So you want to build a model of dark matter?
What goes into dark matter?

DARK MATTER takes a quantum leap forward into a new dimension of post-workout muscle growth called the ANABOLIC AXIS. The Anabolic Axis is the time and point at which insulin levels simultaneously peak with amino acids, creatine and glycogen transport into muscle tissue during the critical 1 hour period immediately after your workout. Dark Matter is the first and only supplement to employ a new technology called Precision Nutrient Infusion, which allows for this synergistic anabolic reaction to occur at the Anabolic Axis. In order to achieve this major breakthrough, MHP scientists bio-engineered new compounds and a revolutionary High Velocity Nano-Physics Technology. These new developments have rendered all post-workout creatines, whey protein/high carbohydrate combos and all other post-workout formulas inferior and outdated. DARK MATTER blasts open the critical "Anabolic Window" faster, wider and longer allowing you to enter the ANABOLIC AXIS for the most powerful anabolic reaction ever experienced!
Most important thing about dark matter

No one knows anything about dark matter!*

*Except for the many things we know about dark matter
DM properties

- None (who needs DM?)
- Gas/brown dwarfs
- neutrinos
- neutralinos
- ...?
Era of data

Cosmics: PAMELA, Fermi, ATIC, HESS, AMS, ACTs, WMAP, Planck...

Direct: CDMS, XENON, LUX, CRESST, COUPP, PICASSO, KIMS...

Production: LHC/Tevatron, Fixed Target, Beam dump
Evidence for DM

- Zwicky measuring galaxies in clusters
- Rubin (and previous)
Evidence for DM
NB: T = 1/time!
What do we know about DM?

- Mostly negative
- Massive (i.e., $w \approx 0$) since $T \approx \text{keV}$
- Neutral
  - $m < M_z \Rightarrow 0.24$ mixing angle
  - $+1$ charge, heavy hydrogen ($4 \times 10^{-17}$ abundance)
What we know about DM

- Cold
  - Typically means cold at freezeout
  - Really means cold at $\sim$ keV (probed by structure)

- Non-interacting: what does that mean?

\[ n\sigma v\tau = 1 \rightarrow \frac{\sigma}{10^{-24} \text{cm}^2} < 10^{-24} \times \frac{\text{TeV}}{m_{DM}} \]
Massless photons (Ackermann et al)

\[ \alpha < 10^{-3} \]

Long range forces/ equivalence principle violations (Kesdu and Kamionkowski) \( B < 0.2 \)

Be careful with massless particles! (but some caveats, e.g., Feng et al)
What you should satisfy

- Massive, neutral, mostly non-interacting, Omega=0.3
- Stable (on Hubble times)
- You’re good to go
## Candidates for DM: Theory Motivated

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Motivation</th>
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</tr>
</thead>
<tbody>
<tr>
<td>axion</td>
<td>promote q to dynamical variable</td>
<td>strong CP problem</td>
</tr>
<tr>
<td>neutralino</td>
<td>mixture of Bino, Wino and up/down Higgsinos</td>
<td>hierarchy problem</td>
</tr>
<tr>
<td>sneutrino</td>
<td>partner of sneutrino (relic abundance and direct detection problems)</td>
<td>hierarchy problem</td>
</tr>
<tr>
<td>LTOP</td>
<td>Little Higgs models, general BSM models</td>
<td>hierarchy problem</td>
</tr>
<tr>
<td>KKDM</td>
<td>First KK resonance, stabilized by KK parity</td>
<td>not the neutralino</td>
</tr>
<tr>
<td>axino</td>
<td>SUSY partner of axion</td>
<td>SCP+HP</td>
</tr>
<tr>
<td>4th gen neutrino</td>
<td>Another generation, but stable</td>
<td>first three generations</td>
</tr>
<tr>
<td>gravitino</td>
<td>LSP decays to gravitino, partner of graviton</td>
<td>HP+unpleasant childhood</td>
</tr>
<tr>
<td>LNSWP</td>
<td>Something stable and weak scale, why not?</td>
<td>The weak scale is there, DM is there</td>
</tr>
</tbody>
</table>

Also qballs, BHs, topological things, and whatever you are working on but I forgot to mention.
### Candidates for DM: “Exp” Motivated

<table>
<thead>
<tr>
<th>Candidate</th>
<th>What is it</th>
<th>Motivation</th>
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<tbody>
<tr>
<td>SIDM</td>
<td>make DM strongly interacting (candidate?)</td>
<td>galaxy structure issues (cusps)</td>
</tr>
<tr>
<td>WDM</td>
<td>warm – keV sterile neutrino</td>
<td>substructure</td>
</tr>
<tr>
<td>Light DM</td>
<td>light (GeV) WIMP</td>
<td>DAMA</td>
</tr>
<tr>
<td>Spin-dependent DM</td>
<td>?</td>
<td>DAMA</td>
</tr>
<tr>
<td>iDM</td>
<td>Mixed sneutrino, split SU(2) doublet, new force</td>
<td>DAMA</td>
</tr>
<tr>
<td>MeVDM</td>
<td>DM with MeV mass</td>
<td>INTEGRAL</td>
</tr>
<tr>
<td>XDM</td>
<td>DM that upscatters with ~ GeV mass force</td>
<td>INTEGRAL, more recently PAMELA/Fermi...</td>
</tr>
<tr>
<td>Decaying DM</td>
<td>DM decays with long lifetime</td>
<td>PAMELA/Fermi</td>
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**All these models are wrong except at most one**
The axion

Consider a scalar field in an expanding universe

\[ \ddot{\phi} + 3H \dot{\phi} + m^2 \phi = 0 \]
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\[
\rho = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} m^2 \phi^2 = \frac{1}{2} (m^2 + 9H^2) \phi_0^2 a^{-6}
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dilutes away rapidly
\[ \phi(t) = \phi_0 e^{-2m^2 t / 3H} = \phi_0 a^{-2m^2 / 3H^2} \sim \text{const} \]

slowly rolling => approximately constant
Coming inside the horizon
Coming inside the horizon

\[
\omega = \frac{-3H \pm \sqrt{9H^2 - 4m^2}}{2} \quad \rightarrow \quad \omega = -\frac{3H}{2} \pm im \quad (m \gg H)
\]
Coming inside the horizon

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\[ \phi(t) = \phi_0 e^{-\frac{3Ht}{2}} e^{\pm imt} = \phi_0 a^{-\frac{3}{2}} e^{\pm imt} \]
Coming inside the horizon

$$\omega = \frac{-3H \pm \sqrt{9H^2 - 4m^2}}{2} \quad \rightarrow \omega = -\frac{3H}{2} \pm im \quad (m \gg H)$$

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Coming inside the horizon

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\omega = \frac{-3H \pm \sqrt{9H^2 - 4m^2}}{2} \quad \rightarrow \quad \omega = -\frac{3H}{2} \pm im \quad (m \gg H)
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\phi(t) = \phi_0 e^{-\frac{3Ht}{2}} e^{\pm imt} = \phi_0 a^{-3/2} e^{\pm imt}
\]

\[
\rho = m^2 \phi_0^2 e^{-3Ht} = \rho_0 a^{-3}
\]

dilutes like matter!
So what’s wrong with that?

\[ H^2 \sim \frac{\rho_{TOT}}{M_{pl}^2} \]

\[ m^2 \sim H^2 \rightarrow m^2 M_{pl}^2 \sim \rho_{TOT} \]
So what's wrong with that?

\[ H^2 \sim \frac{\rho_{TOT}}{M_{pl}^2} \quad m^2 \sim H^2 \rightarrow m^2 M_{pl}^2 \sim \rho_{TOT} \]

starting with Planckian vev means that you dominate the energy density of the universe shortly after coming inside the horizon.
So what's wrong with that?

\[ H^2 \sim \frac{\rho_{TOT}}{M_{pl}^2} \quad \text{and} \quad m^2 \sim H^2 \rightarrow m^2 M_{pl}^2 \sim \rho_{TOT} \]

starting with Planckian vev means that you dominate the energy density of the universe shortly after coming inside the horizon.

that's bad because MRE \(\approx 1\) eV, but DM is DM since before 1 keV!
Axion mass doesn't turn on until the QCD phase transition

\[ m_a^2 \sim \frac{f_{\pi}^2 m_{\pi}^2}{f_a^2} \]

\[ f_a \gtrsim 10^{10} \text{GeV} \quad \text{astrophysics} \]

\[ f_a \sim 10^{12} \text{GeV} \quad \text{relic density} \]
The WIMP

early universe cheat sheet

\[ x = \frac{m}{T} \quad \text{time variable} \]

\[ n_R \sim T^3 \]

\[ n_{NR} \sim (mT)^{3/2}e^{-m/T} \]

\[ H \sim \frac{T^2}{M_{pl}} \quad \text{(radiation domination)} \]
Precision Electroweak Studies
Precision Electroweak Studies

• Just because we haven't detected new particles doesn't mean we don't know much about physics beyond the standard model.
Precision Electroweak Studies

- Just because we haven't detected new particles doesn't mean we don't know much about physics beyond the standard model

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Standard Model</th>
<th>Pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_h$ [GeV]</td>
<td>$125.7 \pm 0.6$</td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td>$M_W$ [GeV]</td>
<td>80.430 ± 0.058</td>
<td>80.376 ± 0.017</td>
<td>1.3</td>
</tr>
<tr>
<td>$M_Z$ [GeV]</td>
<td>91.1876 ± 0.0021</td>
<td>91.1874 ± 0.0021</td>
<td>0.1</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>2.4952 ± 0.0093</td>
<td>2.4958 ± 0.0091</td>
<td>0.7</td>
</tr>
<tr>
<td>$F_{\text{had}}$ [GeV]</td>
<td>1.7444 ± 0.0020</td>
<td>1.7434 ± 0.0020</td>
<td>—</td>
</tr>
<tr>
<td>$F_{\text{em}}$ [MeV]</td>
<td>501.65 ± 0.31</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>$\Gamma_{\text{had}}$ [MeV]</td>
<td>83.984 ± 0.086</td>
<td>83.996 ± 0.021</td>
<td>—</td>
</tr>
<tr>
<td>$\epsilon_{\text{had}}$ [nb]</td>
<td>4.134 ± 0.037</td>
<td>4.146 ± 0.009</td>
<td>2.0</td>
</tr>
<tr>
<td>$R_E$</td>
<td>0.834 ± 0.009</td>
<td>0.835 ± 0.011</td>
<td>1.0</td>
</tr>
<tr>
<td>$R_F$</td>
<td>20.786 ± 0.033</td>
<td>20.756 ± 0.011</td>
<td>0.9</td>
</tr>
<tr>
<td>$R_{\tau}$</td>
<td>20.704 ± 0.045</td>
<td>20.801 ± 0.011</td>
<td>0.8</td>
</tr>
<tr>
<td>$R_{\phi}$</td>
<td>0.21629 ± 0.00066</td>
<td>0.21587 ± 0.00010</td>
<td>0.8</td>
</tr>
<tr>
<td>$R_B$</td>
<td>0.1721 ± 0.0030</td>
<td>0.17230 ± 0.00014</td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)}$</td>
<td>0.0149 ± 0.0025</td>
<td>0.01622 ± 0.0025</td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.0169 ± 0.0013</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.0188 ± 0.0017</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.0992 ± 0.0016</td>
<td>0.1031 ± 0.0008</td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.0707 ± 0.0035</td>
<td>0.0737 ± 0.0006</td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.0726 ± 0.0014</td>
<td>0.1032 ± 0.0008</td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.3334 ± 0.0012</td>
<td>0.3316 ± 0.00014</td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.2238 ± 0.0060</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.1313 ± 0.0026</td>
<td>0.1471 ± 0.0011</td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.1344 ± 0.0060</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.148 ± 0.0015</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.136 ± 0.0015</td>
<td></td>
<td>—</td>
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<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.1436 ± 0.0043</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.292 ± 0.0020</td>
<td>0.3007 ± 0.0001</td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.670 ± 0.0027</td>
<td>0.678 ± 0.005</td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.835 ± 0.0031</td>
<td>0.8356 ± 0.0001</td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.30055 ± 0.00157</td>
<td>0.30075 ± 0.00021</td>
<td>—</td>
</tr>
<tr>
<td>$A_{\phi}^{(0)(0)}$</td>
<td>0.00076 ± 0.000110</td>
<td>0.00076 ± 0.00003</td>
<td>—</td>
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T-Parity (Cheng and Low)

- The problem arises from diagrams like
**T-Parity** (Cheng and Low)

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Need to forbid these diagrams somehow
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```
SM          SM
\|           \|
 SM          SM
```

Need to forbid these diagrams somehow

- Vertex comes from Lagrangian term
T-Parity (Cheng and Low)

- The problem arises from diagrams like

\[ L \supset SM_1 SM_2 BSM \]

Need to forbid these diagrams somehow

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T-Parity (Cheng and Low)

• The problem arises from diagrams like

Need to forbid these diagrams somehow

• Vertex comes from Lagrangian term

\[ \mathcal{L} \supset SM_1 SM_2 BSM \]

• I.e., problem is presence of single BSM field
  - If only even numbers of BSM fields were allowed, this term is forbidden!
• Then process occurs via loop

loops smaller by $\sim \frac{1}{16\pi^2}$

enough to solve problem
• Then process occurs via loop

loops smaller by $\sim \frac{1}{16\pi^2}$

enough to solve problem

Introduce parity at weak scale $\Rightarrow$ stable DM candidates
The WIMP “miracle”

assume thermal equilibrium

\[ \chi \chi \leftrightarrow \bar{f} f \]
The WIMP "miracle"

assume thermal equilibrium

\[ \chi \leftrightarrow \bar{f} \]

\[ \chi \leftarrow \text{time} \rightarrow \bar{f} \]

**miracle** |ˈmirəkl|
---
noun
a surprising and welcome event that is not explicable by natural or scientific laws and is therefore considered to be the work of a divine agency: *the miracle of rising from the grave.*
- a highly improbable or extraordinary event, development, or accomplishment that brings very welcome consequences: *it was a miracle that more people hadn’t been killed or injured* [as adj.]: *a miracle drug.*
- an amazing product or achievement, or an outstanding example of something: *a machine which was a miracle of design.*

ORIGIN Middle English: via Old French from Latin *miraculum* ‘object of wonder,’ from *mirari* ‘to wonder,’ from *mirus* ‘wonderful.’
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\[ \chi \chi \leftrightarrow \bar{f} f \]

\[ \chi \quad f \quad \chi \quad \bar{f} \]

\[ \leftarrow \quad \text{time} \quad \rightarrow \]
assume thermal equilibrium

When $T \ll M_{\text{WIMP}}$, number density falls as $e^{-M/T}$
freezeout \[ T_f \rightarrow n < \sigma v > = H \]
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\begin{align*}
n_{\text{now}} &= n_f \frac{\frac{T^3_{\text{now}}}{T^3_f}}{n} \\
&= \frac{n}{T^3}
\end{align*}

dark matter per photon (is approximately constant)
freezeout

\[ T_f \rightarrow n < \sigma v > = H \]

\[ n_{\text{now}} = n_f \frac{T_{\text{now}}^3}{T_f^3} \]

\[ \frac{n}{T^3} \]

\[ \frac{n_{\text{now}}}{T_{\text{now}}^3} \approx \frac{n_f}{T_f^3} \frac{H_f}{T_f^3 < \sigma v >} = \frac{T_f^2}{T_f^3 M_{\text{pl}} < \sigma v >} \]

dark matter per photon (is approximately constant)
freezeout \quad T_f \rightarrow n < \sigma v > = H

\begin{align*}
n_{\text{now}} &= n_f \frac{T_{\text{now}}^3}{T_f^3} \\
n &= \frac{n}{T^3} \\
n_{\text{now}} \approx n_f \frac{H_f}{T_f^3} &= \frac{T_f^2}{T_f^3 M_{\text{pl}} < \sigma v >} \\
n_{NR} &\sim (mT)^{3/2} e^{-m/T} \quad T_f = m/x_f \quad x_f \sim \text{not infinity}
\end{align*}

dark matter per photon (is approximately constant)
freezeout

\[ T_f \rightarrow n < \sigma v >= H \]

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\[ \frac{n_{\text{now}}}{T_{\text{now}}^3} \approx \frac{n_f}{T_f^3} = \frac{H_f}{T_f^3} < \sigma v > = \frac{T_f^2}{T_f^3 M_{\text{pl}} < \sigma v >} \]

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\[ \rho_{\text{now}} T_{\text{now}}^3 = \frac{m n_{\text{now}}}{T_{\text{now}}^3} \approx \frac{mm^2 x_f^3}{m^3 x_f^2 M_{\text{pl}} < \sigma v >} \]

dark matter per photon (is approximately constant)
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\[ \frac{n_{\text{now}}}{T_{\text{now}}^3} \approx \frac{n_f}{T_f^3} = \frac{H_f}{T_f^3 < \sigma v >} = \frac{T_f^2}{T_f^3 M_{\text{pl}} < \sigma v >} \]

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\[ \rho_{\text{now}} T_{\text{now}}^3 = \frac{m n_{\text{now}}}{T_{\text{now}}^3} \approx \frac{m m^2 x_f^3}{m^3 x_f^2 M_{\text{pl}} < \sigma v >} \]

\[ = \frac{x_f}{M_{\text{pl}} < \sigma v >} \]

dark matter per photon (is approximately constant)
freezeout

$T_f \rightarrow n < \sigma v > = H$

dark matter per photon (is approximately constant)

$n_{now} = n_f \frac{T_{now}^3}{T_f^3}$

$\frac{n_{now}}{T_{now}^3} \approx \frac{n_f}{T_f^3} = \frac{H_f}{T_f^3 < \sigma v >} = \frac{T_f^2}{T_f^3 M_{pl} < \sigma v >}$

$n_{NR} \sim (mT)^{3/2} e^{-m/T}$

$T_f = m/x_f$  \quad $x_f \sim$ not infinity

$\rho_{now} T_{now}^3 = \frac{m n_{now}}{T_{now}^3} \approx \frac{m m^2 x_f^3}{m^3 x_f^2 M_{pl} < \sigma v >}$

$= \frac{x_f}{M_{pl} < \sigma v >}$

just depends on cross section!
The WIMP not-miracle

\[ \Omega h^2 \approx 0.1 \times \left( \frac{3 \times 10^{-26} \text{cm}^3 \text{s}^{-1}}{\langle \sigma v \rangle} \right) \]

\[ \approx 0.1 \times \left( \frac{\alpha^2 / (100 \text{GeV})^2}{\langle \sigma v \rangle} \right) \]

Any weak-scale particle naturally freezes out within a few orders of magnitude of the correct cross section
Three approaches with thermal DM

- Make it (colliders)
- Break it (indirect searches)
- Wait for it (direct searches)
The neutralino combination of Bino, Wino, up/down Higgsino (in MSSM)

\[
\begin{pmatrix}
M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\
0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\
-m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\
m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \\
\end{pmatrix}
\]

\[
\chi_{0,1,2,3} = \sum_{i=\tilde{B},\tilde{W},\tilde{H}_u,\tilde{H}_d} U_i \psi_i
\]

typically “gaugino”-like or “Higgsino”-like
focus point

a funnel

coaannihilation

tail
The CMSSM/mSUGRA neutralino is not your friend

- Common logical path in mSUGRA*

  LEP Higgs mass limit $m_h > 114.4$ GeV  
  SUSY predicts $m_h < m_Z$  
  Need large radiative corrections to quartic to keep $v=246$ GeV  
  Large radiative corrections give contribution to Higgs mass  
  Cancel those corrections with large $\mu$ term  
  $\mu$ term is Higgsino mass  
  LSP is mostly Bino  
  Small elastic scattering cross sections

* No, not every point in mSUGRA, this is just an example
Detecting WIMPS

Like I said, you can

- Make them (at a collider)
- Break them (in the cosmos)
- Wait for them (underground)

How strong are these signals?
Your Canonical WIMP
the 4th generation neutrino

- 4th generation Dirac neutrino is completely ruled out as a WIMP candidate*
- This makes it a very handy study

* unless you tweak some things
Relic abundance

$$\sigma v = \frac{g^4}{512\pi m_\nu^2} (21 + 3t_W^2 + 11t_W^4)$$

$$\Omega h^2 \approx 0.1 \times \left( \frac{m_\nu}{\text{TeV}} \right)^2$$

note: in reality weak scale particles are under-abundant. Want TeV or some suppression in cross section
$\sigma = \frac{G_f^2}{2\pi} \mu^2 \chi_N \left( (1 - 4s^2_W)Z - (A - Z) \right)^2$

$\sigma_n = \frac{G_f^2}{2\pi} \mu_n^2 \frac{\left( (1 - 4s^2_W)Z - (A - Z) \right)^2}{A^2}$

cross section per nucleon

about $10^{-39}$ cm$^2$

very, very ruled out

what about the neutralino?
Consider vector interaction

\[ \chi_1 \sigma_\mu \chi_1 A^\mu \]
Consider vector interaction

\[ \chi_1 \sigma_\mu \chi_1 A^\mu \]
Consider vector interaction

\begin{align*}
\chi_1 \sigma_\mu \chi_1 A^\mu \\
\chi_1 \sigma_\mu \chi_2 A^\mu 
\end{align*}
Consider vector interaction

\[ \chi_1 \sigma_\mu \chi_1 A^\mu \]

Vector interactions for massive Majorana fermions (or real scalars) \textbf{always} require multiple states interaction is off-diagonal
The neutralino is a Majorana fermion

Can have spin-dependent, anapole, and scalar interactions, but not vector interactions

Dominant contributions come from e.g., Higgs, which has coupling $\sim 10^{-3} \Rightarrow 10^{-44} \text{ cm}^2$ or so

Thus neutralino safe, while Dirac fermion is not

Obvious way to save Dirac fermion is to split the components
Important points for dark matter

**Recommended Dosage**

Recommended Use: As a dietary supplement, DARK MATTER is your ultimate post-workout muscle growth accelerator designed to be taken immediately after your workout before you consume any other shake or meal. All other substances will only slow down the precise nutrient infusion of the high velocity nano-physic particles in DARK MATTER, shorten your anabolic window and impede maximum muscle growth and recovery. Add 2 scoops to 20 oz. water in a glass or shaker bottle immediately after training. Stir or shake vigorously. Any other post-workout shake or meal may be taken 1 hour later.
Take away day 1

- DM is well studied, and there are significant limits on its properties
- Easy to get models that fit those requirements
- Many BSM motivated candidates
- Production mechanism $\Rightarrow$ detection mechanism