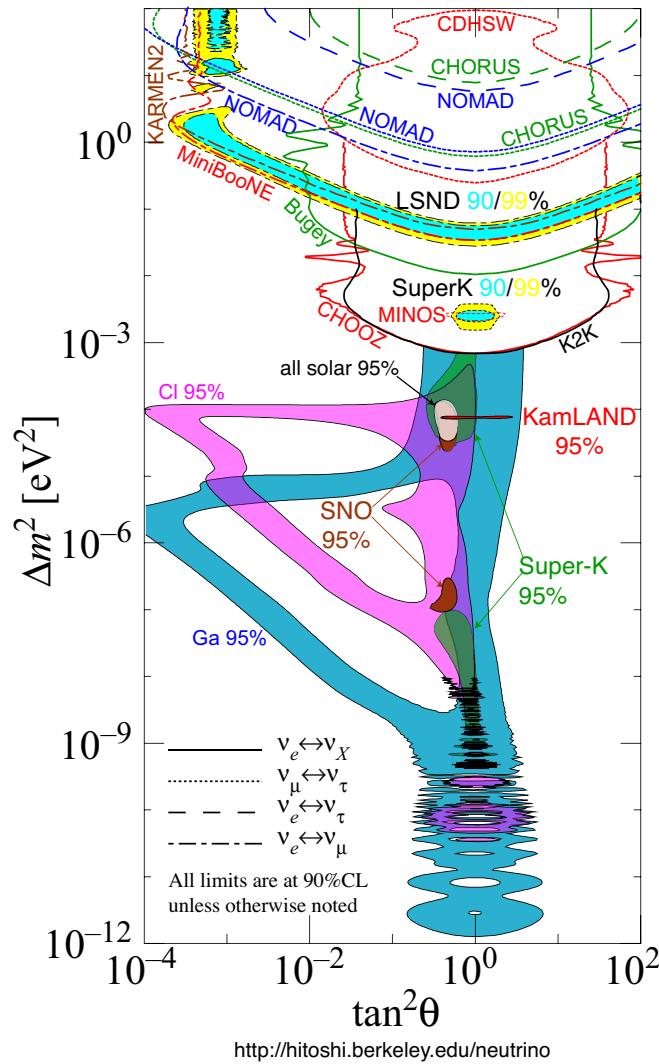


Neutrino Basics



- Neutrinos as a Probe
- Electroweak Interactions of Neutrinos
- Mass, Mixing, and Intrinsic Properties
- Neutrino Oscillations
- Models
- Outstanding Issues

Reference: *The Standard Model and Beyond*, CRC Press

Neutrinos as a Unique Probe: $10^{-33} - 10^{+28}$ cm

- Particle Physics

- $\nu N, \mu N, e N$ scattering: existence/properties of quarks, QCD
- Weak decays ($n \rightarrow p e^- \bar{\nu}_e, \mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$): Fermi theory, parity violation, quark mixing
- Neutral current, Z -pole, atomic parity: electroweak unification, field theory, m_t ; severe constraint on physics to TeV scale
- Neutrino mass: constraint on TeV physics, grand unification, superstrings, extra dimensions; seesaw: $m_\nu \sim m_q^2/M_{\text{GUT}}$

- **Astrophysics/Cosmology**

- Core of Sun
- Supernova dynamics
- Atmospheric neutrinos (cosmic rays)
- Violent events (AGNs, GRBs, cosmic rays)
- Large scale structure (dark matter)
- Nucleosynthesis (big bang - small A ; stars → iron; supernova - large N)
- Baryogenesis
- Simultaneous probes of ν and astrophysics

- **Interior of Earth**

The Standard Model

- Standard model: $SU(2) \times U(1)$ (extended to include ν masses)
+ QCD + general relativity
- Mathematically consistent, renormalizable theory
- Correct to 10^{-16} cm
 - QCD: short distance, long distance symmetries
 - QED, WCC, WNC, W , Z
 - Gauge self-interactions
- Missing: Higgs (or alternative), dark matter, dark energy

The Gauge Group

- Gauge group $SU(3) \times SU(2) \times U(1)$; gauge couplings g_s, g, g'

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$$

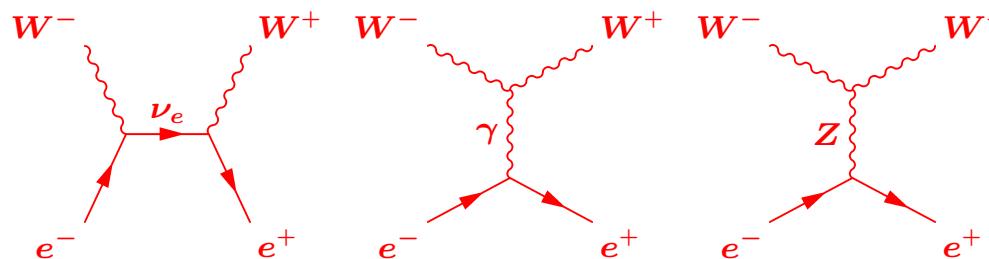
$$\begin{array}{cccc} u_R & u_R & u_R & \nu_{eR}(?) \\ d_R & d_R & d_R & e_R^- \end{array}$$

(L = left-handed, R = right-handed)

- $SU(3)$: $u \leftrightarrow u$ $\leftrightarrow u$, $d \leftrightarrow d$ $\leftrightarrow d$ (8 gluons)
- $SU(2)$: $u_L \leftrightarrow d_L$, $\nu_{eL} \leftrightarrow e_L^-$ (W^\pm); phases (W^0)
- $U(1)$: phases (B)
- Heavy families (c, s, ν_μ, μ^-) , (t, b, ν_τ, τ^-)

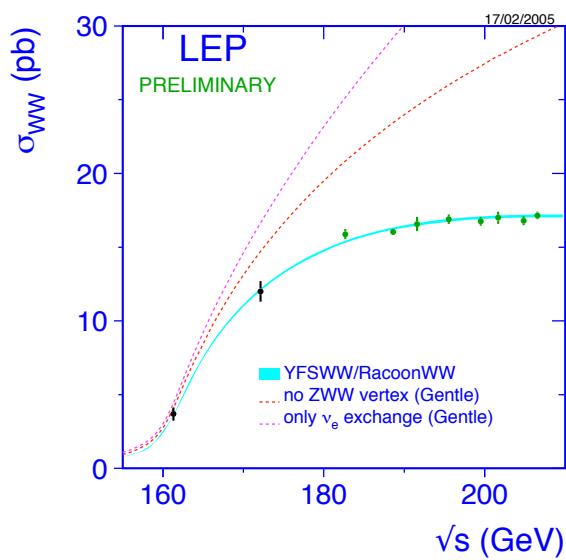
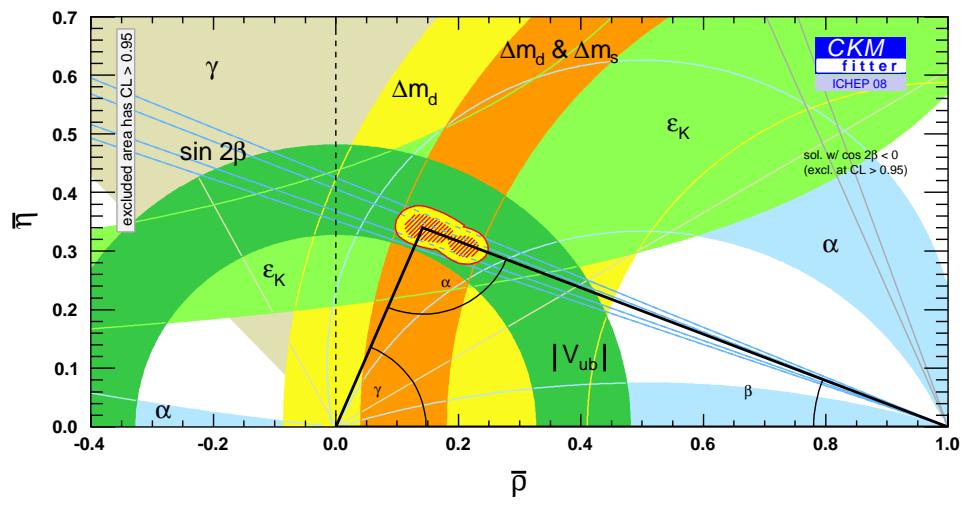
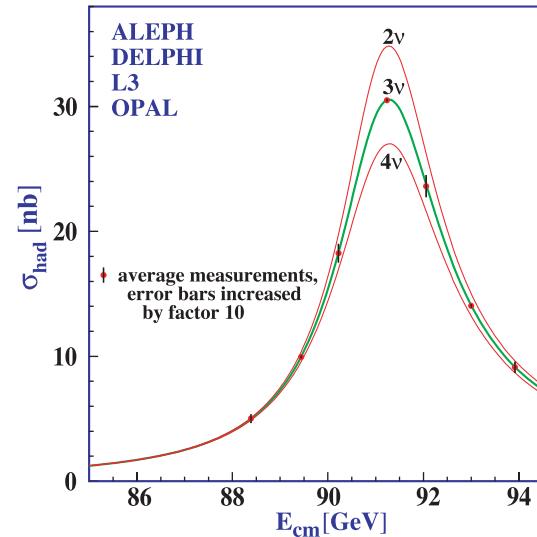
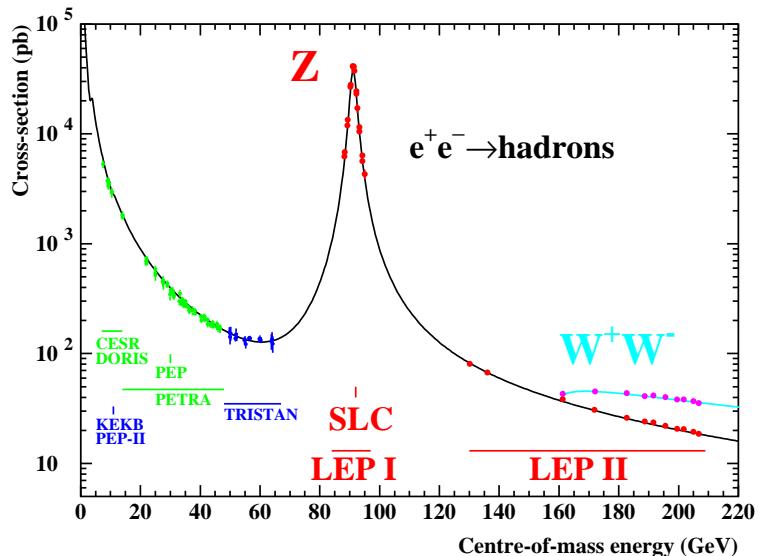
The Electroweak Theory

- QED and weak charged current unified
- Weak neutral current (Z) predicted ($\nu N \rightarrow \nu X$, atomic parity violation)
- Stringent tests of WCC, CP -violation, WNC, Z -pole, beyond
- Fermion gauge and gauge self-interactions

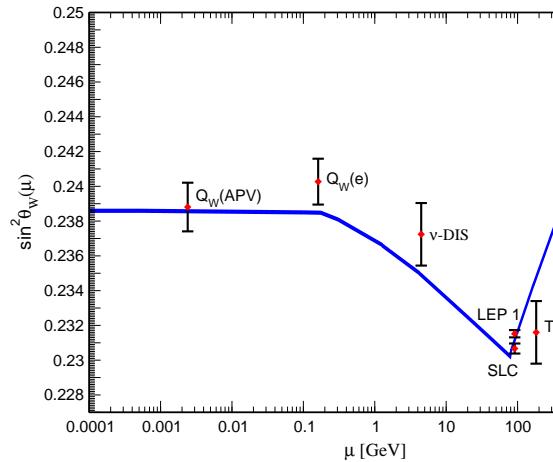


$$\begin{aligned} & -ieq_f \gamma^\mu & & -i\frac{g}{2 \cos \theta_W} \gamma^\mu (g_V^f - g_A^f \gamma^5) & & -i\frac{g}{2\sqrt{2}} \gamma^\mu (1 - \gamma^5) V_{qij} \end{aligned}$$

$$\begin{aligned} & \frac{-ig}{2\sqrt{2}} J_W^{\nu\dagger} & & \frac{-ig}{2\sqrt{2}} J_W^\mu & & \frac{-ig}{2\cos \theta_W} J_Z^\nu & & \frac{-ig}{2\cos \theta_W} J_Z^\mu \end{aligned}$$



- SM correct and unique to zeroth approximation
(gauge principle, group, representations)
- SM correct at loop level
(renorm gauge theory; m_t , α_s , M_H)
- TeV physics severely constrained
(unification versus compositeness)
- Consistent with light elementary Higgs
- Precise gauge couplings
(SUSY gauge unification)



Standard Model Lagrangian after symmetry breaking

$$\begin{aligned}\mathcal{L} = & \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \sum_i \bar{\psi}_i \left(i \not{\partial} - m_i - \frac{m_i H}{\nu} \right) \psi_i \\ & - \frac{g}{2\sqrt{2}} \left(J_W^\mu W_\mu^- + J_W^{\mu\dagger} W_\mu^+ \right) - e J_Q^\mu A_\mu - \frac{g}{2 \cos \theta_W} J_Z^\mu Z_\mu\end{aligned}$$

Mass eigenstate bosons: W^\pm , Z , and A (photon); H (Higgs)

$$\begin{aligned}Z &= -\sin \theta_W B + \cos \theta_W W^0 \\ A &= \cos \theta_W B + \sin \theta_W W^0\end{aligned}$$

Weak angle: $\tan \theta_W \equiv g'/g$

Positron charge: $e = g \sin \theta_W$

Electroweak scale: $\nu \equiv \sqrt{2} \langle \varphi^0 \rangle \simeq 246 \text{ GeV}$

Free Dirac Fermion (four-component)

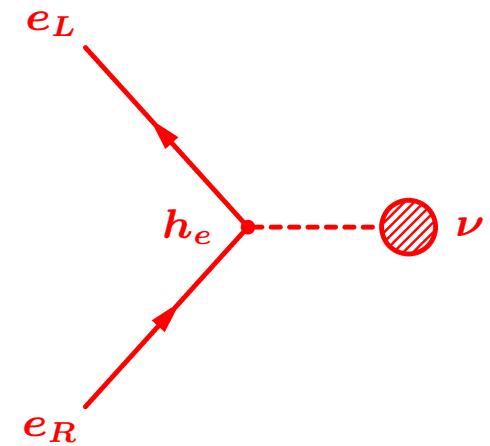
$$\begin{aligned}\mathcal{L} &= \bar{\psi} (i \not{\partial} - m) \psi \\ &= i\bar{\psi}_L \not{\partial} \psi_L + i\bar{\psi}_R \not{\partial} \psi_R - m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)\end{aligned}$$

where

$$\psi_L = \frac{1 - \gamma^5}{2} \psi \quad \psi_R = \frac{1 + \gamma^5}{2} \psi$$

are left (right) chiral projections (chirality = helicity up to $\mathcal{O}(m/E)$)

- ψ_L and ψ_R are (two-component) Weyl spinors
- ψ_L annihilates *L*-particle or creates *R*-antiparticle (opposite for ψ_R)
- m generated by interaction with Higgs in standard model (bare mass forbidden by $SU(2) \times U(1)$)



Particle Mixing

- Consider F fermions of the same charge and color
(e.g., $F = 3$ flavors of charge-2/3 quark)

$$\mathcal{L} = i\bar{u}_L^0 \not{\partial} u_L^0 + i\bar{u}_R^0 \not{\partial} u_R^0 - (\bar{u}_L^0 M^u u_R^0 + \bar{u}_R^0 M^{u\dagger} u_L^0)$$

- $u_L^0 = (u_{1L}^0 u_{2L}^0 u_{3L}^0)^T$ is 3-component column vector of “weak eigenstate” fields (similarly for u_R^0)
- M^u is 3×3 up-quark mass matrix (need not be Hermitian, diagonal, symmetric; can generalize to non-square)

- Diagonalize M^u by unitary A_L^u and A_R^u ($A_L^u = A_R^u$ for Hermitian M^u)

$$A_L^{u\dagger} M^u A_R^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

- “Mass eigenstate” fields

$$u_L = A_L^{u\dagger} u_L^0 = (u_L \ c_L \ t_L)^T$$

$$u_R = A_R^{u\dagger} u_R^0 = (u_R \ c_R \ t_R)^T$$

- Similarly for d, e, ν (will discuss Majorana later)
- Combinations of A_L observable in weak charged current (A_R not observable in standard model)
- (Unobservable) u_L phases arbitrary; u_R phases $\Rightarrow m_i \geq 0$

The Weak Charged Current

Fermi Theory incorporated in SM and made renormalizable

W -fermion interaction:

$$\mathcal{L}^{WCC} = -\frac{g}{2\sqrt{2}} \left(J_W^\mu W_\mu^- + J_W^{\mu\dagger} W_\mu^+ \right)$$

Charge-raising current:

$$\begin{aligned} J_W^{\mu\dagger} &= \sum_{m=1}^3 [\bar{\nu}_m^0 \gamma^\mu (1 - \gamma^5) e_m^0 + \bar{u}_m^0 \gamma^\mu (1 - \gamma^5) d_m^0] \\ &= (\bar{\nu}_1 \bar{\nu}_2 \bar{\nu}_3) \gamma^\mu (1 - \gamma^5) V_\ell \begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix} + (\bar{u} \bar{c} \bar{t}) \gamma^\mu (1 - \gamma^5) V_q \begin{pmatrix} d \\ s \\ b \end{pmatrix} \end{aligned}$$

Pure $V - A \Rightarrow$ maximal P and C violation; CP conserved except for phases in $V_{\ell,q}$

$U \equiv V_\ell^\dagger \equiv A_L^{e\dagger} A_L^\nu$ is 3×3 unitary Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix from mismatch between weak and Yukawa interactions (can ignore when m_ν unimportant)

$V_q = A_L^{u\dagger} A_L^d$ is Cabibbo-Kobayashi-Maskawa (CKM) matrix

Third family almost decouples in $V_q \Rightarrow 2 \times 2$ Cabibbo matrix

$$V_q = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \quad \sin \theta_c \simeq 0.22$$

**Full 3×3 CKM matrix involves 3 angles and 1 CP -violating phase
(after removing unobservable q_L phases)**

$$\begin{aligned}
V_q &\equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}
\end{aligned}$$

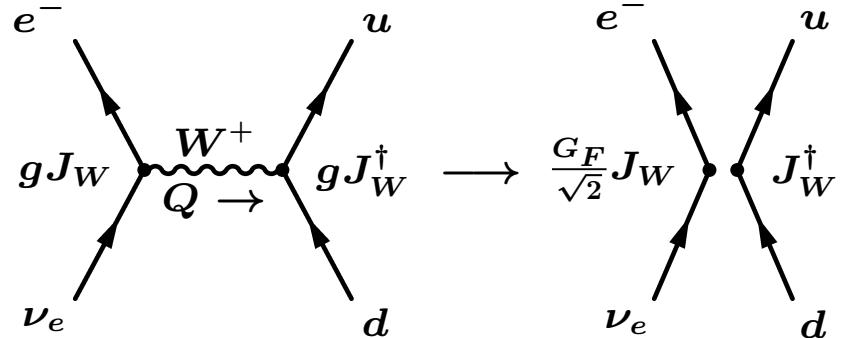
($c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$. δ is the CP -violating phase)

$$|V_{ij}| \sim \begin{pmatrix} 0.9742 & 0.226 & 0.0036 \\ 0.226 & 0.973 & 0.042 \\ 0.0087 & 0.041 & 0.9991 \end{pmatrix}$$

Effective zero-range 4-fermi interaction (Fermi theory)

For $|Q| \ll M_W$, neglect
 Q^2 in W propagator

$$-\mathcal{L}_{\text{eff}}^{WCC} = \frac{G_F}{\sqrt{2}} J_W^\mu J_{W\mu}^\dagger$$



Fermi constant: $\frac{G_F}{\sqrt{2}} \simeq \frac{g^2}{8M_W^2} = \frac{1}{2\nu^2}$

Muon lifetime: $\tau^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} \Rightarrow G_F = 1.16637(5) \times 10^{-5} \text{ GeV}^{-2}$

Weak scale: $\nu = \sqrt{2}\langle 0 | \varphi^0 | 0 \rangle \simeq 246 \text{ GeV}$

Excellent description of β , K , hyperon, heavy quark, μ , and τ decays, $\nu_\mu e \rightarrow \mu^- \nu_e$, $\nu_\mu n \rightarrow \mu^- p$, $\nu_\mu N \rightarrow \mu^- X$

Quantum Electrodynamics (QED)

Incorporated into standard model: $\mathcal{L}^Q = -e J_Q^\mu A_\mu$, $e = g \sin \theta_W$

Electromagnetic current:

$$\begin{aligned} J_Q^\mu &= \sum_{m=1}^3 \left[\frac{2}{3} \bar{u}_m^0 \gamma^\mu u_m^0 - \frac{1}{3} \bar{d}_m^0 \gamma^\mu d_m^0 - \bar{e}_m^0 \gamma^\mu e_m^0 \right] \\ &= \sum_{m=1}^3 \left[\frac{2}{3} \bar{u}_m \gamma^\mu u_m - \frac{1}{3} \bar{d}_m \gamma^\mu d_m - \bar{e}_m \gamma^\mu e_m \right] \end{aligned}$$

Electric charge: $Q = T^3 + Y$, where $Y = \text{weak hypercharge}$

Flavor diagonal: same form in weak and mass bases

Purely vector (parity conserving): L and R fields have same charge

The Weak Neutral Current

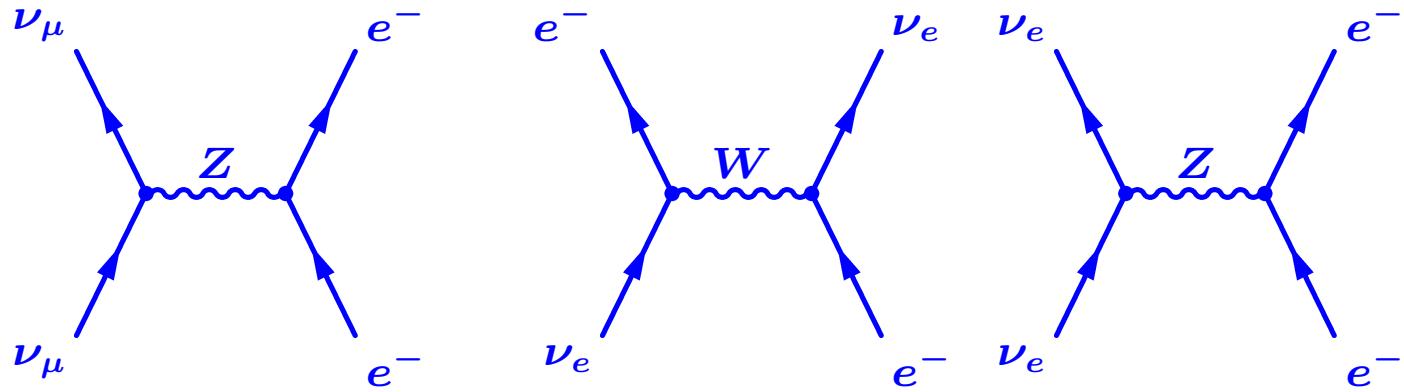
Prediction of $SU(2) \times U(1)$: $\mathcal{L}^{WNC} = -\frac{g}{2 \cos \theta_W} J_Z^\mu Z_\mu$

$$\begin{aligned} J_Z^\mu &= \sum_m [\bar{u}_{mL}^0 \gamma^\mu u_{mL}^0 - \bar{d}_{mL}^0 \gamma^\mu d_{mL}^0 + \bar{\nu}_{mL}^0 \gamma^\mu \nu_{mL}^0 - \bar{e}_{mL}^0 \gamma^\mu e_{mL}^0] \\ &\quad - 2 \sin^2 \theta_W J_Q^\mu \\ &= \sum_m [\bar{u}_{mL} \gamma^\mu u_{mL} - \bar{d}_{mL} \gamma^\mu d_{mL} + \bar{\nu}_{mL} \gamma^\mu \nu_{mL} - \bar{e}_{mL} \gamma^\mu e_{mL}] \\ &\quad - 2 \sin^2 \theta_W J_Q^\mu \end{aligned}$$

Flavor diagonal: same form in weak and mass bases (GIM)

Effective 4-fermi interaction: $-\mathcal{L}_{\text{eff}}^{WNC} = \frac{G_F}{\sqrt{2}} J_Z^\mu J_{Z\mu}$

Electroweak Interactions of Neutrinos



$$-\mathcal{L}_{\text{eff}}^{\nu_a e} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_a \gamma_\mu (1 - \gamma^5) \nu_a] [\bar{e}^- \gamma^\mu (g_V^a - g_A^a \gamma^5) e^-]$$

$$\nu_\mu e : \quad g_V^\mu \sim -\frac{1}{2} + 2 \sin^2 \theta_W, \quad g_A^\mu \sim -\frac{1}{2}$$

$$\nu_e e : \quad g_V^e = 1 + g_V^\mu, \quad g_A^e \sim 1 + g_A^\mu$$

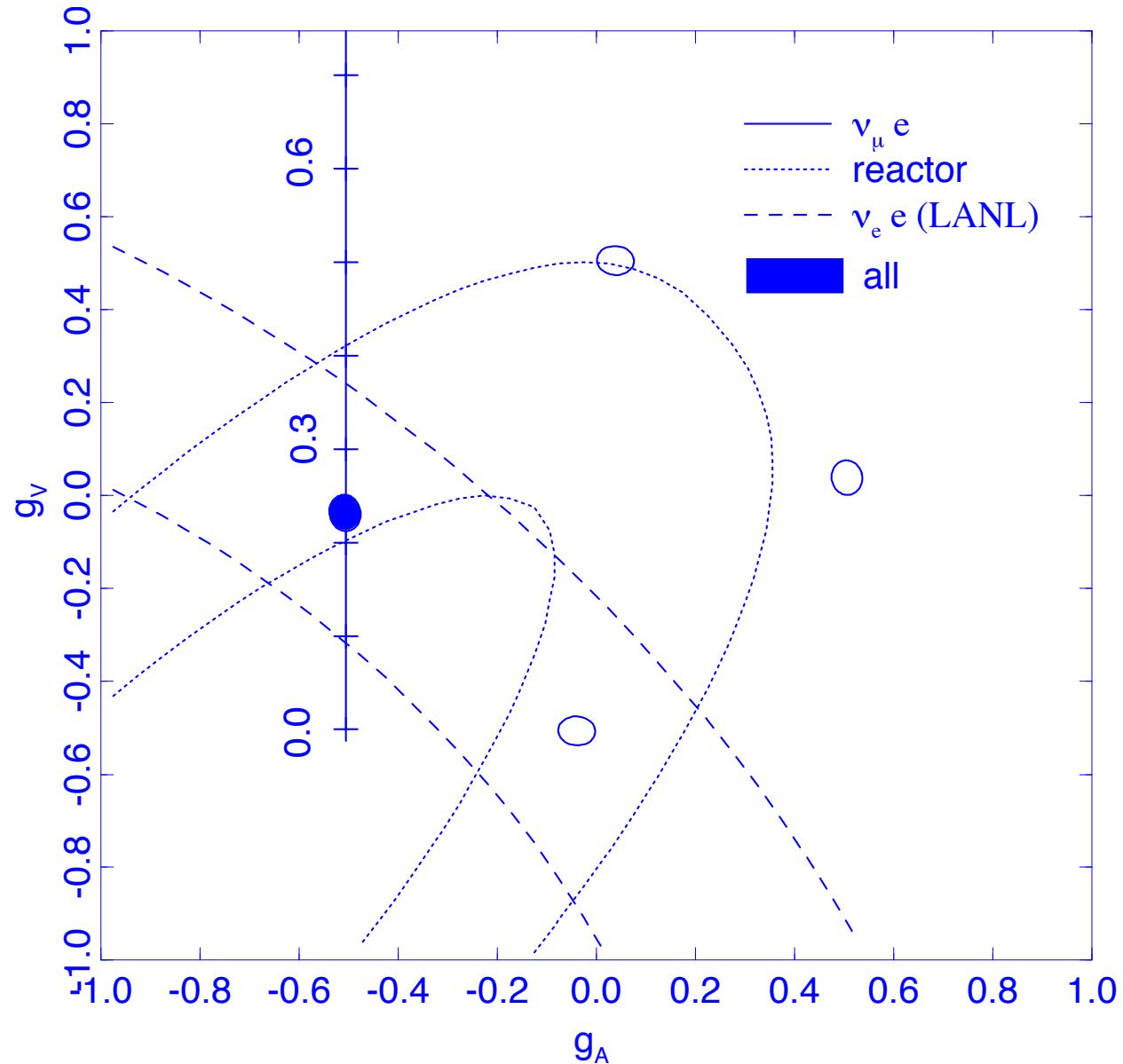
$$\frac{d\sigma_{\nu e}}{dy} = \frac{G_F^2 m_e E_\nu}{2\pi} \left[(g_V + g_A)^2 + (g_V - g_A)^2 (1-y)^2 - (g_V^2 - g_A^2) \frac{m_e}{E_\nu} y \right]$$

$$\sigma_{\nu e} \sim \int_0^1 \frac{d\sigma}{dy} dy \sim \frac{G_F^2 m_e E_\nu}{2\pi} \left[(g_V + g_A)^2 + \frac{1}{3} (g_V - g_A)^2 \right]$$

In lab frame: $y = T_e/E_\nu$ $(0 \leq y \leq (1 + m_e/2E_\nu)^{-1} \sim 1)$

$$\frac{d\sigma_{\bar{\nu} e}}{dy} = \frac{G_F^2 m_e E_\nu}{2\pi} \left[(g_V + g_A)^2 (1-y)^2 + (g_V - g_A)^2 - (g_V^2 - g_A^2) \frac{m_e}{E_\nu} y \right]$$

$$\sigma_{\bar{\nu} e} \sim \frac{G_F^2 m_e E_\nu}{2\pi} \left[\frac{1}{3} (g_V + g_A)^2 + (g_V - g_A)^2 \right]$$



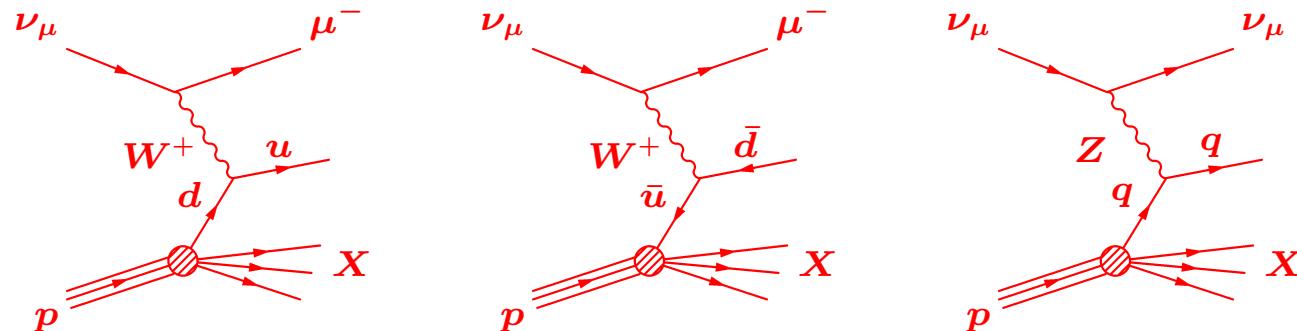
$\nu q \rightarrow \nu q$ (Mainly Deep Inelastic)

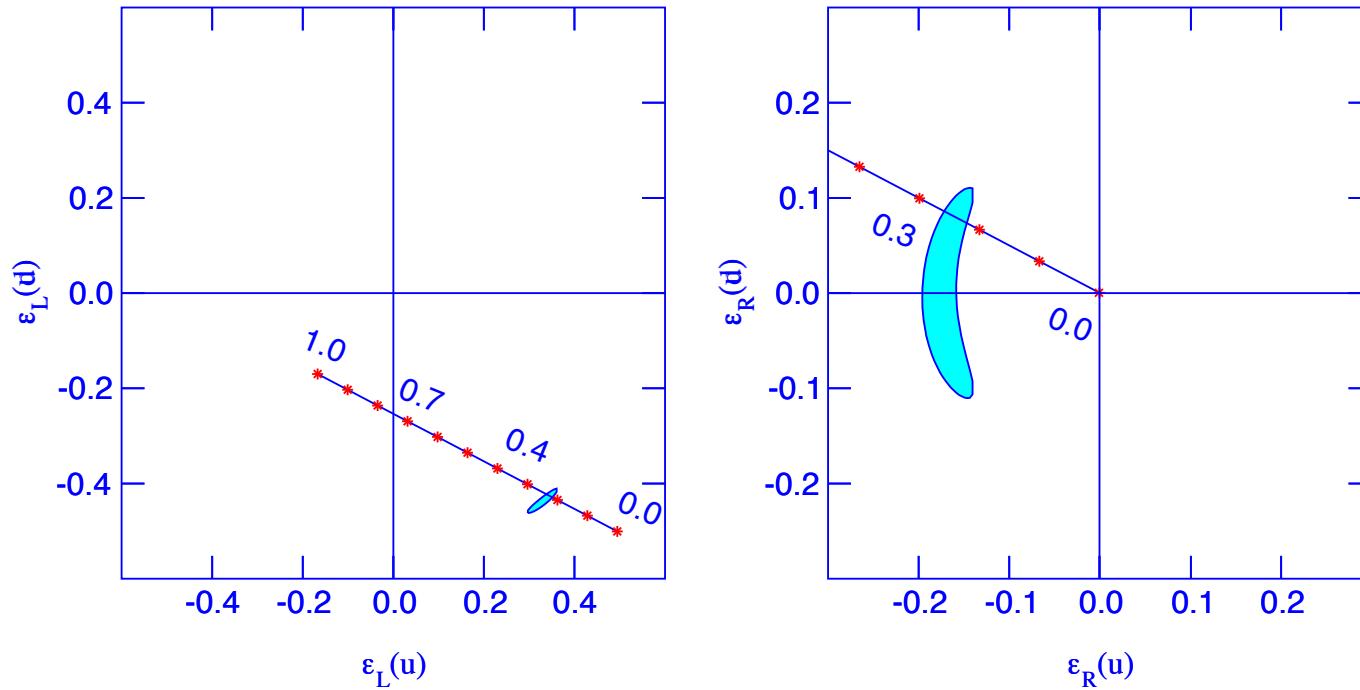
- WCC

$$-\mathcal{L}_{\text{eff}}^{WCC} = \frac{G_F}{\sqrt{2}} \bar{\mu} \gamma^\mu (1 - \gamma^5) \nu_\mu \times \sum_{ij} [\bar{u}_i \gamma_\mu (1 - \gamma^5) V_{ij} d_j]$$

- WNC

$$\begin{aligned} -\mathcal{L}_{\text{eff}}^{WNC} &= \frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1 - \gamma^5) \nu_\mu \\ &\times \sum_i [\epsilon_L(i) \bar{q}_i \gamma_\mu (1 - \gamma^5) q_i + \epsilon_R(i) \bar{q}_i \gamma_\mu (1 + \gamma^5) q_i] \end{aligned}$$





● Standard model

$$\begin{array}{lll} \epsilon_L(u) & \sim & \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \\ \epsilon_L(d) & \sim & -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \end{array} \quad \begin{array}{lll} \epsilon_R(u) & \sim & -\frac{2}{3} \sin^2 \theta_W \\ \epsilon_R(d) & \sim & \frac{1}{3} \sin^2 \theta_W \end{array}$$

Neutrino Oscillations: a first look

- Weak versus mass eigenstates:

$$\underbrace{|\nu_e\rangle}_{\text{weak}} = \underbrace{|\nu_1\rangle \cos \theta + |\nu_2\rangle \sin \theta}_{\text{mass}}, \quad |\nu_\mu\rangle = -|\nu_1\rangle \sin \theta + |\nu_2\rangle \cos \theta$$

- $t = 0$: $|\nu(0)\rangle = |\nu_\mu\rangle$ (from $\pi^+ \rightarrow \mu^+ \nu_\mu$)

- $t > 0$:

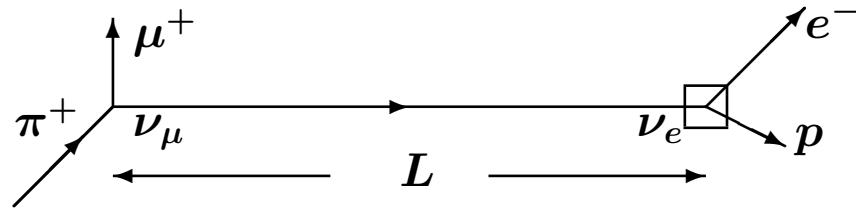
$$\begin{aligned} |\nu(t)\rangle &= -|\nu_1\rangle \sin \theta e^{-iE_1 t} + |\nu_2\rangle \cos \theta e^{-iE_2 t} \\ &\sim \left[-|\nu_1\rangle \sin \theta e^{-i\frac{m_1^2 t}{2E}} + |\nu_2\rangle \cos \theta e^{-i\frac{m_2^2 t}{2E}} \right] e^{-iEt} \end{aligned}$$

$E_i = \sqrt{|\vec{p}|^2 + m_i^2} \sim E + m_i^2/2E$ where $E \sim |\vec{p}|$ for definite $|\vec{p}| \gg m_i$
(same for definite E or wave packets)

- Probability of oscillating to ν_e (observe by $\nu_e n \rightarrow e^- p$)

$$\begin{aligned}
P_{\nu_\mu \rightarrow \nu_e}(L) &= |\langle \nu_e | \nu(t) \rangle|^2 = \sin^2 \theta \cos^2 \theta \left| -e^{-i \frac{m_1^2 t}{2E}} + e^{-i \frac{m_2^2 t}{2E}} \right|^2 \\
&= \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \\
&= \sin^2 2\theta \sin^2 \left[\frac{1.27 \Delta m^2 (\text{eV}^2) L (\text{km})}{E (\text{GeV})} \right]
\end{aligned}$$

$$\Delta m^2 = m_2^2 - m_1^2, \quad L \sim t$$



- Oscillation length:

$$L_{osc} = \frac{4\pi E}{\Delta m^2}$$

- Survival probability:

$$P_{\nu_\mu \rightarrow \nu_\mu}(L) = 1 - P_{\nu_\mu \rightarrow \nu_e}(L)$$

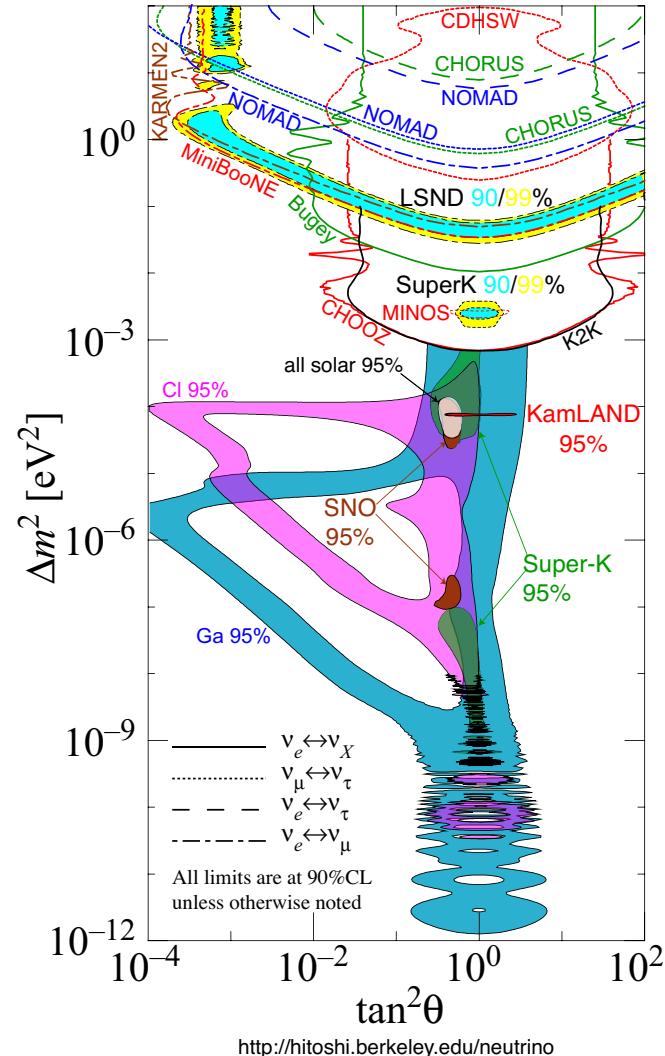
Neutrino Spectra

ν Oscillations

- $P_{\nu_a \rightarrow \nu_b} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$

3 ν Patterns

- Solar: LMA
(SNO, KamLAND, Borexino)
- $\Delta m_{\odot}^2 \sim 8 \times 10^{-5} \text{ eV}^2$, mixing large but nonmaximal
- Atmospheric + K2K + MINOS:
 $|\Delta m_{\text{Atm}}^2| \sim 2.4 \times 10^{-3} \text{ eV}^2$, near-maximal mixing
- Reactor: U_{e3} small ($U \equiv V_\ell^\dagger$)

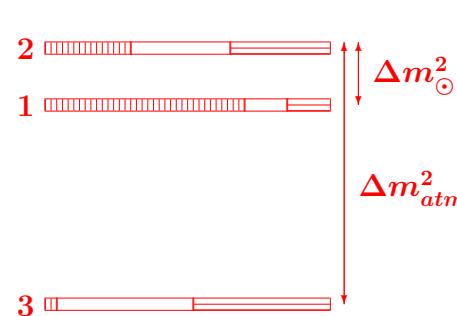
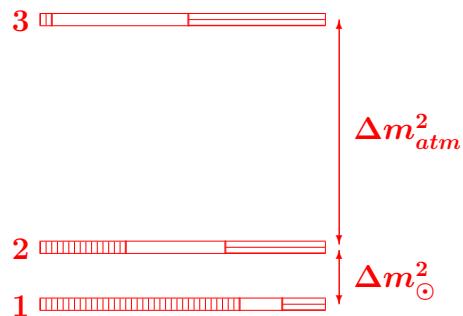


- Mixings: let $\nu_{\pm} \equiv \frac{1}{\sqrt{2}} (\nu_{\mu} \pm \nu_{\tau})$:

$$\nu_3 \sim \nu_+$$

$$\nu_2 \sim \cos \theta_{\odot} \nu_- - \sin \theta_{\odot} \nu_e$$

$$\nu_1 \sim \sin \theta_{\odot} \nu_- + \cos \theta_{\odot} \nu_e$$



- Normal hierarchy
 - Analogous to quarks, charged leptons
 - $\beta\beta_{0\nu}$ rate very small
- Degenerate pattern for $|m| \gg \sqrt{|\Delta m^2|}$
- Inverted hierarchy
 - $\beta\beta_{0\nu}$ if Majorana