no distinction between a massless Dirac and Majorana fermion.

Processes that are proportional to the Majorana nature of the neutrino vanish in the limit $m_\nu \to 0$. Since neutrinos masses are very small, the probability for these to happen is very, very small: $A \propto m_\nu/E$.

The "smoking gun" signature is the observation of LEPTON NUMBER violation. This is easy to understand: Majorana neutrinos are their own antiparticles and, therefore, cannot carry any quantum numbers — including lepton number.

Weak Interactions are Purely Left-Handed (Chirality):

For example, in the scattering process $e^- + X \to \nu_e + X$, the electron neutrino is, in a reference frame where $m \ll E$,

$$|\nu_e\rangle \sim |L\rangle + \left(\frac{m}{E}\right)|R\rangle.$$

If the neutrino is a Majorana fermion, $|R\rangle$ behaves mostly like a "$\bar{\nu}_e$," (and $|L\rangle$ mostly like a "ν_e,") such that the following process could happen:

$$e^- + X \to \nu_e + X, \quad \text{followed by} \quad \nu_e + X \to e^+ + X, \qquad P \simeq \left(\frac{m}{E}\right)^2$$

Lepton number can be violated by 2 units with small probability. Typical numbers: $P \simeq (0.1\ \text{eV}/100\ \text{MeV})^2 = 10^{-18}$. VERY Challenging!

How many new CP-violating parameters in the neutrino sector?

If the neutrinos are Majorana fermions, there are more physical observables in the leptonic mixing matrix.

Remember the parameter counting in the quark sector:

$$\begin{array}{rl} 9 & (3 \times 3 \text{ unitary matrix}) \\ \underline{-5} & (\text{relative phase rotation among six quark fields}) \\ 4 & (\text{3 mixing angles and 1 CP-odd phase}). \end{array}$$

If the neutrinos are Majorana fermions, the parameter counting is quite different: there are no right-handed neutrino fields to "absorb" CP-odd phases:

$$\begin{array}{rl} 9 & (3 \times 3 \text{ unitary matrix}) \\ \underline{-3} & (\text{three right-handed charged lepton fields}) \\ 6 & (\text{3 mixing angles and 3 CP-odd phases}). \end{array}$$

There is CP-invariance violating parameters even in the 2 family case: $4 - 2 = 2$, one mixing angle, one CP-odd phase.

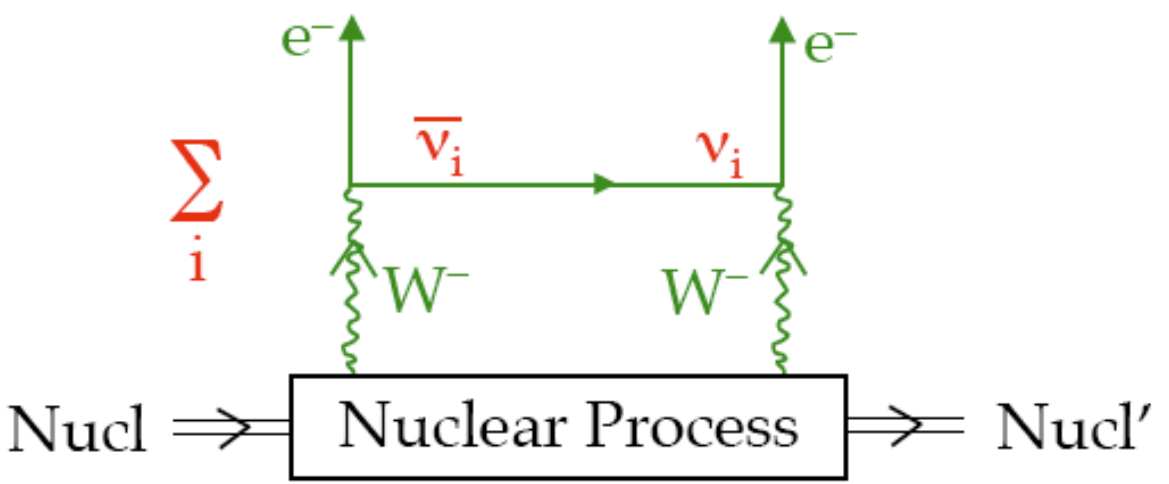
$$V_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & e^{i\alpha_3/2} \end{pmatrix}.$$

It is easy to see that the Majorana phases α_i never show up in neutrino oscillations ($A \propto U_{\alpha i} U^*_{\beta_i}$). Also, only phase differences are physical – overall phase can be ignored.

Furthermore, they only manifest themselves in phenomena that vanish in the limit $m_i \to 0$ – after all they are only physical if we "know" that lepton number is broken.

$$A(\alpha_i) \propto m_i/E \quad \to \quad \text{tiny!}$$

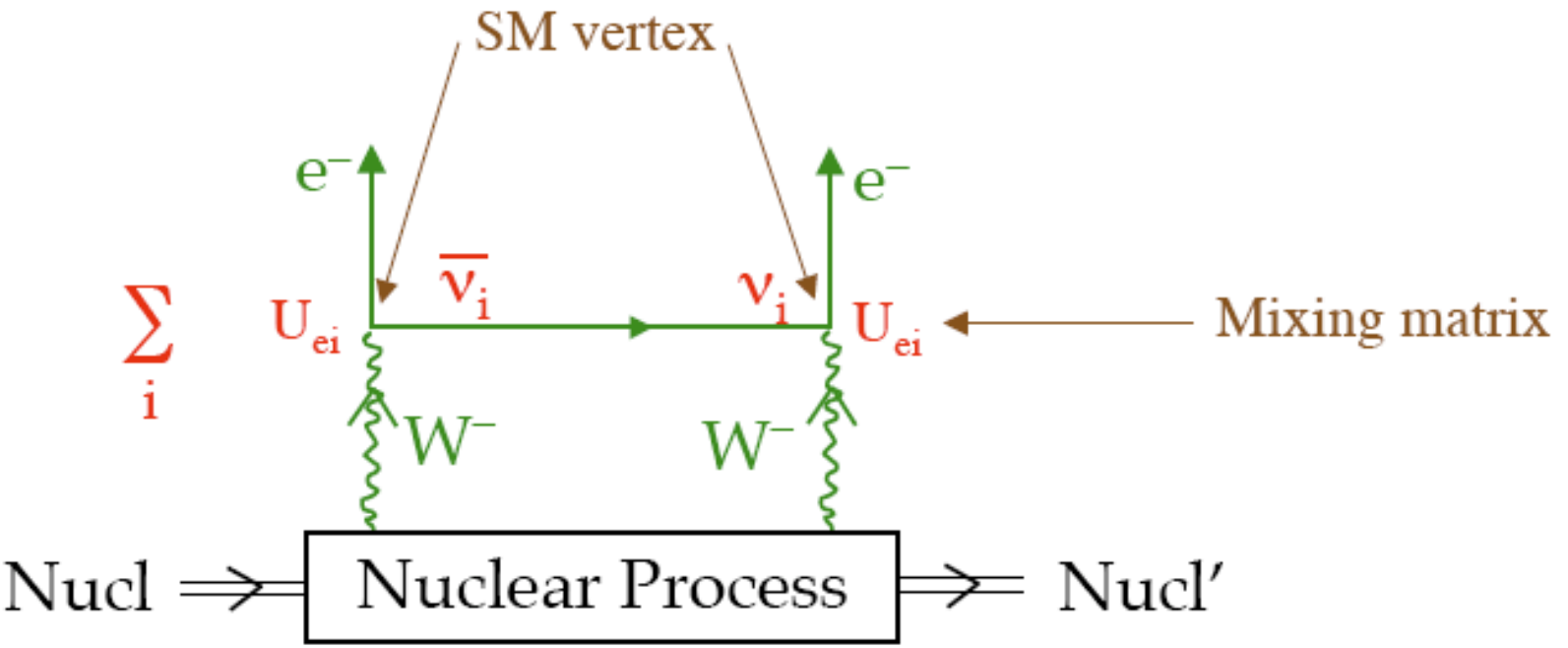
Neutrinoless Double Beta Decay [$0\nu\beta\beta$]



If we start with ***a lot*** of parent nuclei (say, one ton of them), we can cope with the small neutrino masses.

Observation would imply $\not{L}$ and $\overline{\nu}_i = \nu_i$.

In —

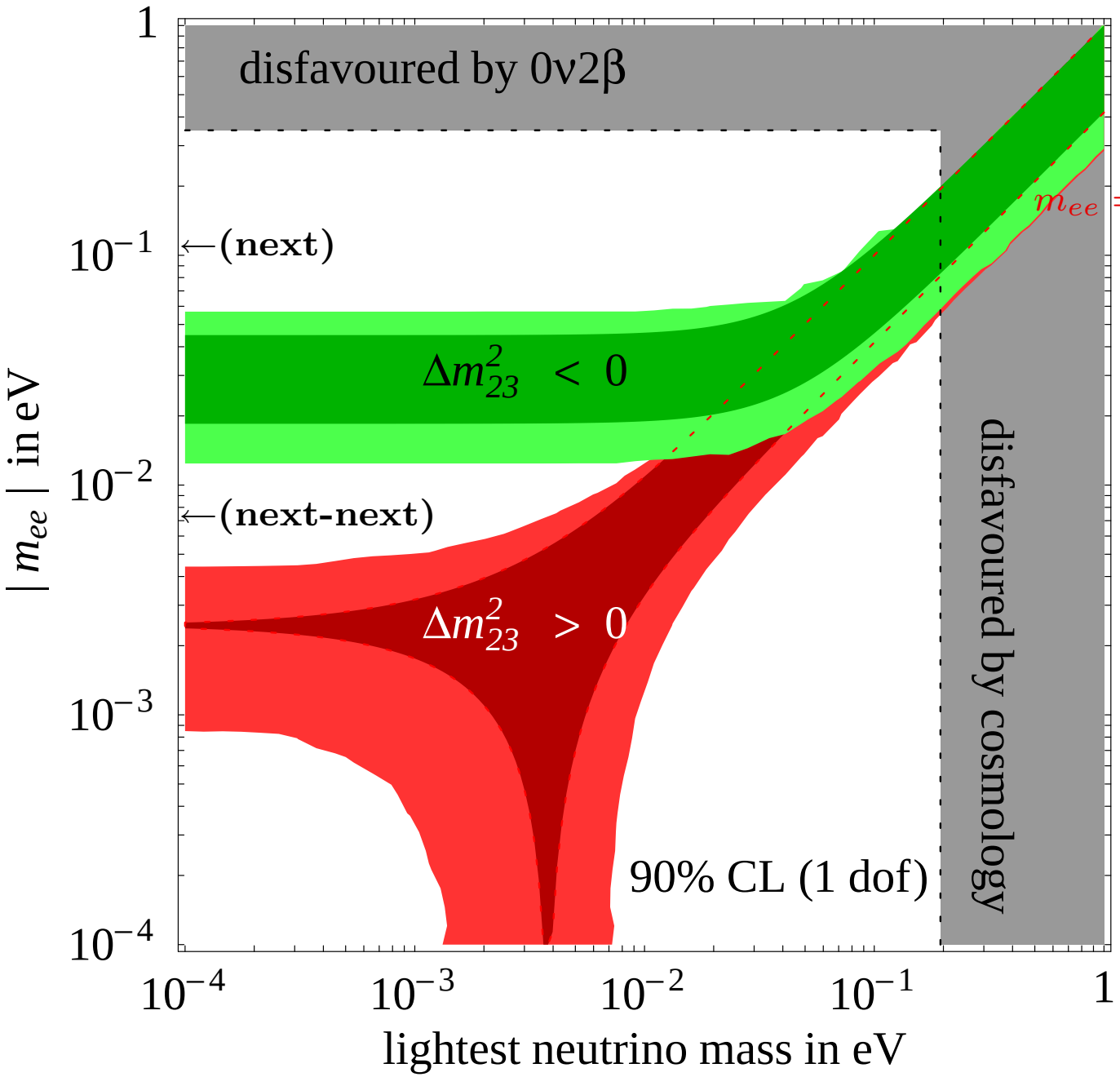


Mass (ν_i)

the $\bar{\nu}_i$ is emitted [RH + O{m_i/E}LH].

Thus, Amp [ν_i contribution] $\propto m_i$

$$\mathrm{Amp}[0\nu\beta\beta] \propto \left|\sum_i m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$



[U^2_{ei} complex numbers]

↓

$$m_{ee} = U^2_{e1} m_1 + U^2_{e2} m_2 + U^2_{e3} m_3$$

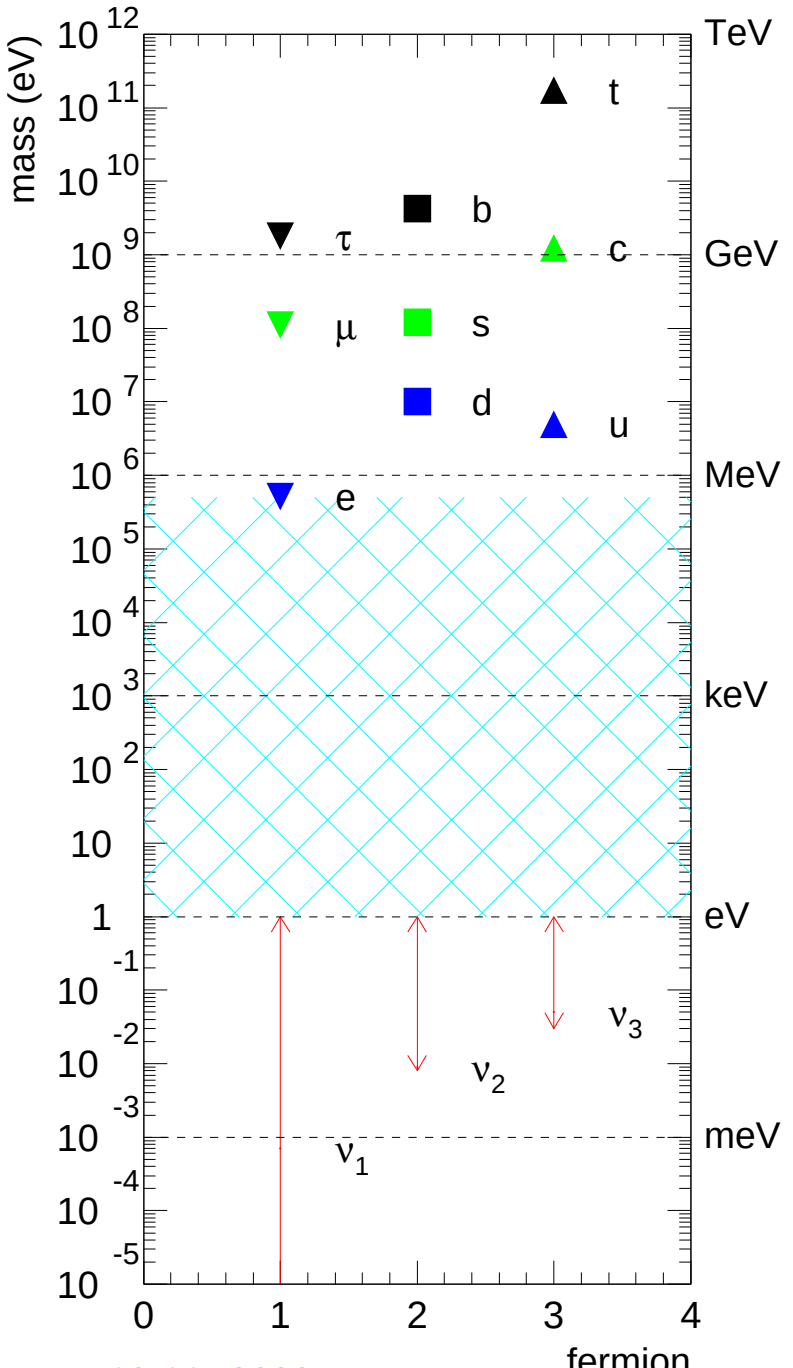
Depends on Majorana Phases

e.g. inverted hierarchy:

$$m_3 \ll m_1 \sim m_2 \sim \sqrt{\Delta m^2_{13}},$$

$$m_{ee} \sim \sqrt{\Delta m^2_{13}} \times \times \left(\cos^2\theta_{12} + e^{i\alpha}\sin^2\theta_{12}\right).$$

$$m_{ee} > \sqrt{\Delta m^2_{13}}\cos 2\theta_{12}$$



NEUTRINOS HAVE MASS

[albeit very tiny ones...]

So What?

Who Cares About Neutrino Masses: "Palpable" Evidence of Physics Beyond the Standard Model*

The SM we all learned in school predicts that neutrinos are strictly massless. Massive neutrinos imply that the the SM is incomplete and needs to be replaced/modified.

Furthermore, the SM has to be replaced by something qualitatively different.

---

* There is only a handful of questions our model for fundamental physics cannot explain properly. These are in order of palpabiloity (these are personal. Feel free to complain)

- What is the physics behind electroweak symmetry breaking? (Higgs *or* not in SM).
- What is the dark matter? (not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (not in SM – Is this "particle physics?").

Standard Model in One Slide, No Equations

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group ($SU(3)_c \times SU(2)_{\rm L} \times U(1)_{\rm Y}$);
- Particle Content (fermions: Q, u, d, L, e, scalars: H).

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done! (after several decades of hard experimental work...)

If you follow these rules, neutrinos have no mass. Something has to give.

What is the New Standard Model? [νSM]

The short answer is – WE DON'T KNOW. Not enough available info!

$$\Updownarrow$$

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the νSM candidates can do. [are they falsifiable?, are they "simple"?, do they address other outstanding problems in physics?, etc]

We need more experimental input, and it looks like it may be coming in the near/intermediate future!

νSM – One Possibility

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu\mathrm{SM}} \supset -y_{ij}\frac{L^i H L^j H}{2\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) + H.c.$$

There is only one dimension five operator [Weinberg, 1979]. If $\Lambda \gg 1$ TeV, it leads to only one observable consequence...

$$\text{after EWSB } \mathcal{L}_{\nu\mathrm{SM}} \supset \frac{m_{ij}}{2}\nu^i\nu^j; \quad m_{ij} = y_{ij}\frac{v^2}{\Lambda}.$$

- Neutrino masses are small: $\Lambda \gg v \to m_\nu \ll m_f$ ($f = e, \mu, u, d$, etc)
- Neutrinos are Majorana fermions – Lepton number is violated!
- νSM effective theory – not valid for energies above at most M.
- What is Λ? First naive guess is that Λ is the Planck scale – does not work. Data require $M \sim 10^{14}$ GeV (related to GUT scale?) [note $y^{\max} \equiv 1$]

What else is this "good for"? Depends on the ultraviolet completion!

[Talks by E. Ma, G. Senjanovic]

Note that this VERY similar to the "discovery" weak interactions. Imagine the following model:

$U(1)_{E\&M} \; + \; e(q=-1), \; \mu(q=-1), \; \nu_e(q=0), \; \nu_\mu(q=0).$

The most general renormalizable Lagrangian explains *all* QED phenomena once all couplings are known (α, m_f).

New physics: the muon decays! $\mu^- \to e^- \bar{\nu}_e \nu_\mu$. This can be interpreted as evidence of effective four fermion theory (nonrenormalizable operators):

$$-\frac{4G_F}{\sqrt{2}} \sum_\gamma g_\gamma \left(\bar{e}\Gamma^\gamma \nu\right)\left(\bar{\nu}\Gamma_\gamma \mu\right), \qquad \Gamma_\gamma = 1, \gamma_5, \gamma_\mu, \ldots$$

Prediction: will discover new physics at an energy scale **below** $\sqrt{1/G_F} \simeq 250$ GeV. We know how this turned out $\Rightarrow$ $W^\pm, Z^0$ discovered slightly below 100 GeV!

The Seesaw Lagrangian

A simple[a], renormalizable Lagrangian that allows for neutrino masses is

$$\mathcal{L}_\nu = \mathcal{L}_{\rm old} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^{3} \frac{M_i}{2} N^i N^i + H.c.,$$

where N_i ($i = 1, 2, 3$, for concreteness) are SM gauge singlet fermions. $\mathcal{L}_\nu$ is the most general, renormalizable Lagrangian consistent with the SM gauge group and particle content, plus the addition of the N_i fields.

After electroweak symmetry breaking, $\mathcal{L}_\nu$ describes, besides all other SM degrees of freedom, six Majorana fermions: **six neutrinos.**

[a] Only requires the introduction of three fermionic degrees of freedom, no new interactions or symmetries.

To be determined from data: λ and M.

The data can be summarized as follows: there is evidence for three neutrinos, mostly "active" (linear combinations of ν_e, ν_μ, and ν_τ). At least two of them are massive and, if there are other neutrinos, they have to be "sterile."

This provides very little information concerning the magnitude of M_i (assume $M_1 \sim M_2 \sim M_3$)

Theoretically, there is prejudice in favor of very large M: $M \gg v$. Popular examples include $M \sim M_{\rm GUT}$ (GUT scale), or $M \sim 1$ TeV (EWSB scale).

Furthermore, $\lambda \sim 1$ translates into $M \sim 10^{14}$ GeV, while thermal leptogenesis requires the lightest M_i to be larger than 10^9 GeV.

we can impose very, very few experimental constraints on M

What We Know About M:

- $M = 0$: the six neutrinos "fuse" into three Dirac states. Neutrino mass matrix given by $\mu_{\alpha i} \equiv \lambda_{\alpha i} v$.

  The symmetry of $\mathcal{L}_\nu$ is enhanced: $U(1)_{B-L}$ is an exact global symmetry of the Lagrangian if all M_i vanish. Small M_i values are 'tHooft natural.

- $M \gg \mu$: the six neutrinos split up into three mostly active, light ones, and three, mostly sterile, heavy ones. The light neutrino mass matrix is given by $m_{\alpha\beta} = \sum_i \mu_{\alpha i} M_i^{-1} \mu_{\beta i}$ $\qquad [m \propto 1/\Lambda \;\Rightarrow\; \Lambda = M/\mu^2]$.

  This the **seesaw mechanism.** Neutrinos are Majorana fermions. Lepton number is not a good symmetry of $\mathcal{L}_\nu$, even though L-violating effects are hard to come by.

- $M \sim \mu$: six states have similar masses. Active–sterile mixing is very large. This scenario is (generically) ruled out by active neutrino data (atmospheric, solar, KamLAND, K2K, etc).

[Aside: Why are Neutrino Masses Small in the $M \neq 0$ Case?]

If $\mu \ll M$, below the mass scale M,

$$\mathcal{L}_5 = \frac{LHLH}{\Lambda}.$$

Neutrino masses are small if $\Lambda \gg \langle H \rangle$. Data require $\Lambda \sim 10^{14}$ GeV.

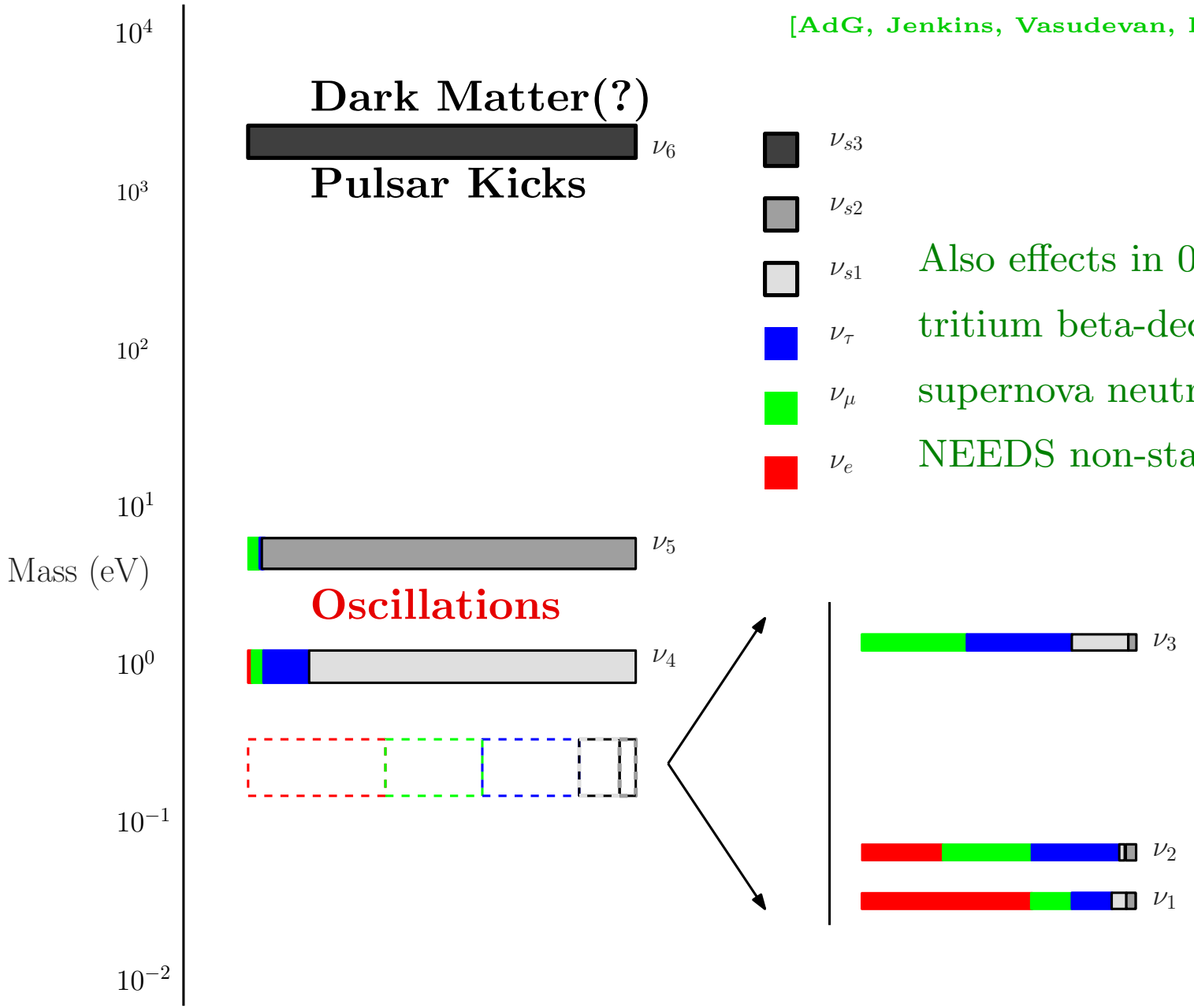
In the case of the seesaw,

$$\Lambda \sim \frac{M}{\lambda^2},$$

so neutrino masses are small if either

- they are generated by physics at a very high energy scale $M \gg v$ (high-energy seesaw); or
- they arise out of a very weak coupling between the SM and a new, hidden sector (low-energy seesaw); or
- cancellations among different contributions render neutrino masses accidentally small ("fine-tuning").

[AdG, Jenkins, Vasudevan, PRD75, 013003 (2007)]



Also effects in $0\nu\beta\beta$,

tritium beta-decay,

supernova neutrino oscillations,

NEEDS non-standard cosmology.

Predictions: **Neutrinoless Double-Beta Decay**

The exchange of Majorana neutrinos mediates lepton-number violating neutrinoless double-beta decay, $0\nu\beta\beta$: $Z \to (Z+2)e^- e^-$.

For light enough neutrinos, the amplitude for $0\nu\beta\beta$ is proportional to the effective neutrino mass

$$m_{ee} = \left| \sum_{i=1}^{6} U_{ei}^2 m_i \right| \sim \left| \sum_{i=1}^{3} U_{ei}^2 m_i + \sum_{i=1}^{3} \vartheta_{ei}^2 M_i \right| .$$

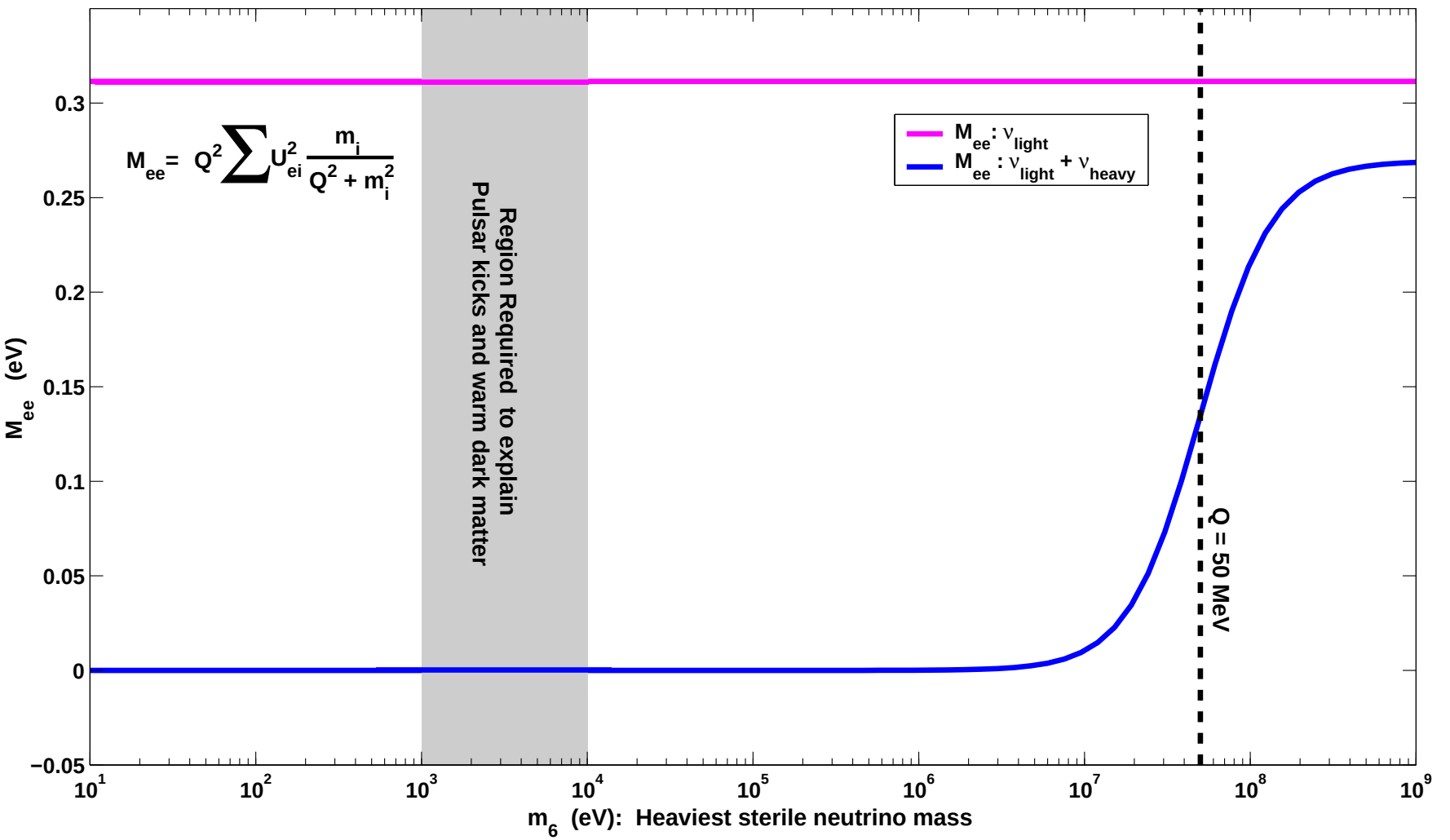
However, upon further examination, $m_{ee} = 0$ in the eV-seesaw. **The contribution of light and heavy neutrinos exactly cancels!** This seems to remain true to a good approximation as long as $M_i \ll 1$ MeV.

$$\left[\mathcal{M} = \begin{pmatrix} 0 & \mu^{\mathrm{T}} \\ \mu & M \end{pmatrix} \quad \to \quad m_{ee} \text{ is identically zero!} \right]$$

[AdG PRD 72, 033005 (2005)]

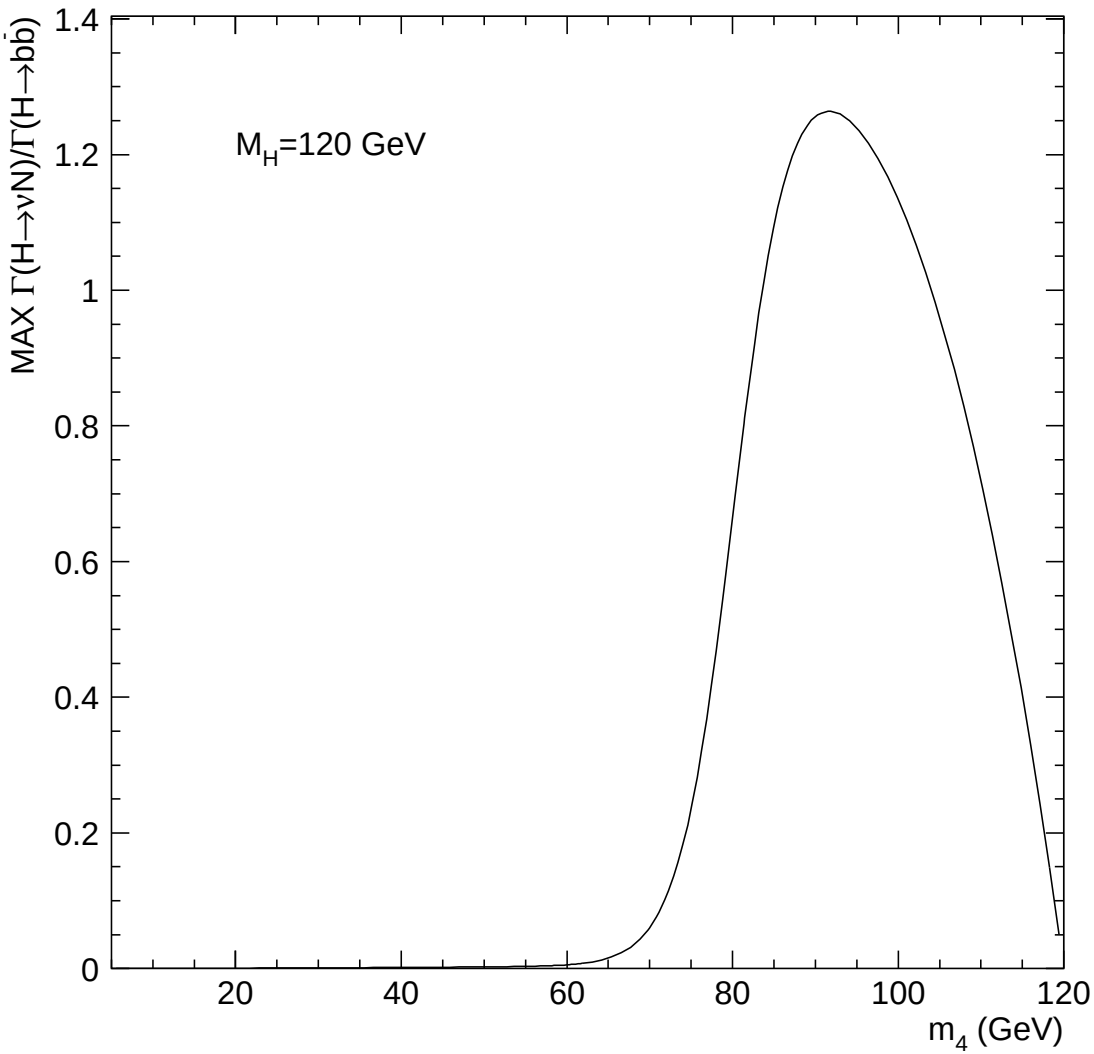
(lack of) sensitivity in $0\nu\beta\beta$ due to seesaw sterile neutrinos

[AdG, Jenkins, Vasudevan, hep-ph/0608147]



Weak Scale Seesaw, and Accidentally Light Neutrino Masses

[AdG arXiv:0706.1732 [hep-ph]]



What does the seesaw Lagrangian predict for the LHC?

Nothing much, unless...

- $M_N \sim 1 - 100$ GeV,
- Yukawa couplings larger than naive expectations.

$\Leftarrow$ $H \to \nu N$ as likely as $H \to b\bar{b}$!

(NOTE: $N \to \ell q' \bar{q}$ or $\ell \ell' \nu$ (prompt)

"Weird" Higgs decay signature!)

ALSO: "Majorana neutrinos at the LHC,"

see talks by Goran, Ernest

Fourth Avenue: Higher Order Neutrino Masses from $\Delta L = 2$ Physics.

Imagine that there is new physics that breaks lepton number by 2 units at some energy scale Λ, but that it does not, in general, lead to neutrino masses at the tree level.

We know that neutrinos will get a mass at some order in perturbation theory – which order is model dependent!

For example:

- SUSY with trilinear R-parity violation – neutrino masses at one-loop;
- Zee model – neutrino masses at one-loop;
- arXiv:0706.1964 and many others – neutrino masses at two loops;
- etc

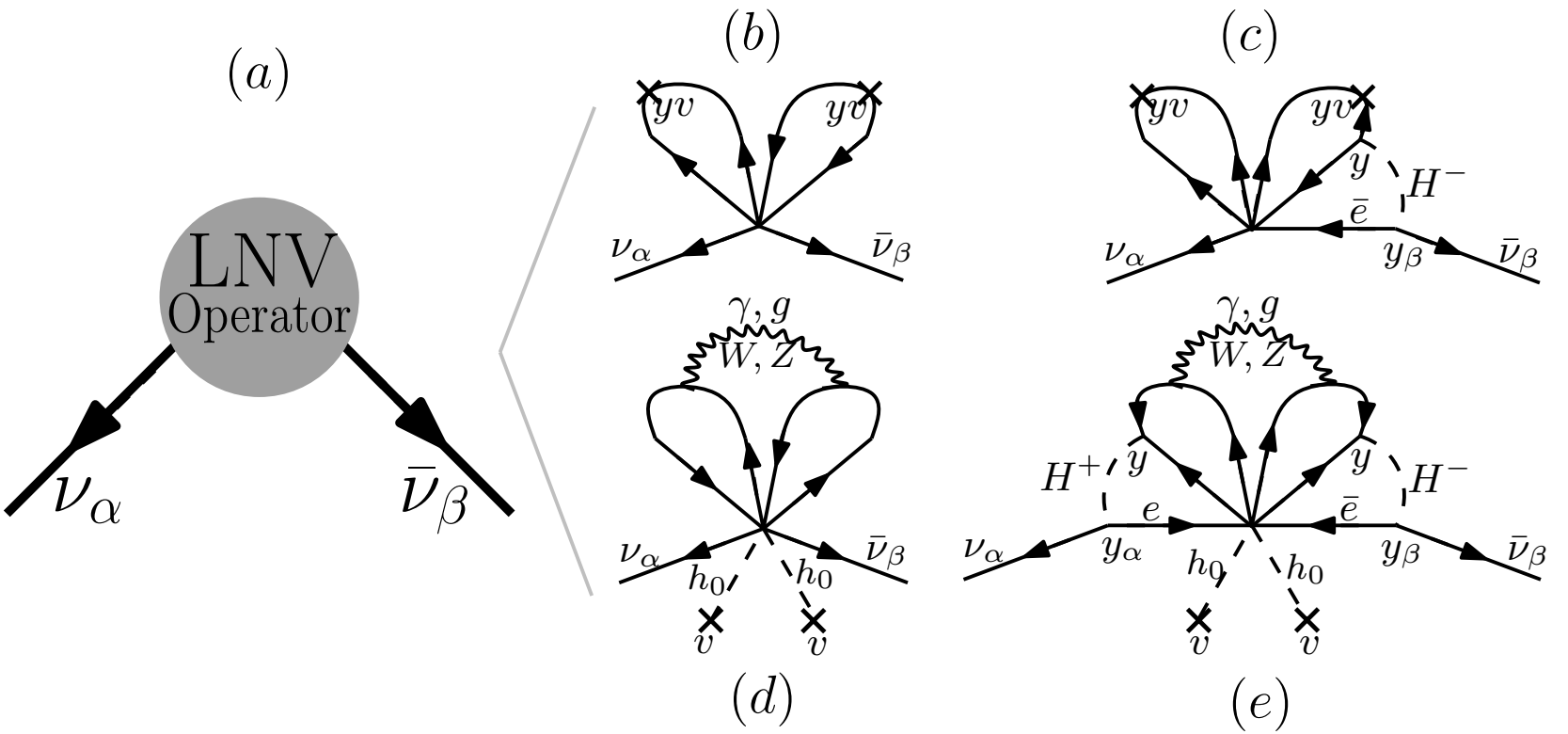
arXiv:0708.1344 [hep-ph]

Effective Operator Approach

(there are 129 of them if you discount different Lorentz structures!)

classified by Babu and Leung in NPB**619**,667(2001)

11_b	$L^iL^jQ^kd^cQ^ld^c\epsilon_{ik}\epsilon_{jl}$	$\frac{y_d}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	2×10^4	$\beta\beta0\nu$
12_a	$L^iL^j\overline{Q}_i\bar{u}^c\overline{Q}_j\bar{u}^c$	$\frac{y_u^2}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	2×10^7	$\beta\beta0\nu$
12_b	$L^iL^j\overline{Q}_k\bar{u}^c\overline{Q}_l\bar{u}^c\epsilon_{ij}\epsilon^{kl}$	$\frac{y_u^2g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^4	$\beta\beta0\nu$
13	$L^iL^j\overline{Q}_i\bar{u}^cL^le^c\epsilon_{jl}$	$\frac{y_\ell y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	2×10^5	$\beta\beta0\nu$
14_a	$L^iL^j\overline{Q}_k\bar{u}^cQ^kd^c\epsilon_{ij}$	$\frac{y_dy_ug^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1×10^3	$\beta\beta0\nu$
14_b	$L^iL^j\overline{Q}_i\bar{u}^cQ^ld^c\epsilon_{jl}$	$\frac{y_dy_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	6×10^5	$\beta\beta0\nu$
15	$L^iL^jL^kd^c\overline{L}_i\bar{u}^c\epsilon_{jk}$	$\frac{y_dy_ug^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1×10^3	$\beta\beta0\nu$
16	$L^iL^je^cd^c\bar{e}^c\bar{u}^c\epsilon_{ij}$	$\frac{y_dy_ug^4}{(16\pi^2)^4}\frac{v^2}{\Lambda}$	2	$\beta\beta0\nu$, LHC
17	$L^iL^jd^cd^c\bar{d}^c\bar{u}^c\epsilon_{ij}$	$\frac{y_dy_ug^4}{(16\pi^2)^4}\frac{v^2}{\Lambda}$	2	$\beta\beta0\nu$, LHC
18	$L^iL^jd^cu^c\bar{u}^c\bar{u}^c\epsilon_{ij}$	$\frac{y_dy_ug^4}{(16\pi^2)^4}\frac{v^2}{\Lambda}$	2	$\beta\beta0\nu$, LHC
19	$L^iQ^jd^cd^c\bar{e}^c\bar{u}^c\epsilon_{ij}$	$y_{\ell_\beta}\frac{y_d^2y_u}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1	$\beta\beta0\nu$, HElnv, LHC, mix
20	$L^id^c\overline{Q}_i\bar{u}^c\bar{e}^c\bar{u}^c$	$y_{\ell_\beta}\frac{y_dy_u^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	40	$\beta\beta0\nu$, mix
21_a	$L^iL^jL^ke^cQ^lu^cH^mH^n\epsilon_{ij}\epsilon_{km}\epsilon_{ln}$	$\frac{y_\ell y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	2×10^3	$\beta\beta0\nu$
21_b	$L^iL^jL^ke^cQ^lu^cH^mH^n\epsilon_{il}\epsilon_{jm}\epsilon_{kn}$	$\frac{y_\ell y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	2×10^3	$\beta\beta0\nu$
22	$L^iL^jL^ke^c\overline{L}_k\bar{e}^cH^lH^m\epsilon_{il}\epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^4	$\beta\beta0\nu$
23	$L^iL^jL^ke^c\overline{Q}_k\bar{d}^cH^lH^m\epsilon_{il}\epsilon_{jm}$	$\frac{y_\ell y_d}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	40	$\beta\beta0\nu$
24_a	$L^iL^jQ^kd^cQ^ld^cH^m\overline{H}_i\epsilon_{jk}\epsilon_{lm}$	$\frac{y_d^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1×10^2	$\beta\beta0\nu$
24_b	$L^iL^jQ^kd^cQ^ld^cH^m\overline{H}_i\epsilon_{jm}\epsilon_{kl}$	$\frac{y_d^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1×10^2	$\beta\beta0\nu$
25	$L^iL^jQ^kd^cQ^lu^cH^mH^n\epsilon_{im}\epsilon_{jn}\epsilon_{kl}$	$\frac{y_dy_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	4×10^3	$\beta\beta0\nu$
26_a	$L^iL^jQ^kd^c\overline{L}_i\bar{e}^cH^lH^m\epsilon_{jl}\epsilon_{km}$	$\frac{y_\ell y_d}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	40	$\beta\beta0\nu$
26_b	$L^iL^jQ^kd^c\overline{L}_k\bar{e}^cH^lH^m\epsilon_{il}\epsilon_{jm}$	$\frac{y_\ell y_d}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	40	$\beta\beta0\nu$
27_a	$L^iL^jQ^kd^c\overline{Q}_i\bar{d}^cH^lH^m\epsilon_{jl}\epsilon_{km}$	$\frac{g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^4	$\beta\beta0\nu$
27_b	$L^iL^jQ^kd^c\overline{Q}_k\bar{d}^cH^lH^m\epsilon_{il}\epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^4	$\beta\beta0\nu$
28_a	$L^iL^jQ^kd^c\overline{Q}_j\bar{u}^cH^l\overline{H}_i\epsilon_{kl}$	$\frac{y_dy_u}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^3	$\beta\beta0\nu$
28_b	$L^iL^jQ^kd^c\overline{Q}_k\bar{u}^cH^l\overline{H}_i\epsilon_{jl}$	$\frac{y_dy_u}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^3	$\beta\beta0\nu$
28_c	$L^iL^jQ^kd^c\overline{Q}_l\bar{u}^cH^l\overline{H}_i\epsilon_{jk}$	$\frac{y_dy_u}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^3	$\beta\beta0\nu$
29_a	$L^iL^jQ^ku^c\overline{Q}_k\bar{u}^cH^lH^m\epsilon_{il}\epsilon_{jm}$	$\frac{y_u^2}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	2×10^5	$\beta\beta0\nu$
29_b	$L^iL^jQ^ku^c\overline{Q}_l\bar{u}^cH^lH^m\epsilon_{ik}\epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^4	$\beta\beta0\nu$
30_a	$L^iL^j\overline{L}_i\bar{e}^c\overline{Q}_k\bar{u}^cH^kH^l\epsilon_{jl}$	$\frac{y_\ell y_u}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	2×10^3	$\beta\beta0\nu$
30_b	$L^iL^j\overline{L}_m\bar{e}^c\overline{Q}_n\bar{u}^cH^kH^l\epsilon_{ik}\epsilon_{jl}\epsilon^{mn}$	$\frac{y_\ell y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	2×10^3	$\beta\beta0\nu$


(a)
LNV
Operator
ν_α
$\bar{\nu}_\beta$
(b)
yv
yv
ν_α
$\bar{\nu}_\beta$
γ, g
W, Z
h_0
h_0
v
v
(c)
yv
yv
y
H^-
$\bar{e}$
ν_α
y_β
$\bar{\nu}_\beta$
γ, g
W, Z
H^+
y
y
H^-
e
$\bar{e}$
ν_α
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$\bar{\nu}_\beta$
v
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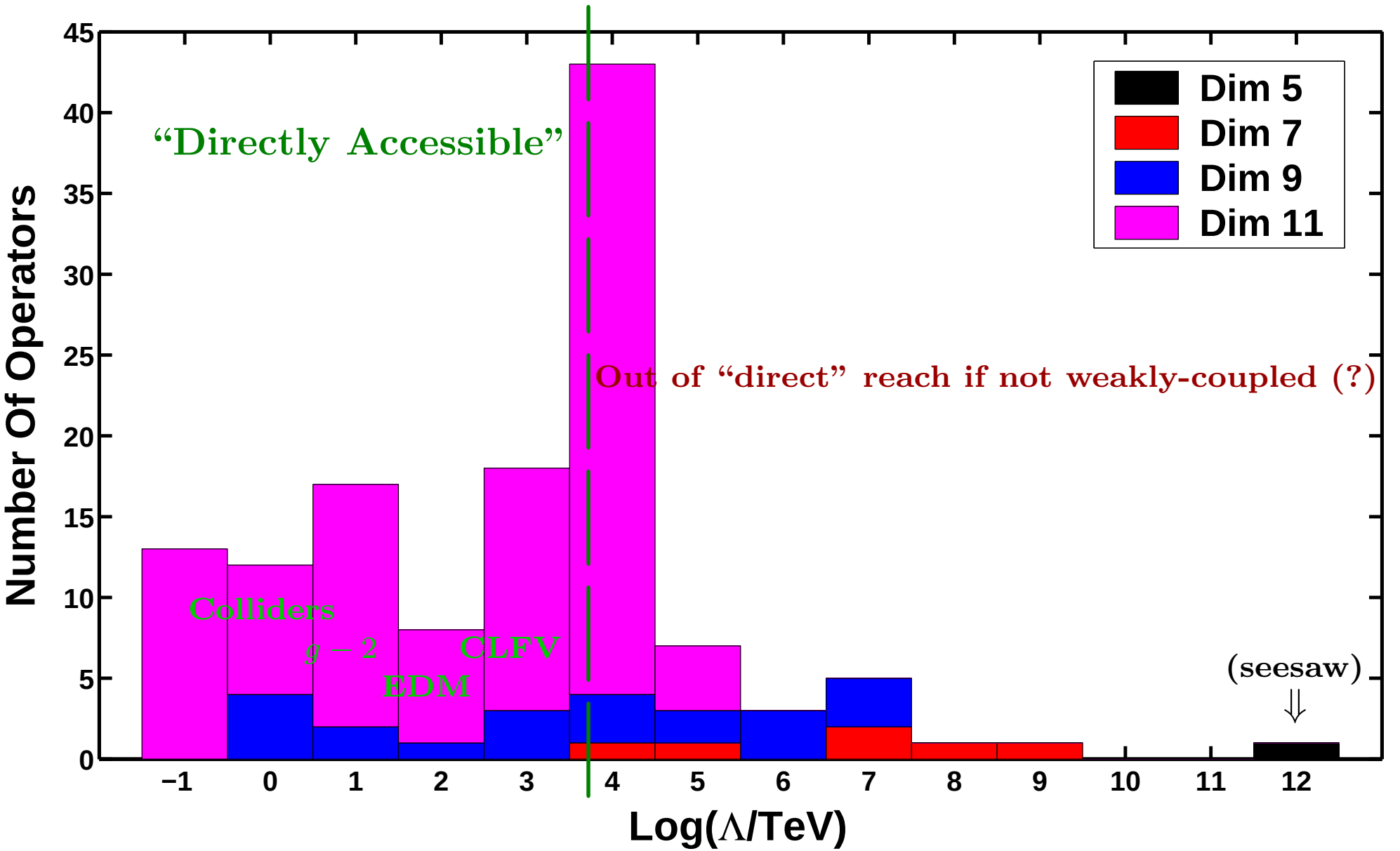
(d)
(e)


Number Of Operators
Log(Λ/TeV)
Dim 5
Dim 7
Dim 9
Dim 11
"Directly Accessible"
Out of "direct" reach if not weakly-coupled (?)
Colliders
g − 2
EDM
CLFV
(seesaw)
⇓
0
5
10
15
20
25
30
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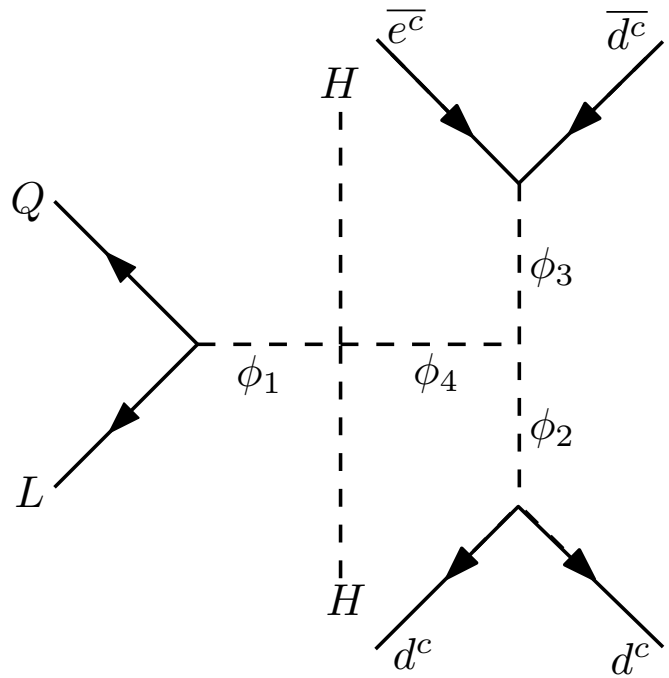
[arXiv:0708.1344 [hep-ph]]



Order-One Coupled, Weak Scale Physics

Can Also Explain Naturally Small

Majorana Neutrino Masses:

Multi-loop neutrino masses from lepton number violating new physics.

$$-\mathcal{L}_{\nu\mathrm{SM}} \supset \sum_{i=1}^{4} M_i \phi_i \bar{\phi}_i + i y_1 Q L \phi_1 + y_2 d^c d^c \phi_2 + y_3 e^c d^c \phi_3 + \lambda_{14} \bar{\phi}_1 \phi_4 H H + \lambda_{234} M \phi_2 \bar{\phi}_3 \phi_4 + h.c.$$

$m_\nu \propto (y_1 y_2 y_3 \lambda_{234}) \lambda_{14} / (16\pi)^4$ $\rightarrow$ neutrino masses at 4 loops, requires $M_i \sim 100$ GeV!

WARNING: For illustrative purposes only. Details still to be worked out. Scenario most likely ruled out by charged-lepton flavor-violation, LEP, Tevatron, and HERA.

How Do We Learn More?

In order to learn more, we need more information. Any new data and/or idea is welcome, including

- searches for charged lepton flavor violation;

  ($\mu \to e\gamma$, $\mu \to e$-conversion in nuclei, etc)

- searches for lepton number violation;

  (neutrinoless double beta decay, etc)

- precision measurements of the neutrino oscillation parameters;

  (Daya Bay, NOνA, etc)

- searches for fermion electric/magnetic dipole moments

  (electron edm, muon $g-2$, etc);

- precision studies of neutrino – matter interactions;

  (Minerνa, NuSOnG, etc)

- collider experiments:

  (LHC, etc)

  – *Can* we "see" the physics responsible for neutrino masses at the LHC? – YES!

    Must we see it? – NO, but we won't find out until we try!

  – we need to understand the physics at the TeV scale before we can really understand the physics behind neutrino masses (is there low-energy SUSY?, etc).

CONCLUSIONS

The venerable Standard Model has finally sprung a leak – neutrinos are not massless!

1. we have a very successful parametrization of the neutrino sector, and we have identified what we know we don't know.
2. neutrino masses are very small – we don't know why, but we think it means something important.
3. lepton mixing is very different from quark mixing – we don't know why, but we think it means something important.
4. we need a minimal νSM Lagrangian. In order to decide which one is "correct" (required in order to attack 2. and 3. above) we must uncover the faith of baryon number minus lepton number ($0\nu\beta\beta$ is the best [only?] bet).

5. We need more experimental input – and more seems to be on the way (this is a truly data driven field right now). We only started to figure out what is going on.

6. The fact that neutrinos have mass may be intimately connected to the fact that there are more baryons than antibaryons in the Universe. How do we test whether this is correct?

7. There is plenty of room for surprises, as neutrinos are very narrow but deep probes of all sorts of physical phenomena. Remember that neutrino oscillations are "quantum interference devices" – potentially very sensitive to whatever else may be out there (e.g., $M_{\rm seesaw} \simeq 10^{14}$ GeV).

BACK-UP MATERIAL:

Short Description of the MSW Effect in the Sun

The MSW Effect

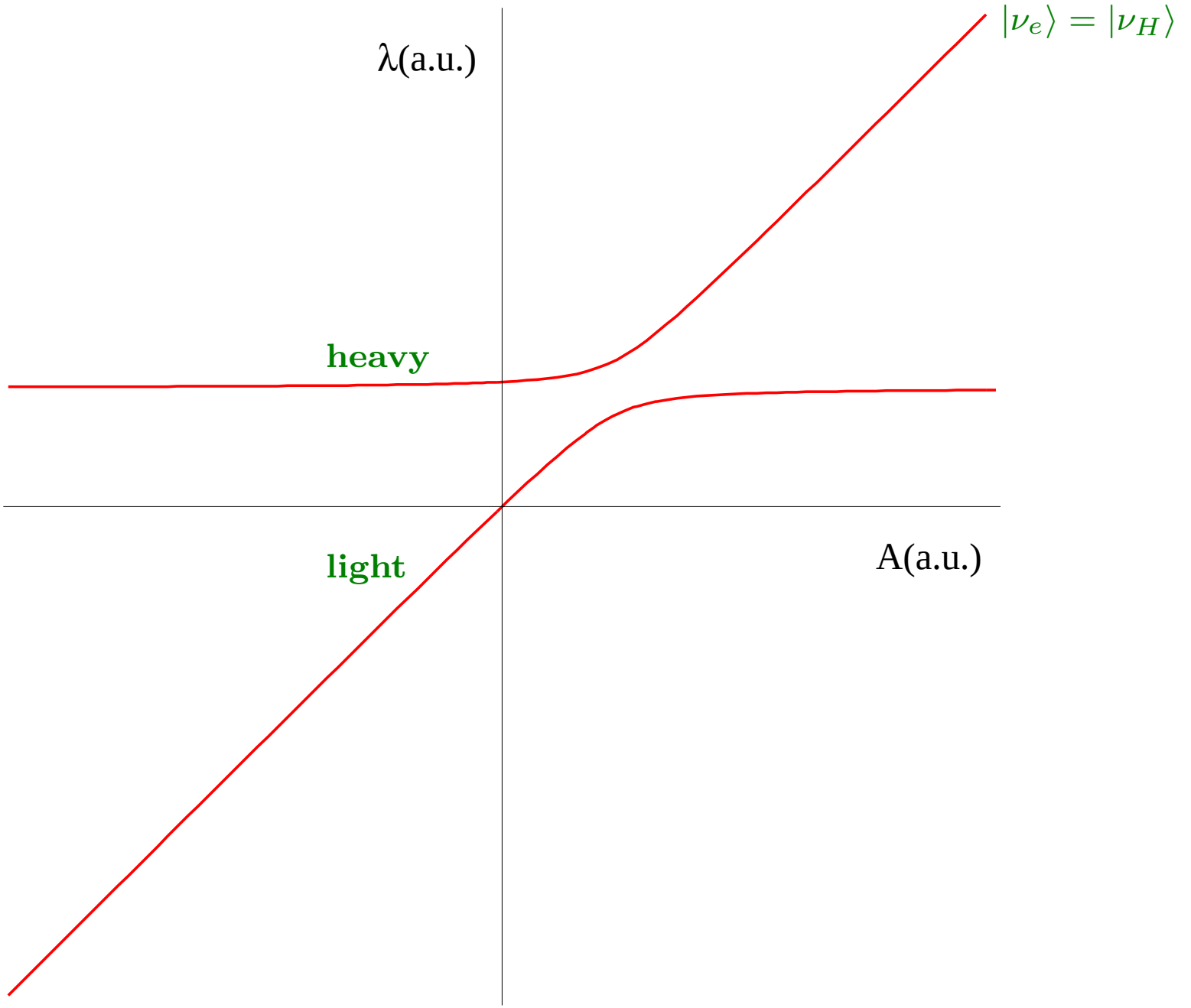
Curiously enough, the oldest neutrino puzzle is the one that is most subtle to explain. This is because solar neutrinos traverse a strongly varying matter density on their way from the center of the Sun to the surface of the Earth.

For the Hamiltonian

$$\left[\Delta \begin{pmatrix} \sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta \end{pmatrix} + A \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right],$$

it is easy to compute the eigenvalues as a function of A:

(remember, $\Delta = \Delta m^2/2E$)


$|\nu_e\rangle = |\nu_H\rangle$
λ(a.u.)
heavy
light
A(a.u.)

A decreases "slowly" as a function of L $\Rightarrow$ system evolves adiabatically.

$$|\nu_e\rangle = |\nu_{2M}\rangle \text{ at the core } \rightarrow |\nu_2\rangle \text{ in vacuum,}$$
$$P_{ee}^{\text{Earth}} = |\langle\nu_e|\nu_2\rangle|^2 = \sin^2\theta.$$

Note that $P_{ee} \simeq \sin^2\theta$ applies in a wide range of energies and baselines, as long as the approximations mentioned above apply —ideal to explain the energy independent suppression of the ^{8}B solar neutrino flux!

Furthermore, large average suppressions of the neutrino flux are allowed if $\sin^2\theta \ll 1$. Compare with $\bar{P}_{ee}^{\text{vac}} = 1 - 1/2\sin^2 2\theta > 1/2$.

One can expand on the result above by loosening some of the assumptions. $|\nu_e\rangle$ state is produced in the Sun's core as an *incoherent* mixture of $|\nu_{1M}\rangle$ and $|\nu_{2M}\rangle$. Introduce adiabaticity parameter P_c, which measures the probability that a $|\nu_{iM}\rangle$ matter Hamiltonian state will *not* exit the Sun as a $|\nu_i\rangle$ mass-eigenstate.

$$
\begin{aligned}
|\nu_e\rangle \quad &\rightarrow \quad |\nu_{1M}\rangle, \text{ with probability } \cos^2\theta_M, \\
&\rightarrow \quad |\nu_{2M}\rangle, \text{ with probability } \sin^2\theta_M,
\end{aligned}
$$

where θ_M is the matter angle at the neutrino **production point**.

$$
\begin{aligned}
|\nu_{1M}\rangle \quad &\rightarrow \quad |\nu_1\rangle, \text{ with probability } (1-P_c), \\
&\rightarrow \quad |\nu_2\rangle, \text{ with probability } P_c, \\
|\nu_{2M}\rangle \quad &\rightarrow \quad |\nu_1\rangle \text{ with probability } P_c, \\
&\rightarrow \quad |\nu_2\rangle \text{ with probability } (1-P_c).
\end{aligned}
$$

$P_{1e} = \cos^2\theta$ and $P_{2e} = \sin^2\theta$ so

$$
\begin{aligned}
P_{ee}^{\text{Sun}} = \quad & \cos^2\theta_M \left[(1-P_c)\cos^2\theta + P_c \sin^2\theta\right] \\
& + \sin^2\theta_M \left[P_c \cos^2\theta + (1-P_c)\sin^2\theta\right].
\end{aligned}
$$

For $N_e = N_{e0} e^{-L/r_0}$, P_c, (crossing probability), is exactly calculable

$$
P_c = \frac{e^{-\gamma \sin^2\theta} - e^{-\gamma}}{1 - e^{-\gamma}}, \quad \gamma = 2\pi r_0 \Delta. \tag{1}
$$

Adiabatic condition: $\gamma \gg 1$, when $P_c \rightarrow 0$.