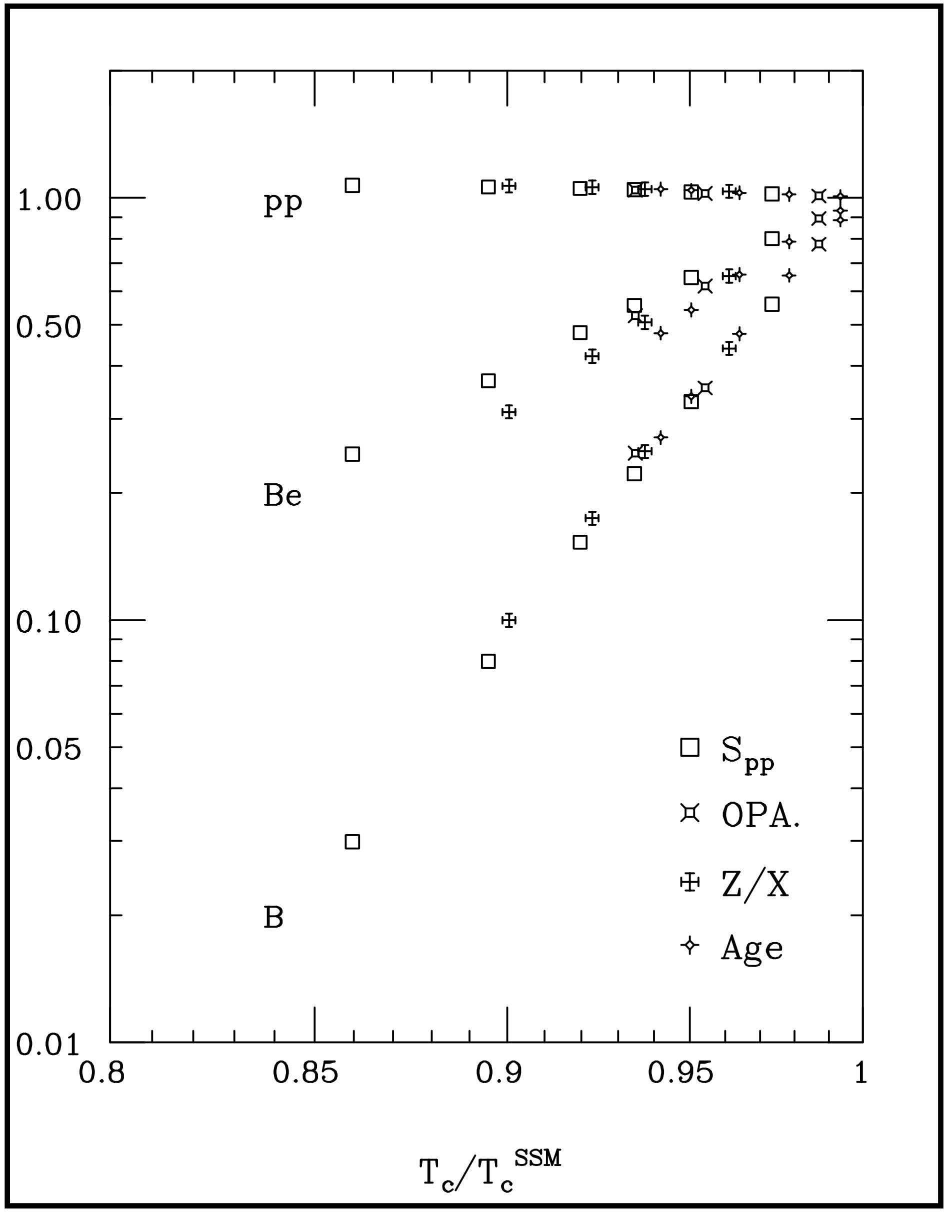
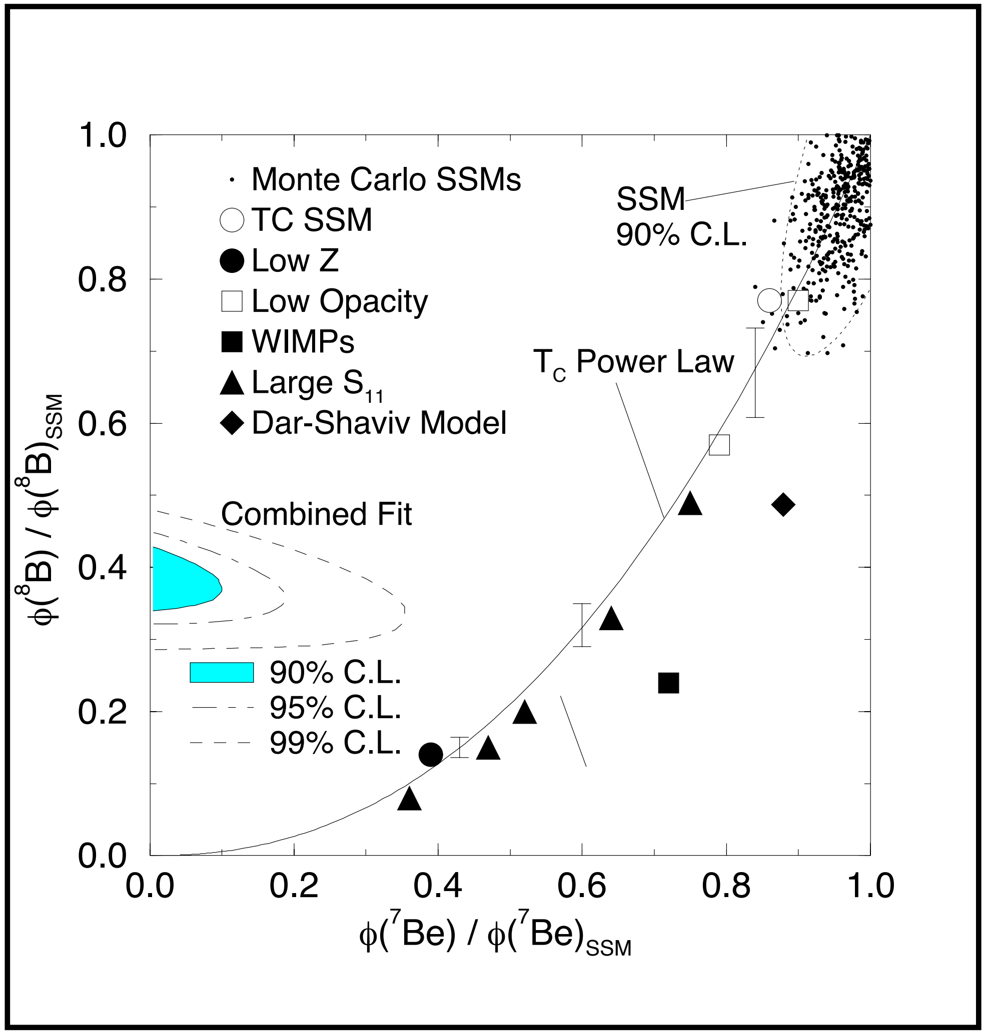
Solar Neutrinos II: New Physics

- Helioseismology
- Matter-enhanced neutrino oscillations
- SNO and Super-Kamiokande

Cl/Ga/Kamioka measured the three major solar ν fluxes and found they were difficult to reconcile with the SSM



Castellani et al.



Hata et al.

(and Heeger and Robertson)

- Under the assumption that the experiments were measuring EC line sources and continuous β-decay sources with allowed shapes

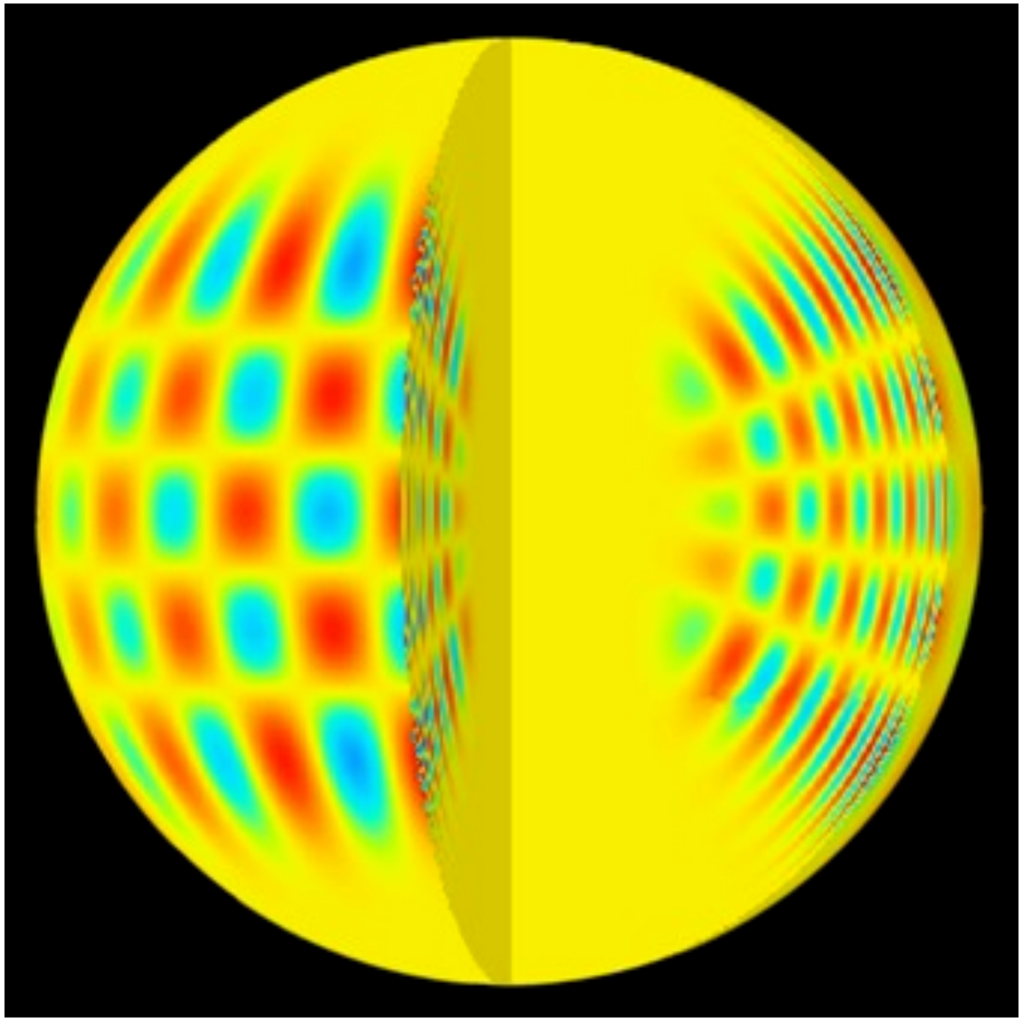
$$\phi(pp) \sim 9.9\phi^{\mathrm{SSM}}(pp)$$
$$\phi(^{7}\mathrm{Be}) \sim 0$$
$$\phi(^{8}\mathrm{B}) \sim 0.43\phi^{\mathrm{SSM}}(^{8}\mathrm{B})$$

- Steady-state models where the fluxes are largely controlled by the average temperature of the core cannot produce this pattern

$$\frac{\phi(^{8}\mathrm{B})}{\phi(\mathrm{pp})} \sim T^{23} << \frac{\phi^{\mathrm{SSM}}(^{8}\mathrm{B})}{\phi^{\mathrm{SSM}}(\mathrm{pp})} \Rightarrow T < T^{\mathrm{SSM}} \text{ cooler Sun}$$
$$\frac{\phi(^{7}\mathrm{Be})}{\phi(^{8}\mathrm{B})} \sim T^{-12} << \frac{\phi^{\mathrm{SSM}}(^{7}\mathrm{Be})}{\phi^{\mathrm{SSM}}(^{8}\mathrm{B})} \Rightarrow T > T^{\mathrm{SSM}} \text{ hotter Sun}$$

so the pattern is contradictory

In parallel, a second precise probe of the solar interior was being developed: helioseismology, the measurement and analysis of Doppler shifts of photospheric absorption lines. Amplitudes ~30m and velocities ~0.1m/s.



Turbulence within Sun's convective zone acts as a random driver of sound waves propagating through the gas

Specific frequencies are enhanced as standing waves -- normal modes whose frequencies depend on solar physics

n=14 l=20 m=16 p-mode (acoustic)

How does this probe the SSM? For a spherical star

$$p(r),\ \rho(r),\ T(r),\ s(r),\ \phi_{\rm gravity}(r),\ \epsilon_{\rm nuclear\ energy}(r)$$

Introduce adiabatic indices describing power-law behavior of p, T with ρ

$$\Gamma_1 \equiv \left(\frac{\partial \log p}{\partial \log \rho}\right)_s \qquad \Gamma_3 - 1 \equiv \left(\frac{\partial \log T}{\partial \log \rho}\right)_s$$

Define a total derivative $\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}$

Write down the equations for

motion: $\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} p - \rho \vec{\nabla} \phi$ continuity: $\frac{D\rho}{Dt} + \rho \vec{\nabla} \cdot \vec{v} = 0$

gravitational potential: $\vec{\nabla}^2 \phi = 4\pi G \rho$

energy conservation: $\frac{1}{p}\frac{Dp}{Dt} - \Gamma_1 \frac{1}{\rho}\frac{D\rho}{Dt} = \frac{\Gamma_3 - 1}{p}(\rho\epsilon - \vec{\nabla} \cdot \vec{F})$

(internal energy corrected for any work done due to volume change)

where $\vec{F}$ is the energy flux. Static interior solution ⇒ SSM

- Now look for a variation around the SSM solution

$$\rho(\vec{r},t) = \rho_0(r) + \rho'(\vec{r},t)$$

where displacements are small, $\delta(\vec{r}), \qquad v = \frac{\partial}{\partial t}\delta(\vec{r})$

- Plug into the stellar evolution equations (see Balantekin, WH nucl-th9903038)
  - ◇ try normal mode solution $\rho'(\vec{r},t) \sim \rho'(r) Y_{lm}(\theta,\phi) e^{-i\omega t}$
  - ◇ introduce the adiabatic sound speed c, $p = \frac{1}{\Gamma_1}\rho c^2$
  - ◇ and the field $\Psi(r) = c^2\sqrt{\rho}\ \vec{\nabla}\cdot\delta\vec{r}$

and one finds that the equations reduce to a Schroedinger-like form

$$\frac{d^2\Psi(r)}{dr^2} + \frac{1}{c^2}\underbrace{\left[\omega^2 - \omega_{\rm co}^2 - \frac{l(l+1)c^2}{r^2}\left(1 - \frac{N^2}{\omega^2}\right)\right]}_{}\Psi(r) \sim 0$$

$\omega^2_{\rm eff}>0$ propagating $\omega^2_{\rm eff}<0$ damped

an eigenvalue problem, governed by two frequencies

the bouyancy frequency $$N(r) = \sqrt{\frac{Gm(r)}{r}\left(\frac{1}{\Gamma_1}\frac{d\log\, p}{dr} - \frac{d\log\, \rho}{dr}\right)}$$

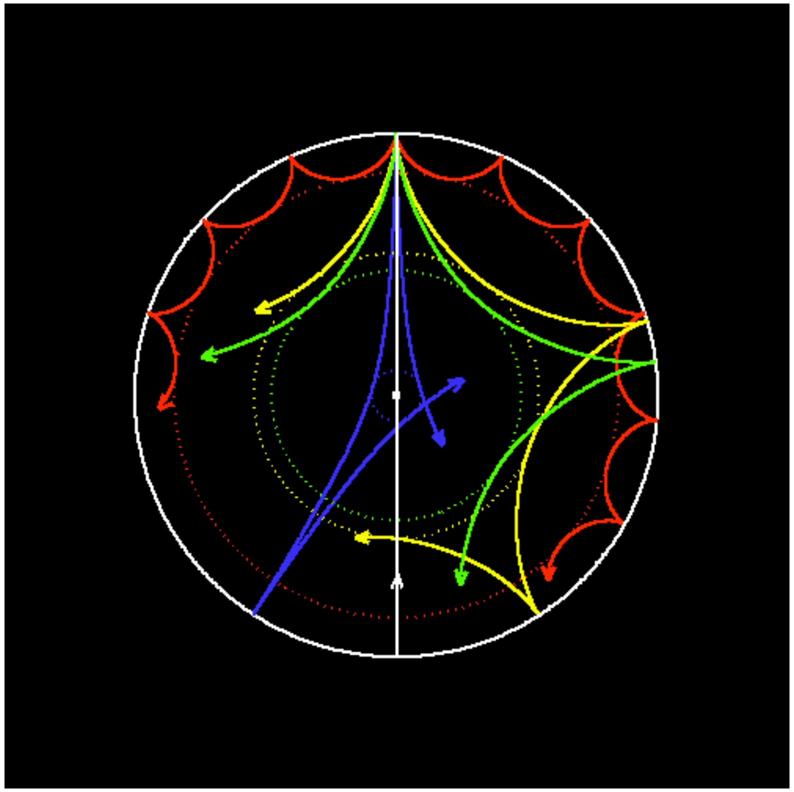
vanishes in the convective zone, ~constant in radiative zone

acoustic cutoff frequency $$\omega_{\rm co} = \frac{c}{2H}\sqrt{1 - 2\frac{dH}{dr}} \text{ where } H^{-1} = -\frac{1}{\rho}\frac{d\rho}{dr}$$

propagating modes are those where one does not "see" a change in the density over a wavelength

p-modes: surface modes, $N = 0, \quad \omega > \omega_{co}, \quad \omega > \frac{l(l+1)c^2}{r^2}$

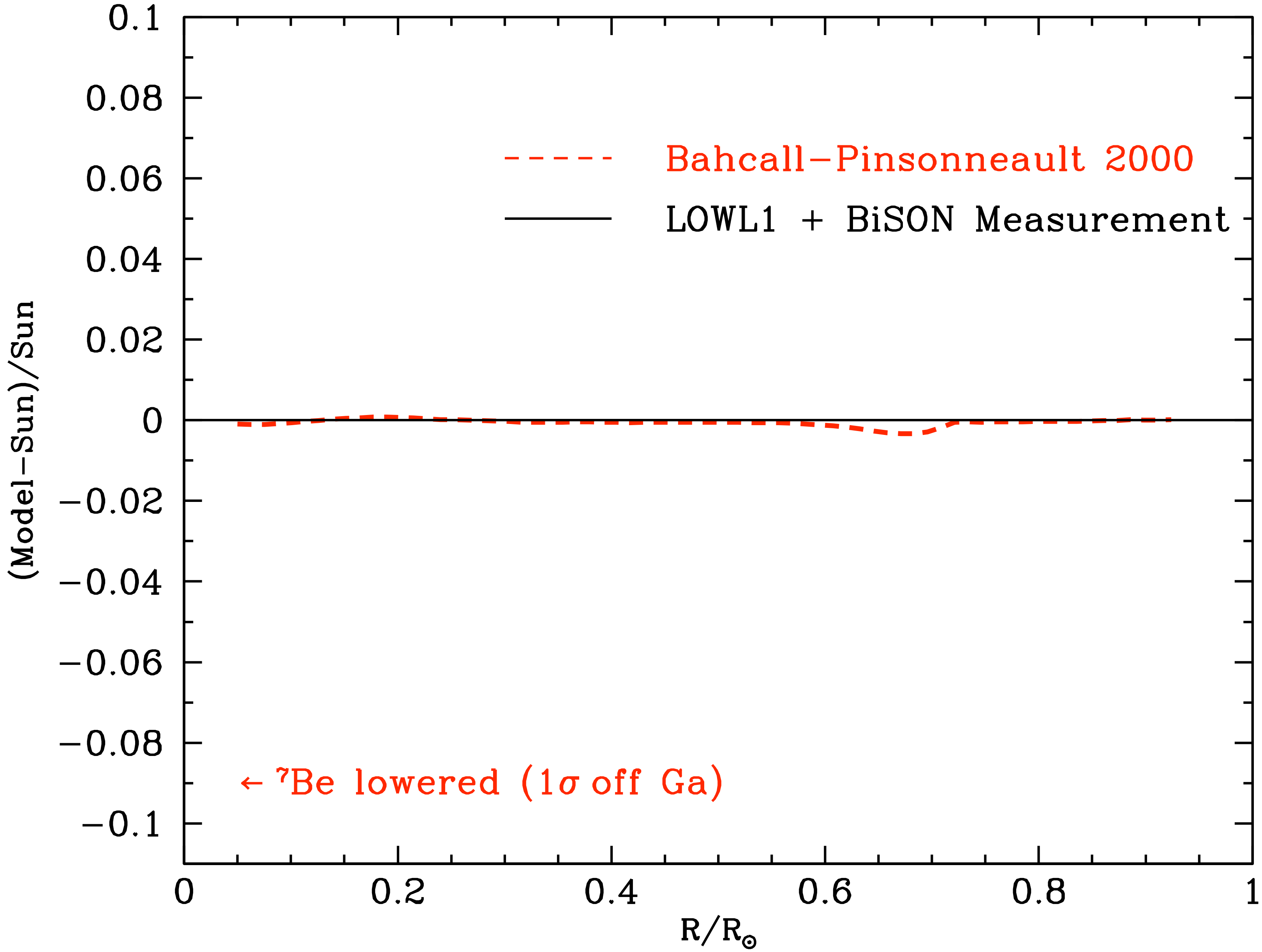
different modes propagate to different depths depending on *l* the turning-point defined by $$\frac{\omega^2 - \omega_{\rm co}^2}{l(l+1)} = \frac{c(r)^2}{r^2}$$



so large-l acoustic modes (p-modes) less penetrating

similar arguments for the gravity modes, those that propagate in the radiative zone, controlled by the bouyancy frequency ($\omega < N$ guarantees propagation at sufficiently small r):
difficult to see, because the surface is in the forbidden region

sound speed c(r) derived from mode inversion, compared to SSM



Bahcall: agreement at 0.2% over 80% of Sun a more severe test than the νs

The neutrino flux discrepancy -- the fact it was not compatible with any adjustment of T in steady-state solar models --combined with the SSM success in helioseismology made a "new-physics" solution more credible

Another development that changed viewpoints was a theoretical step, the recognition that solar matter could enhance ν oscillations

Vacuum flavor oscillations: mass and weak eigenstates

flavor states				mass states
	$\|\nu_e>$	$\leftrightarrow$	$\|\nu_L>\quad m_L$	
	$\|\nu_\mu>$		$\|\nu_H>\quad m_H$	

Noncoincident bases ⇒ oscillations down stream:

$$\begin{aligned} |v_e> &= \cos\theta|\nu_L> + \sin\theta|\nu_H> \\ |v_\mu> &= -\sin\theta|\nu_L> + \cos\theta|\nu_H> \end{aligned}$$

$$\begin{aligned} |\nu_e^k> &= |\nu^k(x=0,t=0)> \qquad E^2 = k^2 + m_i^2 \\ |\nu^k(x\sim ct,t)> &= e^{ikx}\left[e^{-iE_Lt}\cos\theta|\nu_L> + e^{-iE_Ht}\sin\theta|\nu_H>\right] \\ |<\nu_\mu|\nu^k(t)>|^2 &= \sin^2 2\theta \sin^2\left(\frac{\delta m^2}{4E}t\right), \qquad \delta m^2 = m_H^2 - m_L^2 \end{aligned}$$

ν_μ appearance downstream ⇔ vacuum oscillations

(some cheating here: wave packets)

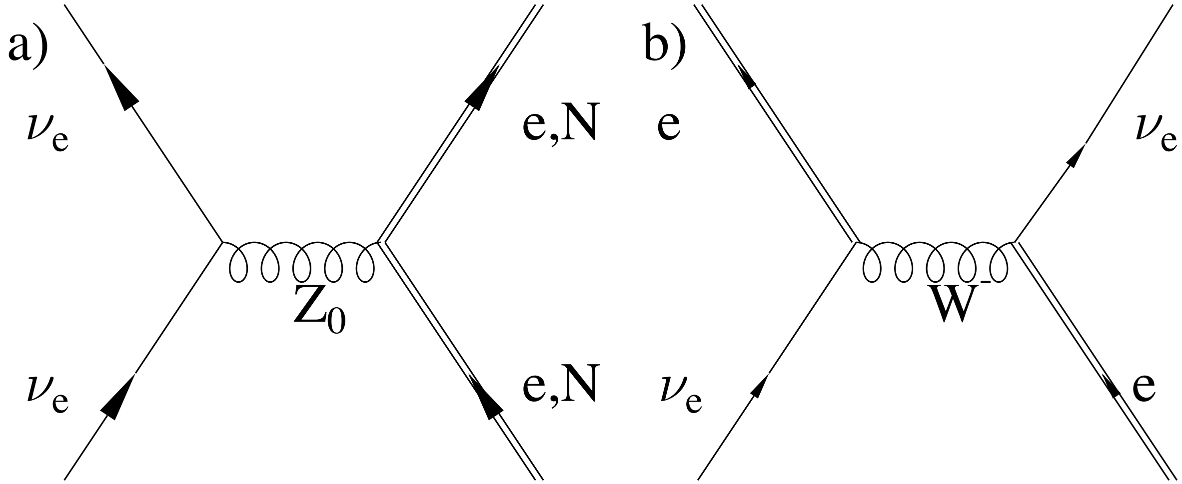
Can slightly generalize this

$$|\nu(0)\rangle \;\rightarrow\; a_e(0)|\nu_e\rangle + a_\mu(0)|\nu_\mu\rangle$$

yielding

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\underbrace{\begin{pmatrix} -\delta m^2\cos 2\theta & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{pmatrix}}_{\text{vacuum } m_\nu^2 \text{ matrix}}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

solar matter generates a flavor asymmetry



- modifies forward scattering amplitude
- explicitly dependent on solar electron density
- makes the electron neutrino heavier at high density

$$m^2_{\nu_e} = 4E\sqrt{2}G_F\ \rho_e(x)$$

inserting this into mass matrix generates the 2-flavor MSW equation

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2 \cos 2\theta + 4E\sqrt{2}G_F\rho_e(x) & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

or equivalently

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2 \cos 2\theta + 2E\sqrt{2}G_F\rho_e(x) & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & -2E\sqrt{2}G_F\rho_e(x) + \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

the m_ν^2 matrix's diagonal elements vanish at a critical density

$$\rho_c: \quad \delta m^2 \cos 2\theta \equiv 2E\sqrt{2}G_F\rho_c$$

Alternately this in terms of local mass eigenstates

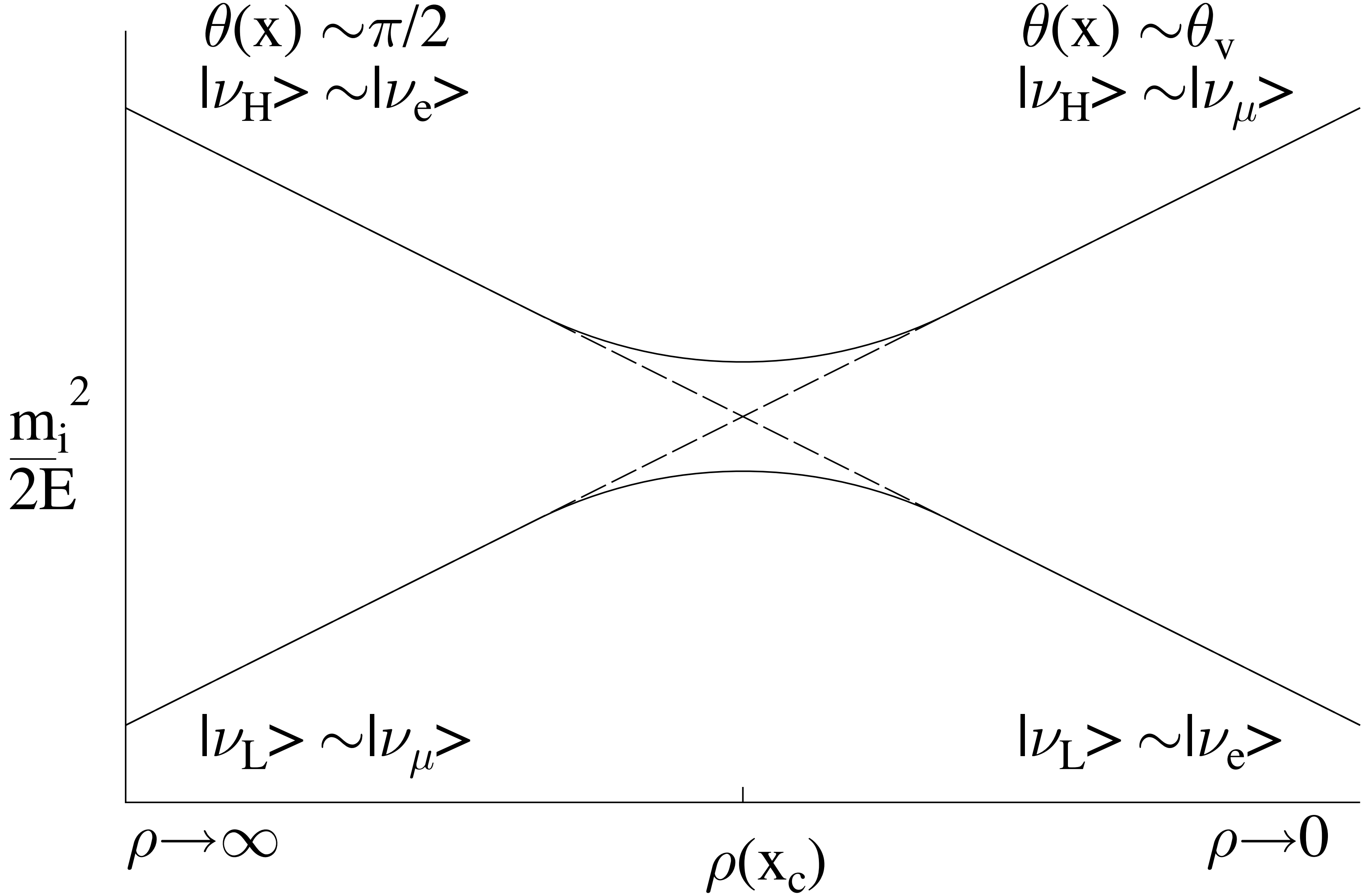
$$|\nu(x)\rangle = a_H(x)|\nu_H(x)\rangle + a_L(x)|\nu_L(x)\rangle$$

$$i\frac{d}{dx}\begin{pmatrix} a_H(x) \\ a_L(x) \end{pmatrix} = \frac{1}{4E}\begin{bmatrix} m_H^2(x) & i\alpha(x) \\ -i\alpha(x) & m_L^2(x) \end{bmatrix}\begin{pmatrix} a_H(x) \\ a_L(x) \end{pmatrix}$$

observe:

- mass splittings small at ρc: avoided level crossing
- $\nu_H(x) \sim \nu_e$ at high density
- if vacuum θ small, $\nu_H(0) \sim \nu_\mu$ in vacuum

thus there is a local mixing angle θ(x) that rotates from $\sim \pi/2 \rightarrow \theta_v$ as $\rho_e(x)$ goes from $\infty \rightarrow 0$



$\theta(x) \sim \pi/2$
$|\nu_H> \sim |\nu_e>$
$\theta(x) \sim \theta_v$
$|\nu_H> \sim |\nu_\mu>$
$\frac{m_i^2}{2E}$
$|\nu_L> \sim |\nu_\mu>$
$|\nu_L> \sim |\nu_e>$
$\rho \rightarrow \infty$
$\rho(x_c)$
$\rho \rightarrow 0$

- it must be that $\alpha(x) \sim \frac{d\rho}{dx}$
- if derivative gentle (change in density small over one local oscillation length) we can ignore: matrix then diagonal, easy to integrate

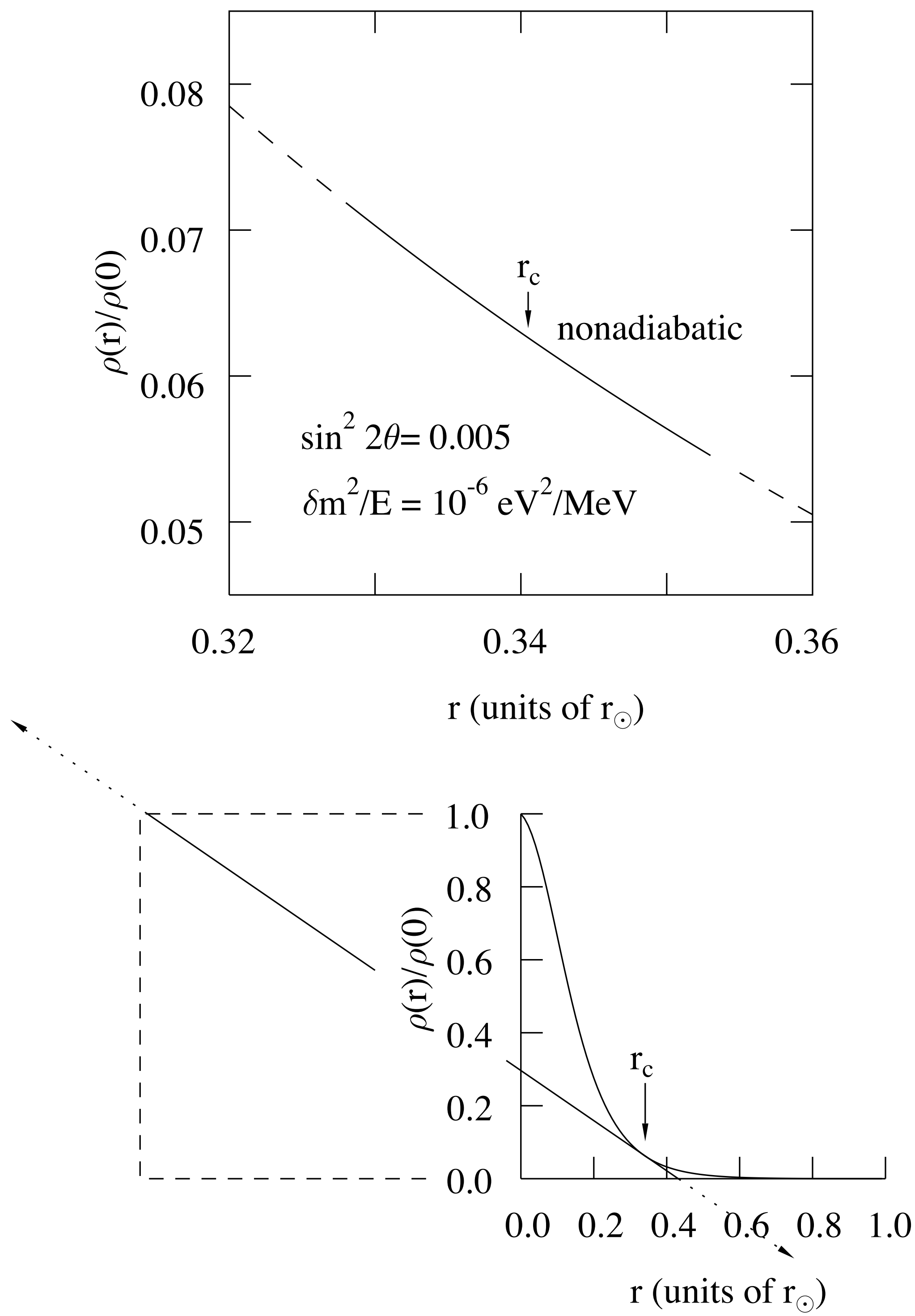
$$\Rightarrow \; P_{\nu_e}^{adiabatic} = \frac{1}{2} + \frac{1}{2}\cos 2\theta_v \cos 2\theta_i \rightarrow 0 \text{ if } \theta_v \sim 0, \theta_i \sim \pi/2$$

- most adiabatic behavior is near the crossing point: small splitting ⇒ large local oscillation length ⇒ can "see" density gradient

- derivative at ρ_c governs nonadiabatic behavior (Landau Zener)

$$P_{\nu_e}^{LZ} = \frac{1}{2} + \frac{1}{2}\cos 2\theta_v \cos 2\theta_i (1 - 2P_{hop})$$

so $\rightarrow 1 \text{ if } \theta_v \sim 0, \theta_i \sim \pi/2, P_{hop} \sim 1$



we can do this problem analytically

$$P_{hop}^{linear} = e^{-\pi\gamma_c/2} \qquad \gamma_c = \frac{\sin^2 2\theta}{\cos 2\theta}\frac{\delta m^2}{2E}\frac{1}{\left|\frac{1}{\rho_c}\frac{d\rho}{dx}\right|}$$

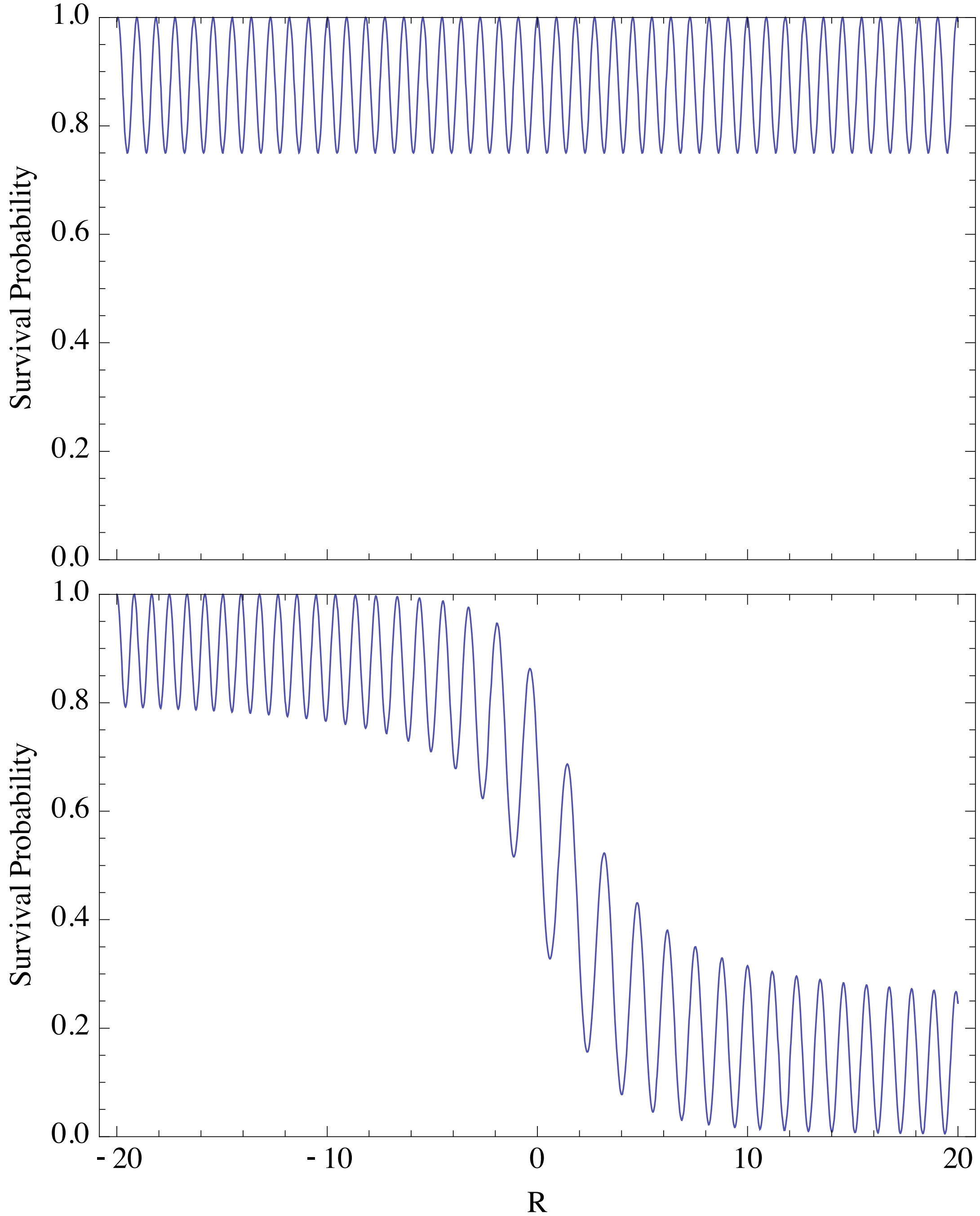
$\Upsilon_c >> 1 \Leftrightarrow$ adiabatic, so strong flavor conversion

$\Upsilon_c << 1 \Leftrightarrow$ nonadiabatic, little flavor conversion

so two conditions for strong flavor conversion:

sufficient density to create a level crossing
adiabatic crossing of that critical density

MSW mechanism is about passing through a level crossing



Mathematica HW problem

a) vacuum oscillations θ=15°
R from -20 to +20

R in units of $\dfrac{4E\cos 2\theta}{\delta m^2 \sin^2 2\theta}$

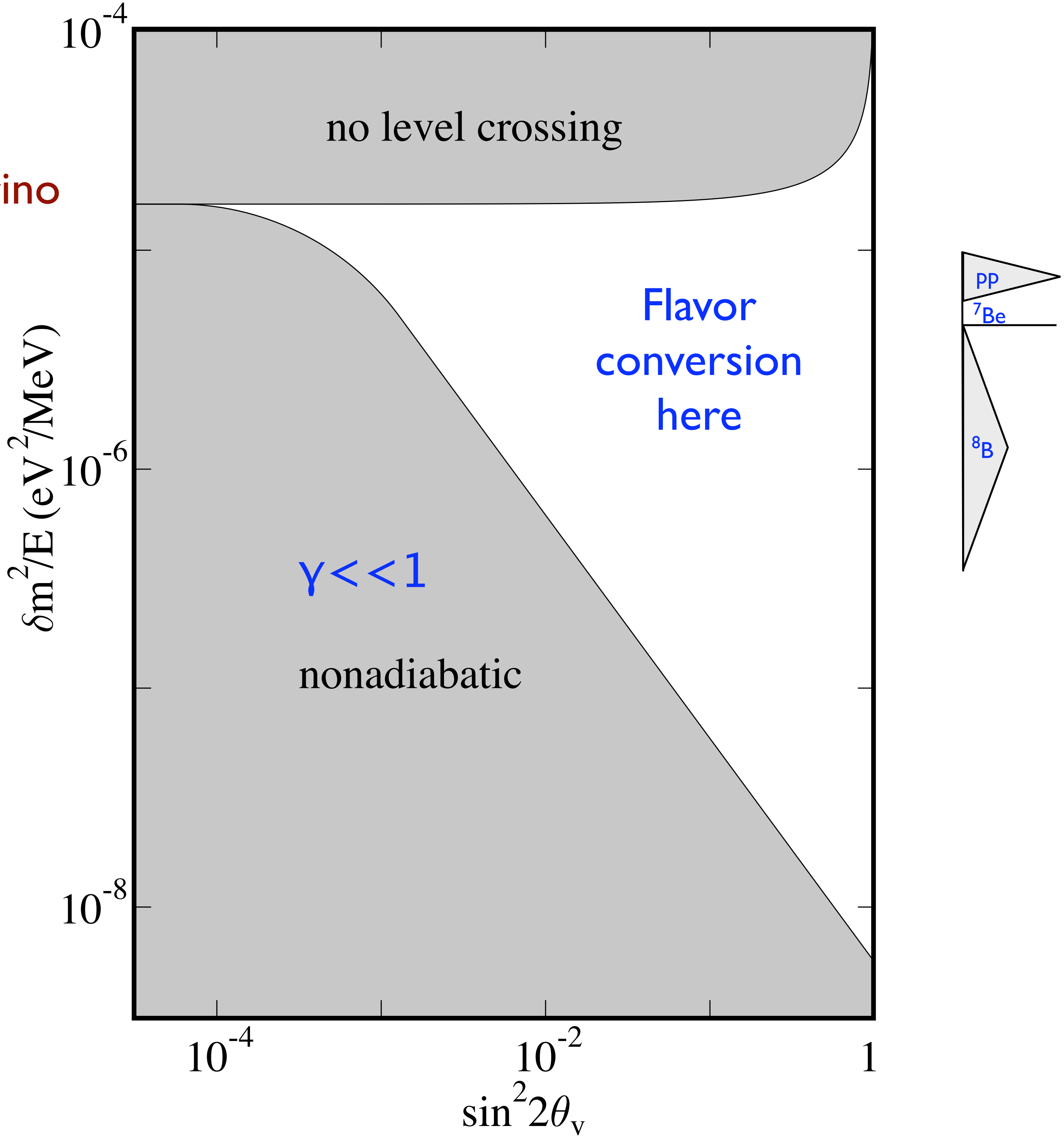
b) matter oscillations

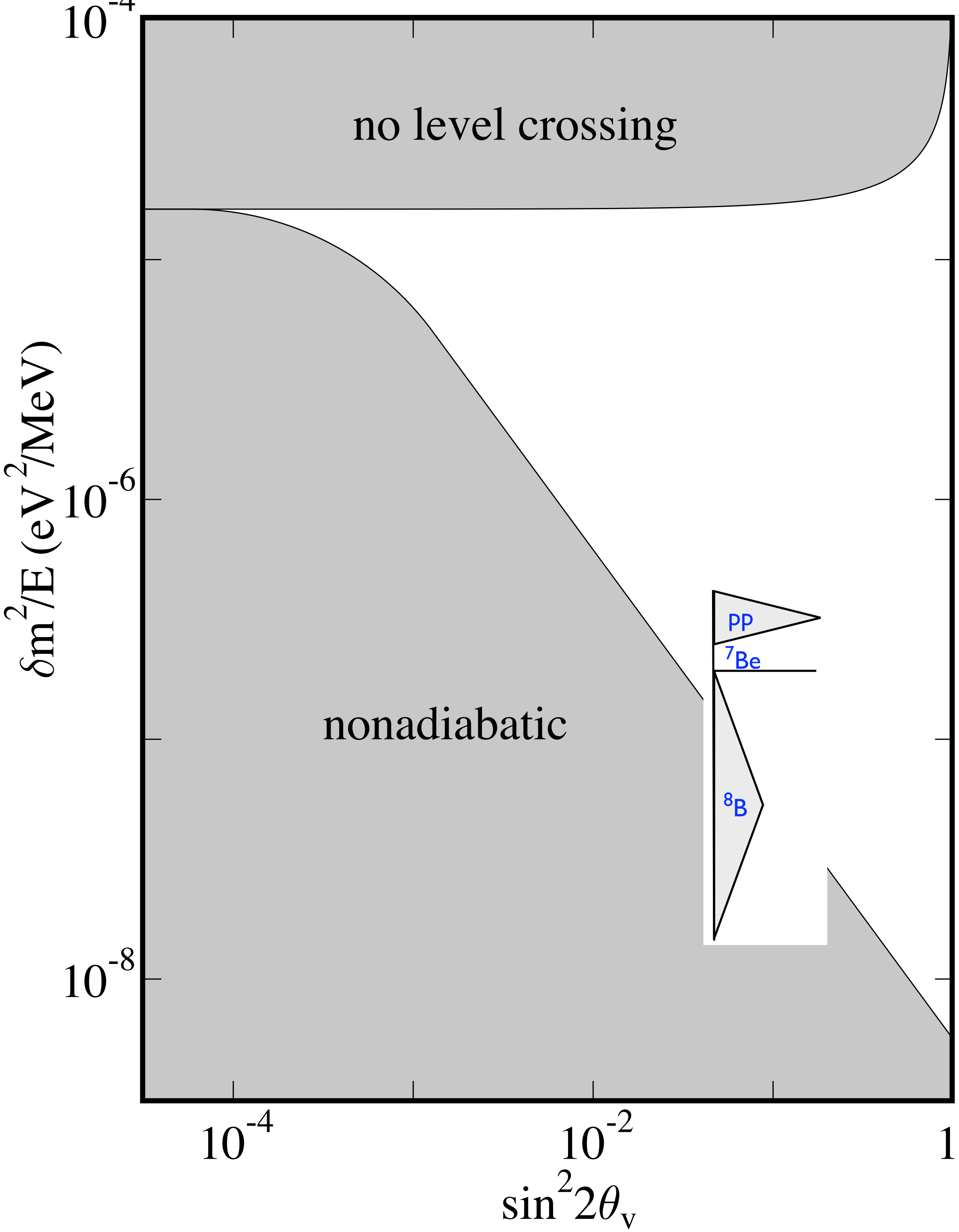
add $\rho_e(R) \propto 1 - \dfrac{2}{\pi}\arctan aR$

normalize so that crossing occurs at R = 0

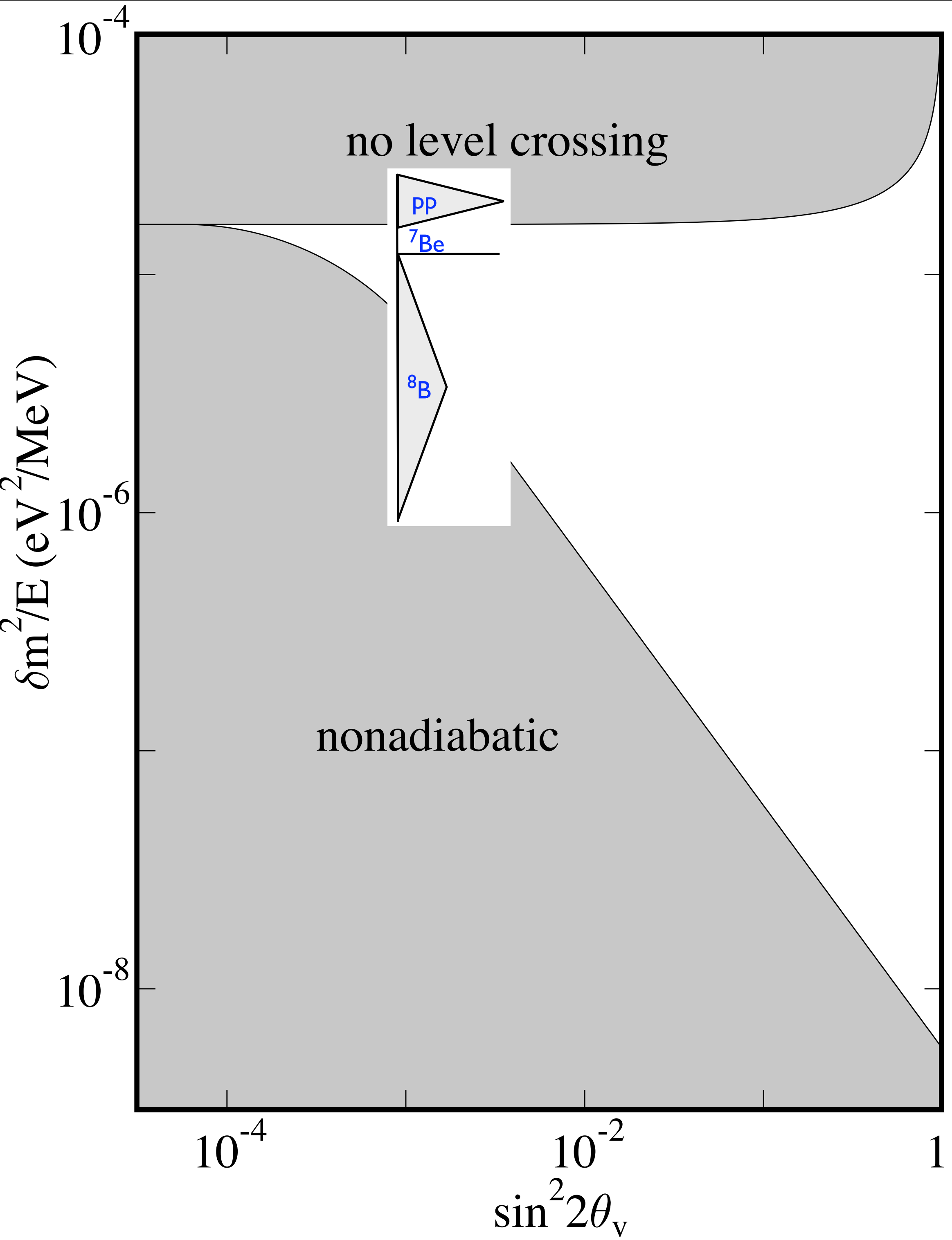
note $\rho_e(R) \to 0$ as $R \to \infty$

So ν_e is produced as a heavy eigenstate, then propagates toward the vacuum, where it is the light eigenstate solving the solar neutrino problem

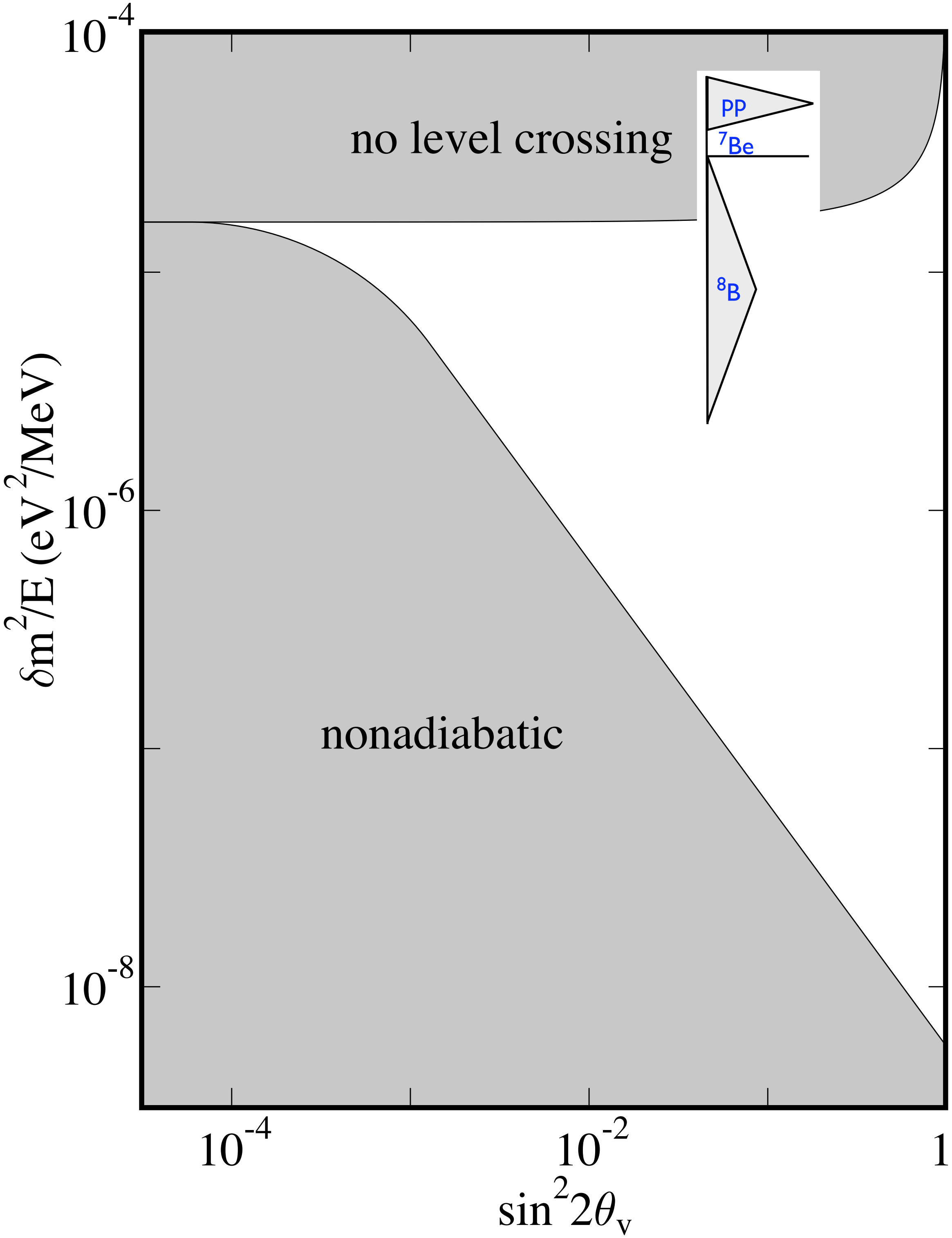




Low solution



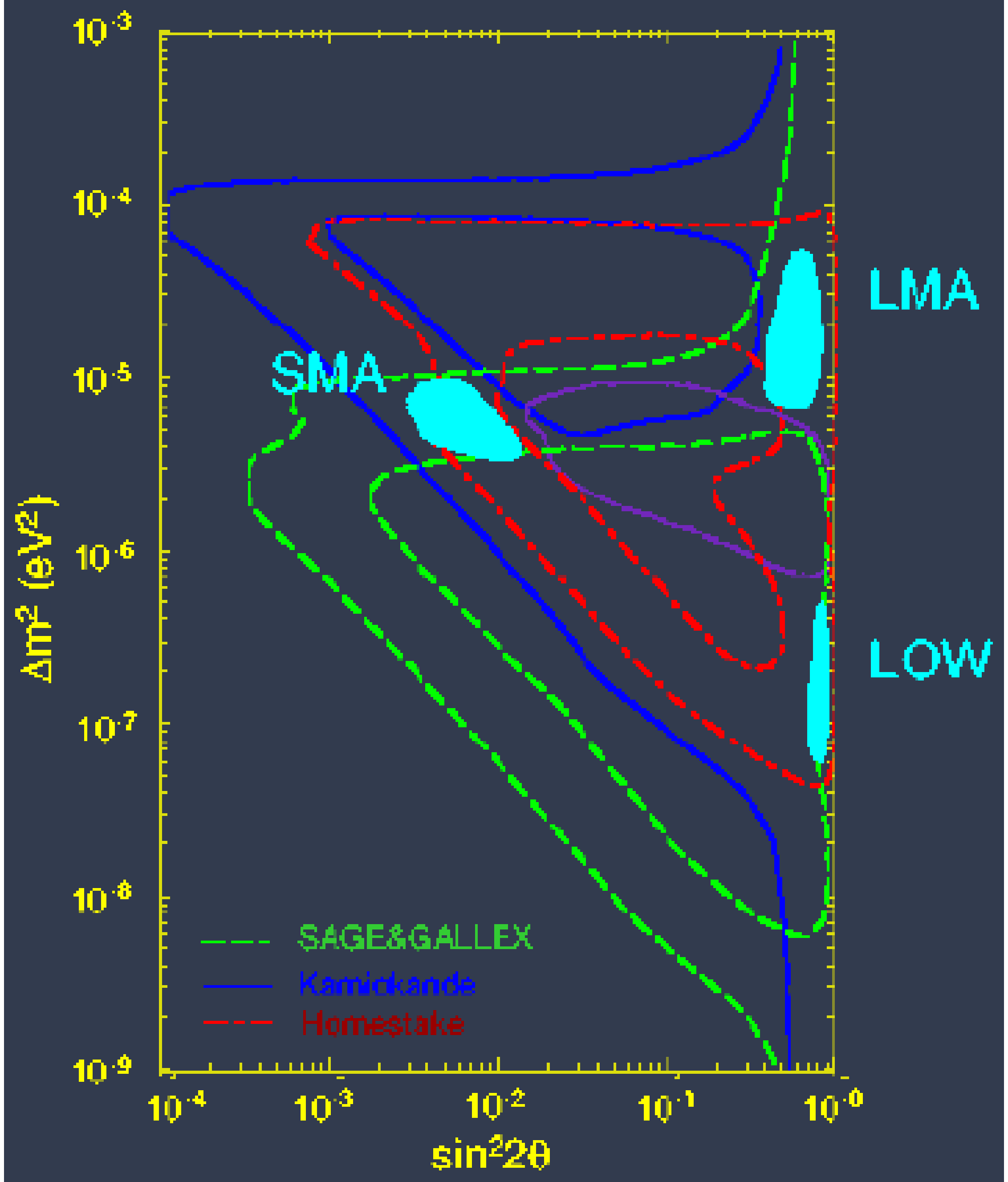
Small angle solution



Large angle solution

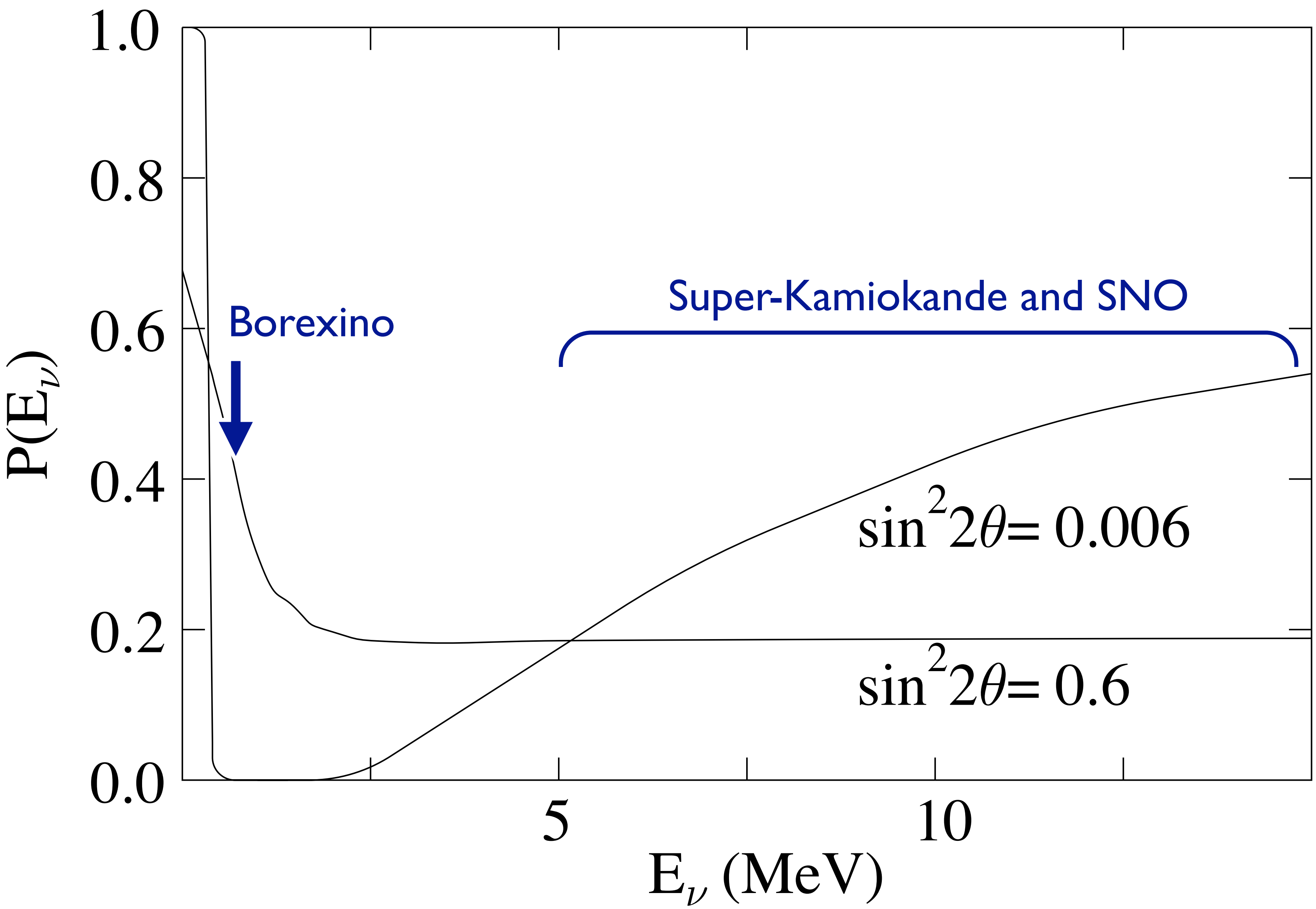
this is the solution matching SNO and SuperK results + Ga/Cl/KII

$\tan^2\theta_v \sim 0.40$



LMA
SMA
LOW
SAGE&GALLEX
Kamiokande
Homestake
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
Δm^2 (eV^2)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
$\sin^2 2\theta$

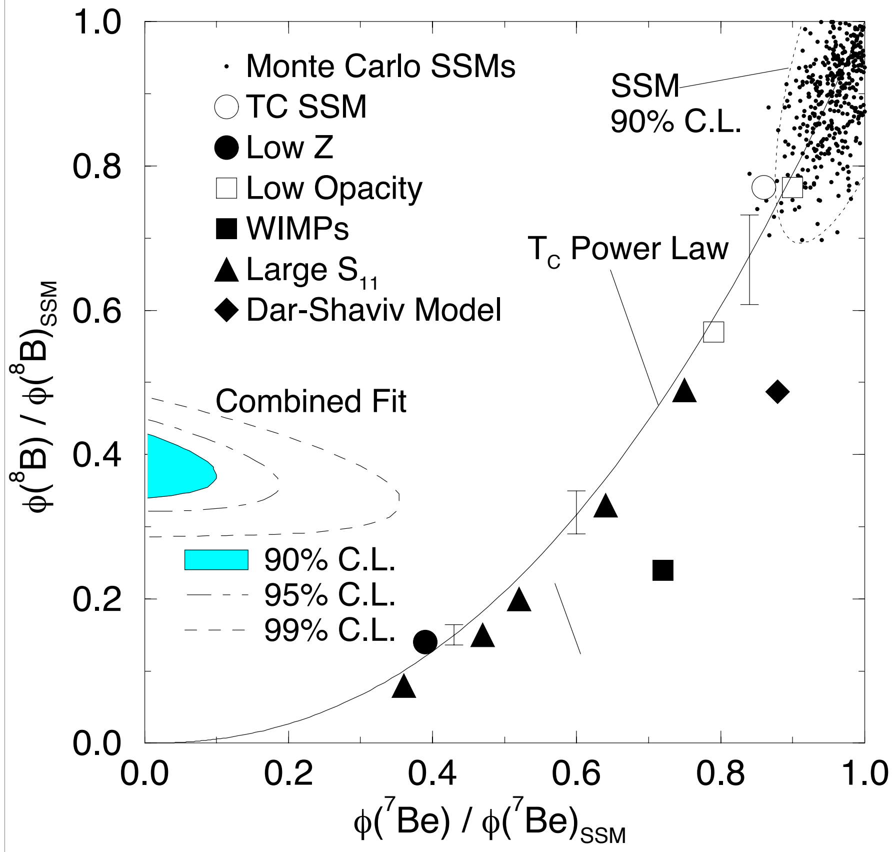
distinctive energy-dependent suppression



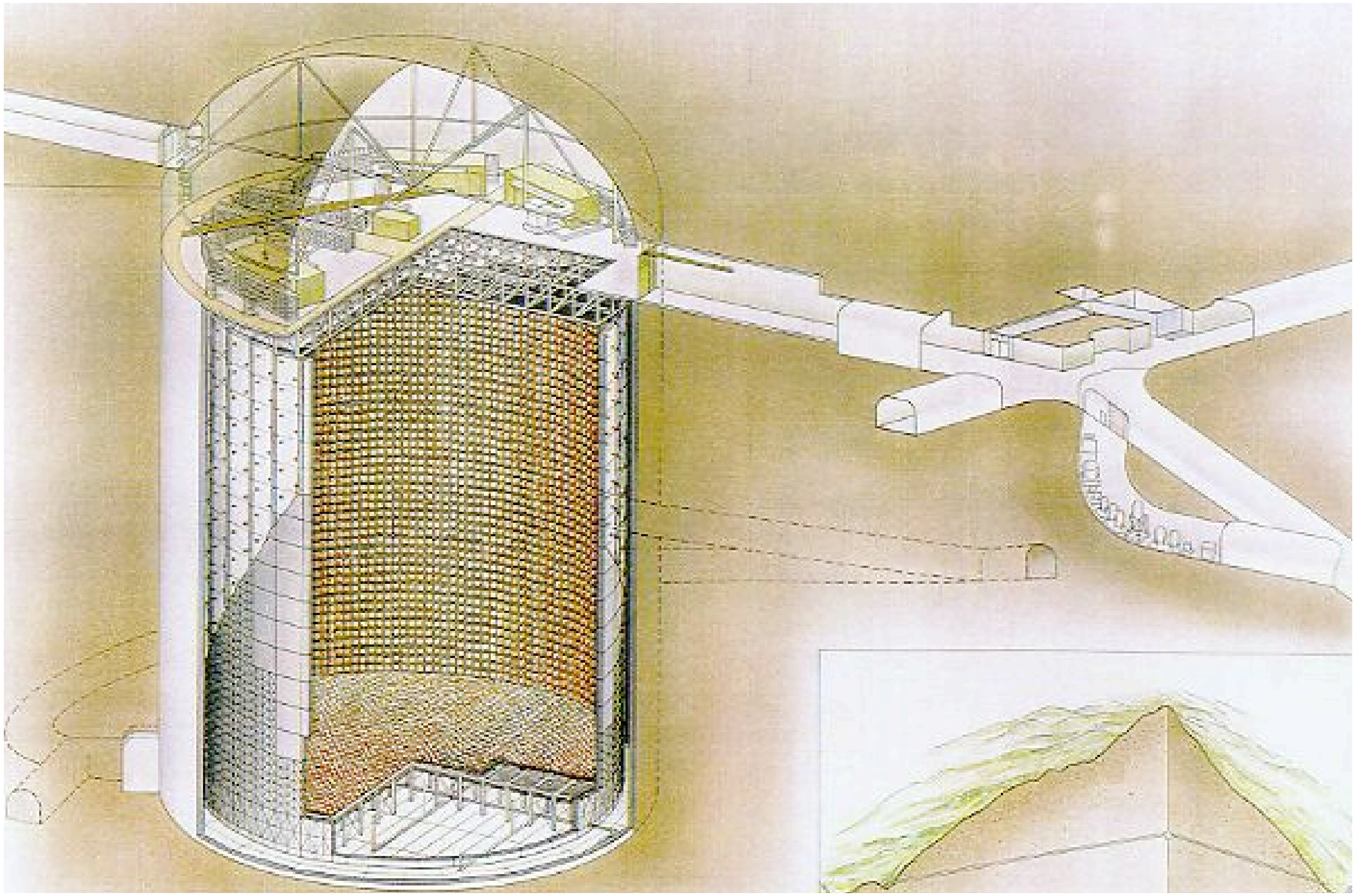
Neutrino oscillations had been one of the early suggestions for solving the solar neutrino puzzle (Pontecorvo) -- but the apparent need for nearly complete mixing of three neutrino species to produce the needed factor-of-three reduction in the Cl counting rate seemed a stretch. The known quark mixing angles are small.

The MSW mechanism provided a means for suppressing the flux even if the mixing angle were small;

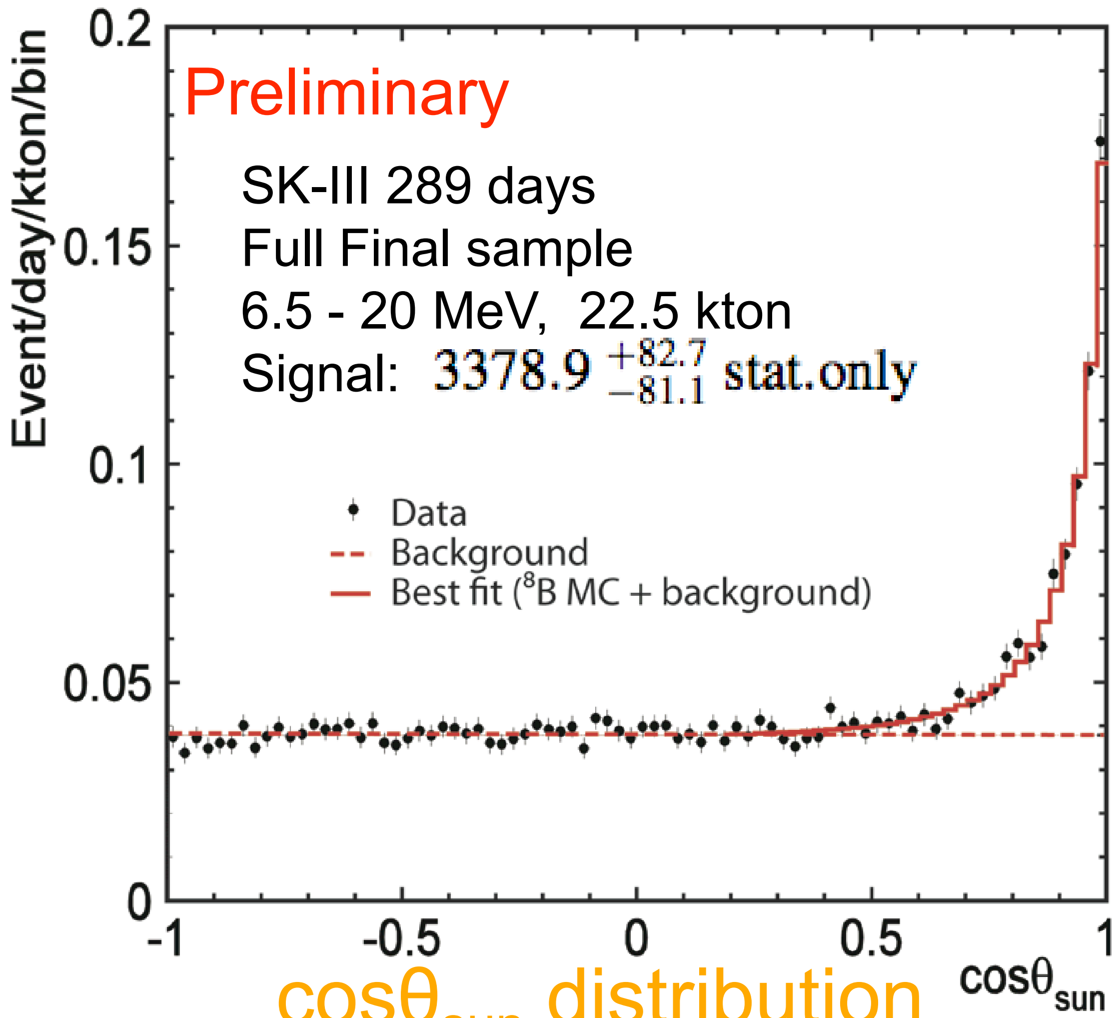
and the energy-dependent reductions that the data seemed to demand.



By the mid-1980s planning was underway for two next-generation experiments to resolve the solar neutrino puzzle, Super-Kamiokande and SNO



SK increase in fiducial volume (from 0.68 to 22 ktons) provided the potential to see spectrum distortions or day-night matter effects -- reconstructed from spectrum of scattered electrons in $\nu_e(\nu_x) + e \rightarrow \nu_e'(\nu_x') + e'$

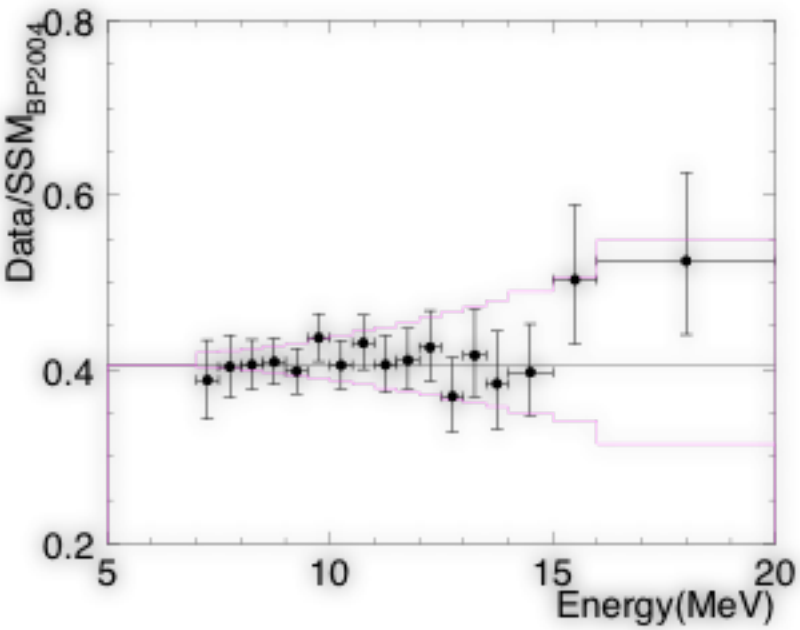


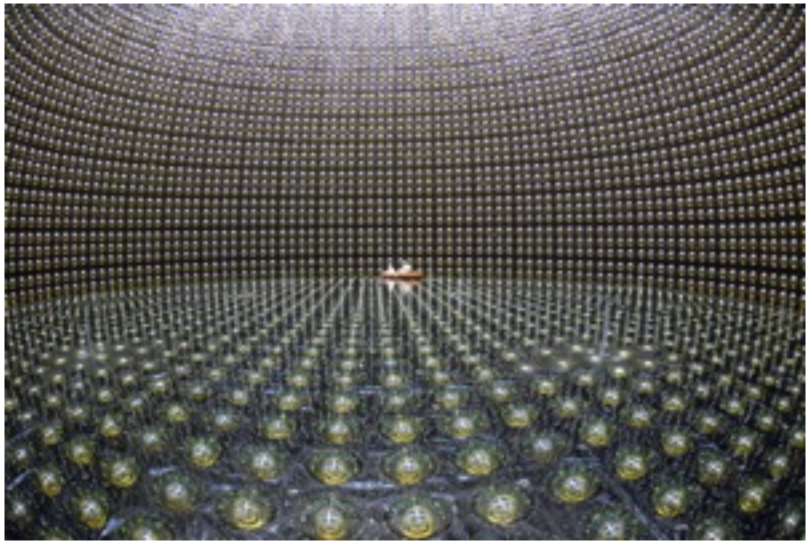
$\cos\theta_{sun}$ distribution

SKI (1996 →), SKII (2002 →), SKIII (2006 →), SKIV (2008 →)

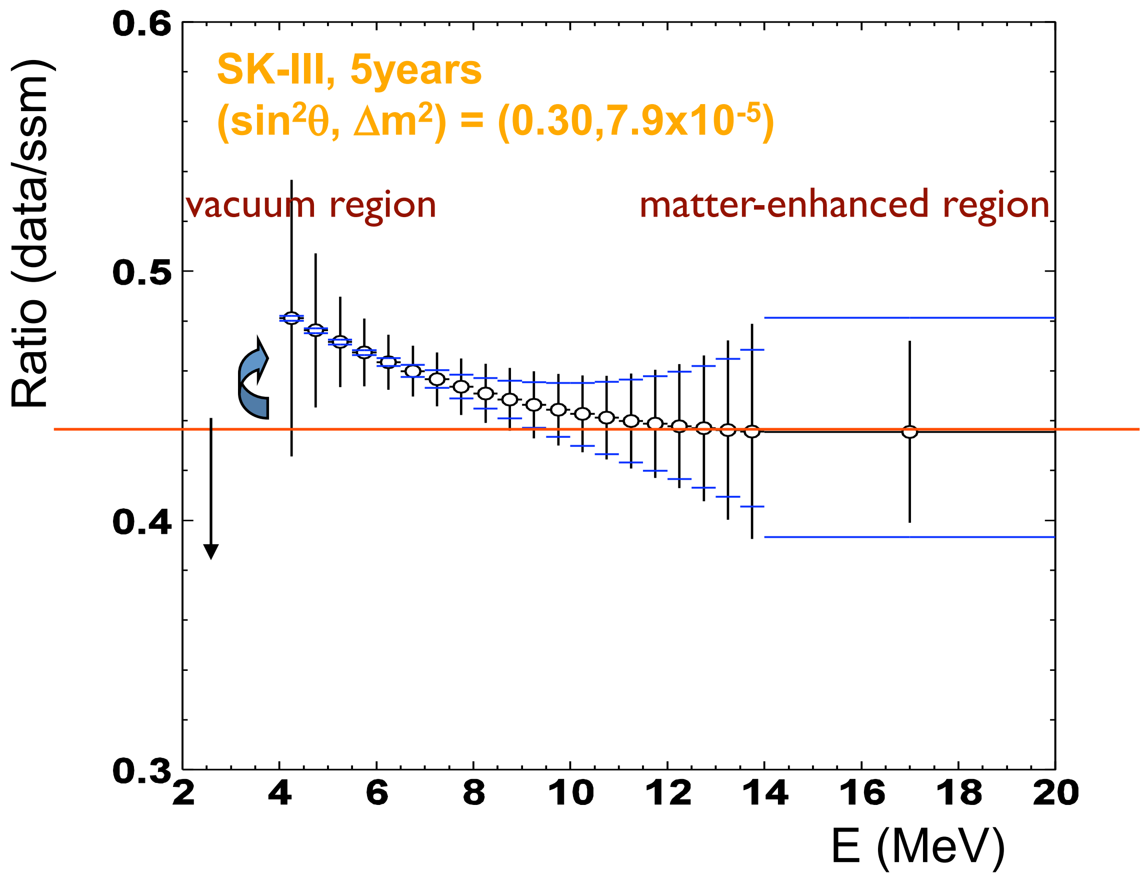
$$\phi(^8\mathrm{B}) = \left[2.38 \pm 0.05(\mathrm{stat})^{+0.16}_{-0.15}(\mathrm{sys})\right] \cdot 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$ SKII

$$\delta(\mathrm{day/night}) = -6.3 \pm 4.2(\mathrm{stat}) \pm 3.7(\mathrm{sys})\%$$

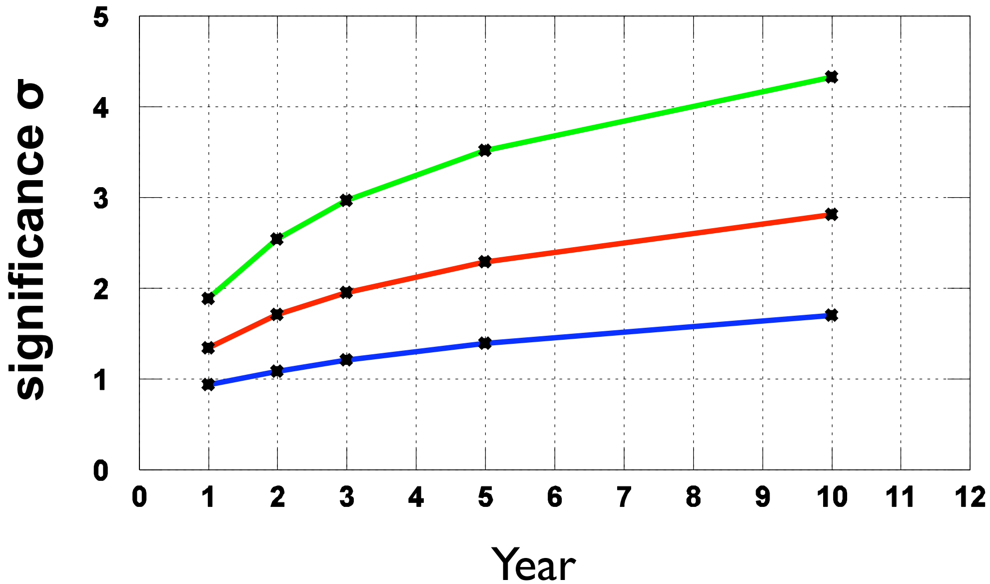




Ratio of SKII observed to SSM energy spectrum.
Purple: 1σ level of energy-correlated systematic errors.



Low-energy turnup predicted is potentially a 10% effect, detectable with proper attention to energy-correlated systematic errors, and with a reduced threshold



Phase	Energy response	Energy threshold
SK-I	6p.e./MeV	5.0MeV
SK-II	3p.e./MeV	7.0MeV
SK-III	6p.e./MeV	5.0MeV→4.5MeV
SK-IV	6p.e./MeV	4.0MeV

Sudbury Neutrino Observatory

- Suggested by Herb Chen in mid-1980s: replacement of the ordinary water in a Cerenkov detector with heavy water

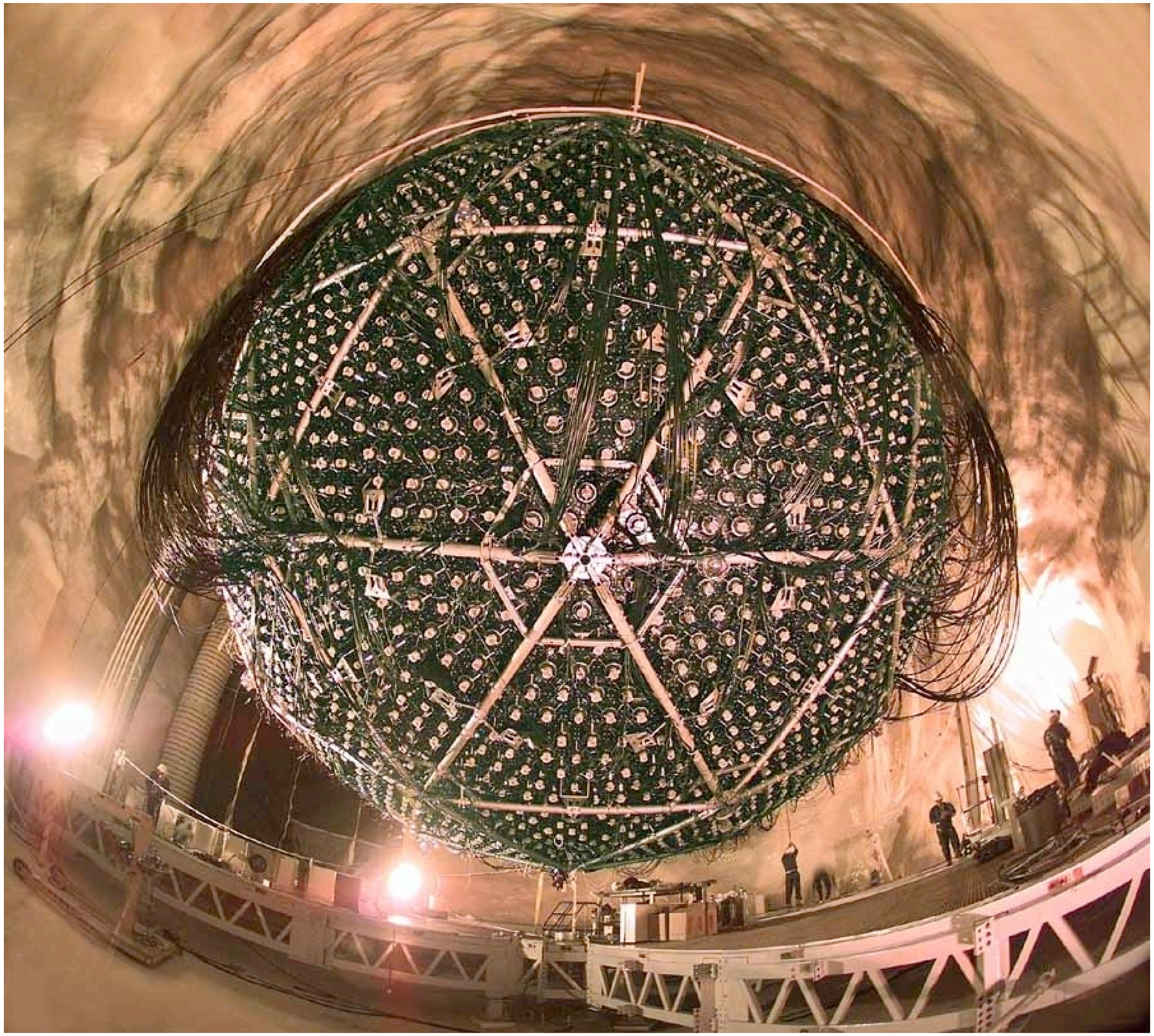
- Provides three complementary detection channels

	reaction	detected
1)	$\nu_x + e \rightarrow \nu_x' + e'$	scattered electron
2)	$\nu_e + D \rightarrow p + p + e'$	produced electron
3)	$\nu_x + D \rightarrow p + n + \nu_x'$	produced neutron

1) isolated from the forward-peaking of the scattering: energy shared among outgoing leptons -- sensitive to $\nu_e s$, reduced sensitive to $\nu_{\mu,\tau} s$

2) detected by the scattered electron, hard spectrum with $E_e \sim E_\nu - 1.44$ MeV, as GT strength concentrated near threshold; angular distribution $1 - 1/3\ \cos\theta$; only sensitive to $\nu_e s$

3) detected by capture of the produce neutron: total cross section measured; sensitive equal to νs of any flavor



- Detection of electrons weakly correlated with direction, and especial of neutrons, placed exception requirements on background reduction
    - ◇ cavity at exceptional depth of 2 kilometers to reduce muons
    - ◇ construction under cleanroom conditions: tiny quantities of dust in 12-story cavity would have produced neutrons above the expected solar rate, 8/day

- Experiment proceeded in three phases, depending on the neutral current detection scheme
    - ◇ capture on deuterium d(n,γ) producing a 6.25 MeV γ — Phase I
    - ◇ capture on 2 tons of dissolved salt:

      $^{35}\mathrm{Cl}(n,\gamma)$ 8.6 MeV energy release — Phase II
    - ◇ capture in ^{3}He proportional counters — Phase III

- Recent low-energy re-analysis of Phases I and II, reaching to electron kinetic energies of 3.5 MeV

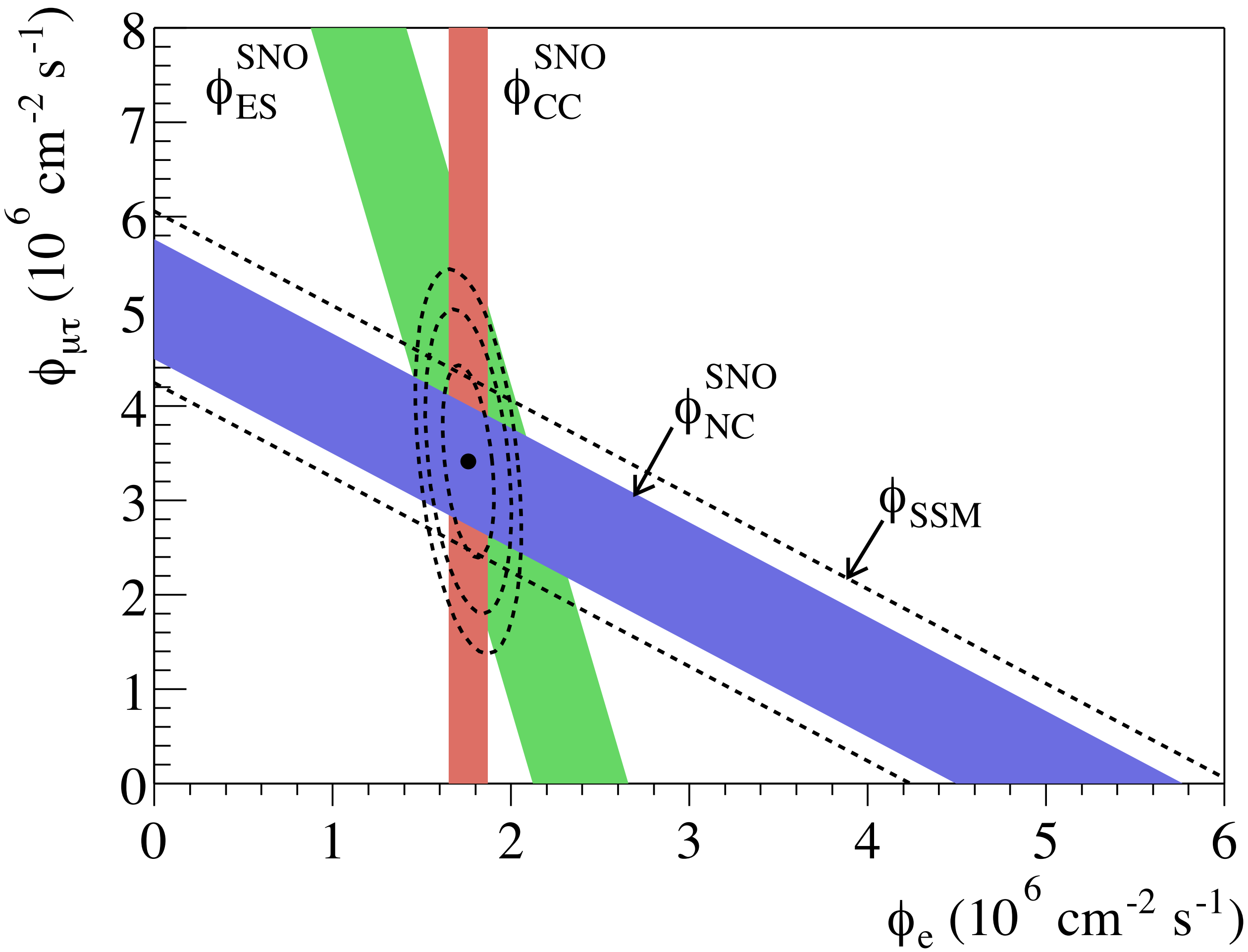


Figure 2: Flux of ^{8}B solar neutrinos is divided into ν_μ/ν_τ and ν_e flavors by the SNO analysis. The diagonal bands show the total ^{8}B flux as predicted by the SSM (dashed lines) and that measured with the NC reaction in SNO (solid band). The widths of these bands represent the $\pm1\sigma$ errors. The bands intersect in a single region for $\phi(\nu_e)$ and $\phi(\nu_\mu/\nu_\tau)$, indicating that the combined flux results are consistent with neutrino flavor transformation assuming no distortion in the ^{8}B neutrino energy spectrum.

- SNO thus definitively resolved the solar neutrino problem
- The detector is dismantled, making space for SNO+, but the analysis continues. The best-fit combined-analysis two-flavor parameters are

$$\delta m^2_{12} = 7.59^{+0.20}_{-0.21} \times 10^{-5}\ \mathrm{eV}^2$$
$$\theta_{12} = 34.06^{+1.16}_{-0.84}\ \mathrm{degrees}$$

- The SSM was found to be consistent with the measurements

$$\phi(^8\mathrm{B}) = \left(5.046^{+0.169}_{-0.152}(\mathrm{stat})^{+0.107}_{-0.123}(\mathrm{syst})\right) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

$$\mathrm{BPS08(OP;GS)}\quad 5.95 \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$
$$\mathrm{BPS08(OP;AGS)}\quad 4.72 \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

``I was called right after the[SNO] announcement was made by someone from the New York Times and asked how I felt. Without thinking I said `I feel like dancing I'm so happy.' ... It was like a person who had been sentenced for some heinous crime, and then a DNA test is made and it's found that he isn't guilty. That's exactly the way I felt." JNB