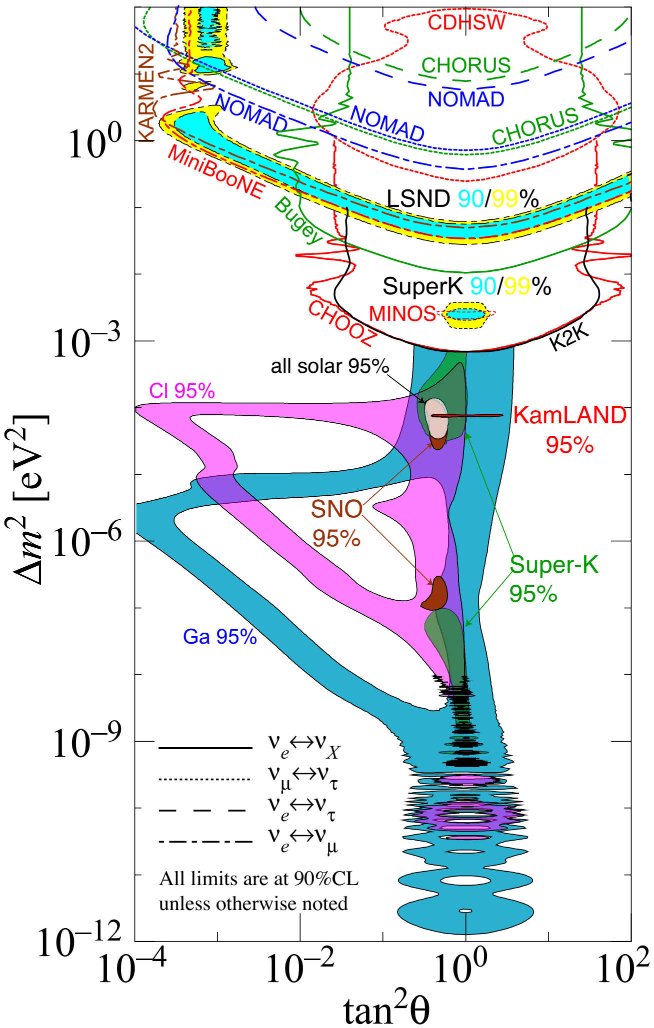
Neutrino Basics



http://hitoshi.berkeley.edu/neutrino

- **Neutrinos as a Probe**
- **Electroweak Interactions of Neutrinos**
- **Mass, Mixing, and Intrinsic Properties**
- **Neutrino Oscillations**
- **Models**
- **Outstanding Issues**

Reference: ***The Standard Model and Beyond*, CRC Press**

Mass, Mixing, and Intrinsic Properties

- Weyl fermion
  - Minimal (two-component) fermionic degree of freedom
  - $\psi_L \leftrightarrow \psi_R^c$ by CP ($\psi_R^c \sim \psi_L^\dagger$)
- Active Neutrino (a.k.a. ordinary, doublet)
  - in $SU(2)$ doublet with charged lepton $\rightarrow$ normal weak interactions
  - $\nu_L \leftrightarrow \nu_R^c$ by CP
- Sterile Neutrino (a.k.a. singlet, right-handed)
  - $SU(2)$ singlet; no interactions except by mixing, Higgs, or BSM
  - $\nu_R \leftrightarrow \nu_L^c$ by CP
  - Almost always present: Are they light? Do they mix?

- **Fermion Mass**
  - **Transition between right and left Weyl spinors:**

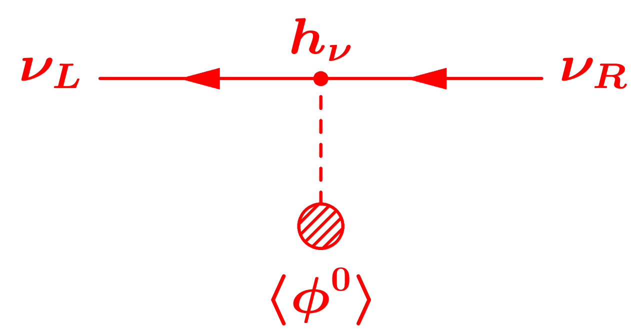
    $$m\bar{\psi}_L\psi_R + m^*\bar{\psi}_R\psi_L \rightarrow m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

    ($m \geq 0$ by $\psi_{L,R}$ phase changes)

- **Dirac Mass**
  - **Connects two distinct Weyl spinors** (usually active to sterile):

    $$m_D\left(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L\right) = m_D\bar{\nu}_D\nu_D$$

  - **Dirac field:** $\nu_D \equiv \nu_L + \nu_R$
  - **4 components,** $\Delta L = 0$
  - $\Delta t_L^3 = \pm\frac{1}{2} \rightarrow$ **Higgs doublet**
  - **Why small?** (Large dimensions? Higher-dimensional operators? String instantons?)



$$m_D = h_\nu\nu = \sqrt{2}h_\nu\langle\varphi^0\rangle$$

- **Majorana Mass**

  - **Connects Weyl spinor with itself:**

    $\frac{m_T}{2}\left(\bar{\nu}_L \nu_R^c + \bar{\nu}_R^c \nu_L\right) = \frac{m_T}{2}\bar{\nu}_M \nu_M$ (active)

    $\frac{m_S}{2}\left(\bar{\nu}_L^c \nu_R + \bar{\nu}_R \nu_L^c\right) = \frac{m_S}{2}\bar{\nu}_{M_S} \nu_{M_S}$ (sterile)

  - **Majorana fields:**

    $\nu_M \equiv \nu_L + \nu_R^c = \nu_M^c$

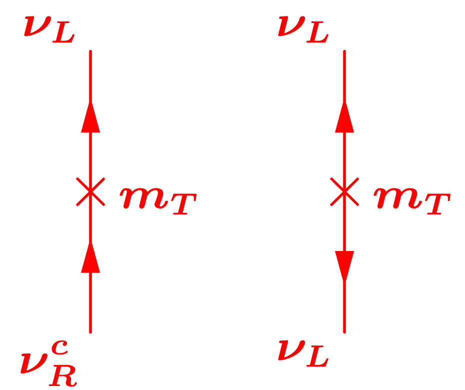
    $\nu_{M_S} \equiv \nu_L^c + \nu_R = \nu_{M_S}^c$

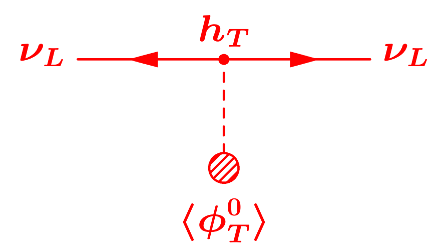
  - **2 components, $\Delta L = \pm 2$, self-conjugate**

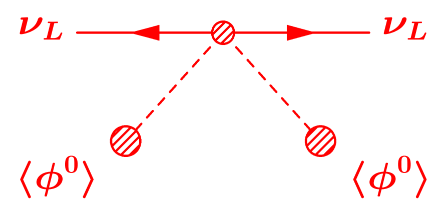
  - **Active: $\Delta t_L^3 = \pm 1$** (triplet or higher-dimensional operator)

  - **Sterile: $\Delta t_L^3 = 0$** (singlet or bare mass)

  - **Phase of ν_L or ν_L^c fixed by $m_{T,S} \geq 0$**

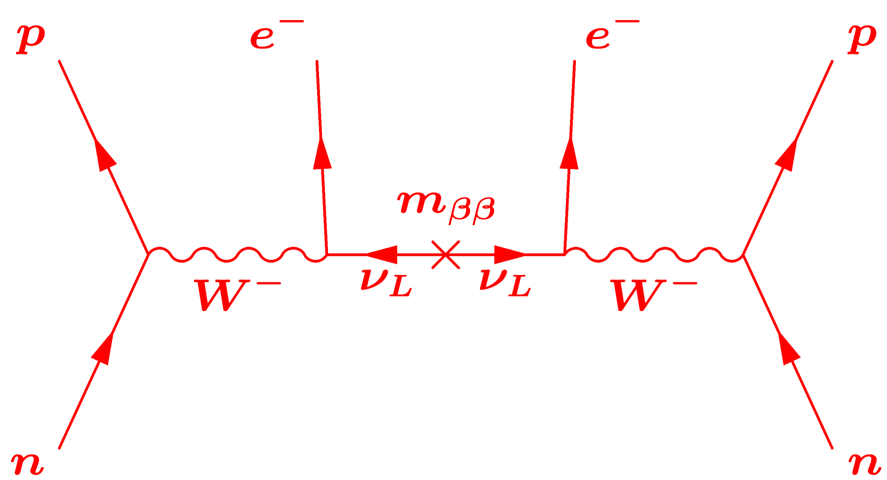






Neutrinoless Double Beta Decay ($\beta\beta_{0\nu}$)

- $\Delta L = 2$: Majorana mass only ($m_{\beta\beta} \sim m_T$)



- Other mechanisms, e.g., R_P violation in supersymmetry

Mixed Models

- Can have simultaneous Majorana and Dirac mass terms

$$-\mathcal{L} = \frac{1}{2} \underbrace{\left(\bar{\nu}_L^0 \quad \bar{\nu}_L^{0c} \right)}_{\text{weak eigenstates}} \begin{pmatrix} m_T & m_D \\ m_D & m_S \end{pmatrix} \begin{pmatrix} \nu_R^{0c} \\ \nu_R^0 \end{pmatrix} + h.c.$$

$$m_T: \quad |\Delta L| = 2, \qquad |\Delta t_L^3| = 1 \qquad \text{(Majorana)}$$

$$m_D: \quad |\Delta L| = 0, \qquad |\Delta t_L^3| = \frac{1}{2} \qquad \text{(Dirac)}$$

$$m_S: \quad |\Delta L| = 2, \qquad |\Delta t_L^3| = 0 \qquad \text{(Majorana)}$$

–

- **Mass eigenvalues:**

$$A_L^{\nu\dagger} \underbrace{\begin{pmatrix} m_T & m_D \\ m_D & m_S \end{pmatrix}}_{M=M^T} A_R^{\nu} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

- **(Majorana) mass eigenstates:** $\nu_{iM} = \nu_{iL} + \nu_{iR}^c = \nu_{iM}^c, \quad i = 1, 2$

$$\begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix} = A_L^{\nu\dagger} \begin{pmatrix} \nu_L^0 \\ \nu_L^{0c} \end{pmatrix}, \qquad \begin{pmatrix} \nu_{1R}^c \\ \nu_{2R}^c \end{pmatrix} = A_R^{\nu\dagger} \begin{pmatrix} \nu_R^{0c} \\ \nu_R^0 \end{pmatrix},$$

– $M = M^T$ (unlike Dirac mass matrix) $\Rightarrow A_L^\nu = A_R^{\nu *}$

Special Cases

$$-\mathcal{L} = \frac{1}{2} \begin{pmatrix} \bar{\nu}_L^0 & \bar{\nu}_L^{0c} \end{pmatrix} \begin{pmatrix} m_T & m_D \\ m_D & m_S \end{pmatrix} \begin{pmatrix} \nu_R^{0c} \\ \nu_R^0 \end{pmatrix} + h.c.$$

- **Majorana** $(m_D = 0)$:

$$m_1 = m_T : \qquad \nu_{1L} = \nu_L^0, \qquad \nu_{1R}^c = \nu_R^{0c}$$

$$m_2 = m_S : \qquad \nu_{2L} = \nu_L^{0c}, \qquad \nu_{2R}^c = \nu_R^0$$

- Dirac $(m_T = m_S = 0)$:

$$m_1 = +m_D: \qquad \nu_{1L} = \frac{1}{\sqrt{2}}(\nu_L^0 + \nu_L^{0c}), \qquad \nu_{1R}^c = \frac{1}{\sqrt{2}}(\nu_R^{0c} + \nu_R^0)$$

$$m_2 = -m_D: \qquad \nu_{2L} = \frac{1}{\sqrt{2}}(\nu_L^0 - \nu_L^{0c}), \qquad \nu_{2R}^c = \frac{1}{\sqrt{2}}(\nu_R^{0c} - \nu_R^0)$$

  – Dirac ν (4 components) equivalent to two degenerate $(|m_1| = |m_2|)$ Majorana ν's with $m_1 = -m_2$ and 45° mixing (cancel in $\beta\beta_{0\nu}$)

  – Useful description for Dirac limit of general case

  – Recover usual Dirac expression

$$-\mathcal{L} = \frac{m_D}{2}(\bar{\nu}_{1L}\nu_{1R}^c - \bar{\nu}_{2L}\nu_{2R}^c) + h.c. = m_D(\bar{\nu}_L^0\nu_R^0 + \bar{\nu}_R^0\nu_L^0)$$

  – No $\nu_L^0 - \nu_L^{0c}$ or $\nu_R^{0c} - \nu_R^0$ mixing $\Rightarrow L$ conserved

- **Seesaw** (a.k.a. minimal or Type I seesaw) $(m_S \gg m_{D,T})$:

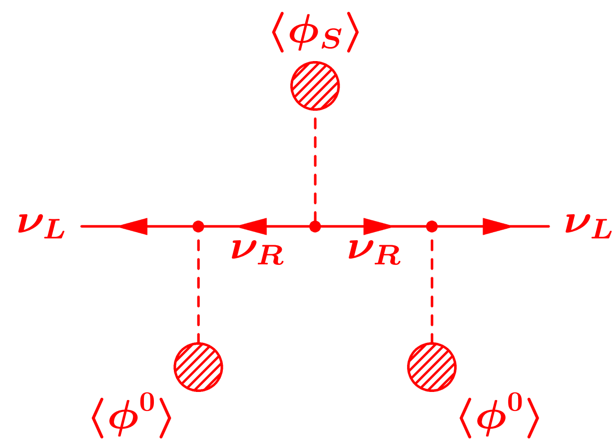
$$m_1 \sim m_T - \frac{m_D^2}{m_S} : \qquad \nu_{1L} \sim \nu_L^0 - \frac{m_D}{m_S}\nu_L^{0c} \sim \nu_L^0,$$

$$m_2 \sim m_S : \qquad \nu_{2L} \sim \frac{m_D}{m_S}\nu_L^0 + \nu_L^{0c} \sim \nu_L^{0c}$$

  - E.g., $m_T = 0$, $m_D = \mathcal{O}(m_{u,e,d})$, $m_S = \mathcal{O}(M_X \sim 10^{14}$ GeV):

$$|m_1| \sim m_D^2/m_S \ll m_D$$

$$\nu_{1M} \sim \nu_L^0 + \nu_R^{0c} \text{ (active)}$$



  - Lower m_S possible (even TeV)
  - Heavy ($\sim$sterile) ν_{2M} decouples at low energy
  - ν_{2M} decays $\Rightarrow$ leptogenesis

Active-sterile ($\nu_L^0 - \nu_L^{0c}$) mixing (LSND)

- No active-sterile mixing for Majorana, Dirac, or seesaw
- m_D and m_S (and/or m_T) both small and comparable:
  - Mechanism?
- Pseudo-Dirac ($m_T,\ m_S \ll m_D$):
  - Small mass splitting, small L violation, e.g.,

$$m_T = \epsilon,\ m_S = 0 \quad \Rightarrow \quad |m_{1,2}| = m_D \pm \epsilon/2$$

- Reactor and accelerator disappearance limits?
- Cosmological implications?

Extension to Three Families

- **Dirac mass:** $-\mathcal{L}_D = \bar{\nu}^0_L M_D \nu^0_R + \bar{\nu}^0_R M_D^\dagger \nu^0_L$

$$\nu^0_L \equiv \begin{pmatrix} \nu^0_{1L} \\ \nu^0_{2L} \\ \nu^0_{3L} \end{pmatrix} = A^\nu_L \nu_L, \qquad \nu^0_R \equiv \begin{pmatrix} \nu^0_{1R} \\ \nu^0_{2R} \\ \nu^0_{3R} \end{pmatrix} = A^\nu_R \nu_R$$

  - $M_D =$ arbitrary 3×3 Dirac mass matrix

$$A^{\nu\dagger}_L M_D A^\nu_R = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

– Leptonic weak charge lowering current

$$J_W^{\ell\mu} = 2\bar{e}_L\gamma^\mu V_\ell^\dagger \nu_L = (\bar{e}\ \bar{\mu}\ \bar{\tau})\gamma^\mu(1-\gamma^5)V_\ell^\dagger \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

– $U \equiv V_\ell^\dagger = A_L^{e\dagger} A_L^\nu$ is Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix (leptonic analog of CKM)

– Choose ν_L, e_L phases: remove 5 unobservable phases from $V_\ell^\dagger$ (choose ν_R, e_R phases for real non-negative masses)

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{reactor limits}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar}}$$

– $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, $\delta = CP$-violating phase (if $s_{13} \neq 0$) ($s_{ij}, \delta \neq$ CKM angles/phase)

- More on unobservable phases:
  - Most general unitary 3×3 PMNS matrix $\hat{U}$: 3 angles, 6 phases

$$J_W^{\ell\mu} = (\bar{e}\ \bar{\mu}\ \bar{\tau})\gamma^\mu(1-\gamma^5)\hat{U}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

$$\hat{U} = \begin{pmatrix} e^{-i\beta_1} & 0 & 0\\ 0 & e^{-i\beta_2} & 0\\ 0 & 0 & e^{-i\beta_3}\end{pmatrix}\underbrace{U}_{s_{ij},\delta}\begin{pmatrix} e^{i\alpha_1} & 0 & 0\\ 0 & e^{i\alpha_2} & 0\\ 0 & 0 & e^{i\alpha_3}\end{pmatrix}$$

  - β_i, α_j *not* determined by diagonalization ($\hat{U}^\dagger\hat{U} = \hat{U}\hat{U}^\dagger = I$)
  - Only depends on $\alpha_j - \beta_i \Rightarrow$ can choose $\alpha_3 = 0$
  - Redefine $\nu_{jL} \to e^{-i\alpha_j}\nu_{jL}, \quad e_{iL} \to e^{-i\beta_i}e_{iL} \Rightarrow \hat{U} \to U$
  - (Independent) e_{iR}, ν_{jR} phases chosen so that $m_{ei}, m_{\nu j} \geq 0$

- Majorana mass: $-\mathcal{L}_T = \frac{1}{2}\left(\bar{\nu}_L^0 M_T \nu_R^{0c} + \bar{\nu}_R^{0c} M_T^\dagger \nu_L^0\right)$
  - Diagonalize: $M_T \Rightarrow A_L^{\nu\dagger} M_D A_R^\nu$ by $\nu_L^0 = A_L^\nu \nu_L$, $\nu_R^{0c} = A_R^\nu \nu_R^c$
  - But $\bar{\psi}_{aL}\psi_{bR}^c = \bar{\psi}_{bL}\psi_{aR}^c$, where $\psi_R^c = \mathcal{C}\bar{\psi}_L^T$
  - Therefore $M_T = M_T^T \;\Rightarrow\; A_L^\nu = A_R^{\nu *}$
  - Phases determined by $m_i \geq 0 \;\Rightarrow$ two unremovable *Majorana phases* in U (observable in $\beta\beta_{0\nu}$?)

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric, } s_{23}^2 \sim \frac{1}{2}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{s_{13}^2 \lesssim 0.035,\ \delta = ?} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar, } s_{12}^2 \sim 0.3} \underbrace{\begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Majorana only}}$$

  - Same results for active sector in seesaw model

- **General case** (3 active and 3 sterile):

$$-\mathcal{L} = \frac{1}{2} \begin{pmatrix} \bar{\nu}_L^0 & \bar{\nu}_L^{0c} \end{pmatrix} \begin{pmatrix} M_T & M_D \\ M_D^T & M_S \end{pmatrix} \begin{pmatrix} \nu_R^{0c} \\ \nu_R^0 \end{pmatrix} + h.c.$$

  - M_D, $M_T = M_T^T$, $M_S = M_S^T$ are 3×3
  - 6 Majorana mass eigenstates
  - Majorana, Dirac, seesaw, active-sterile mixing (LSND) limits
  - Can also have > 3 or < 3 sterile