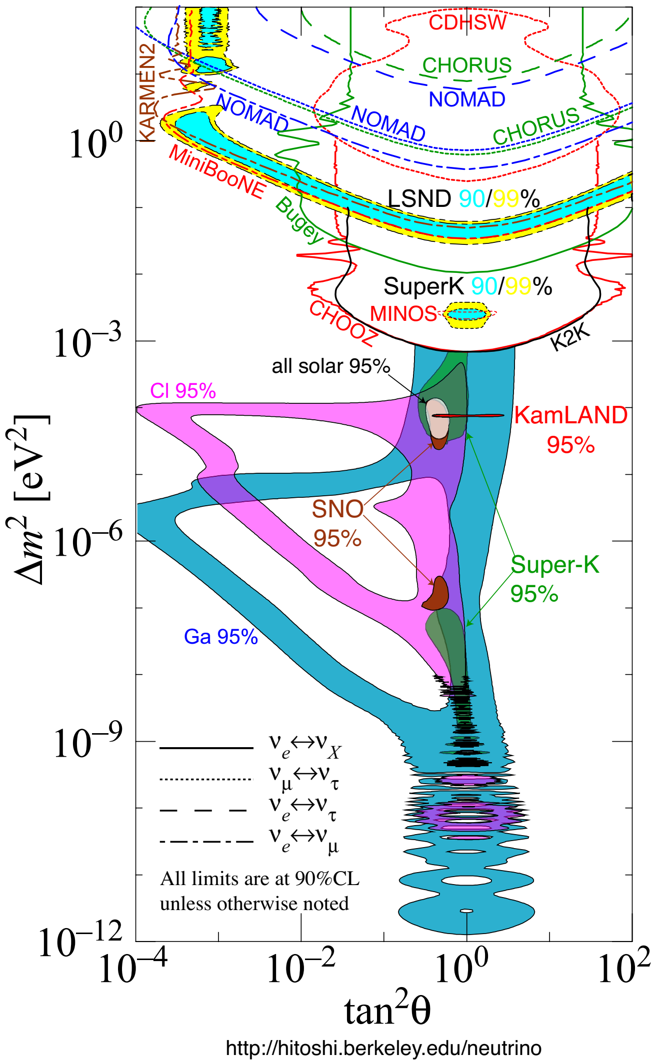
Neutrino Basics

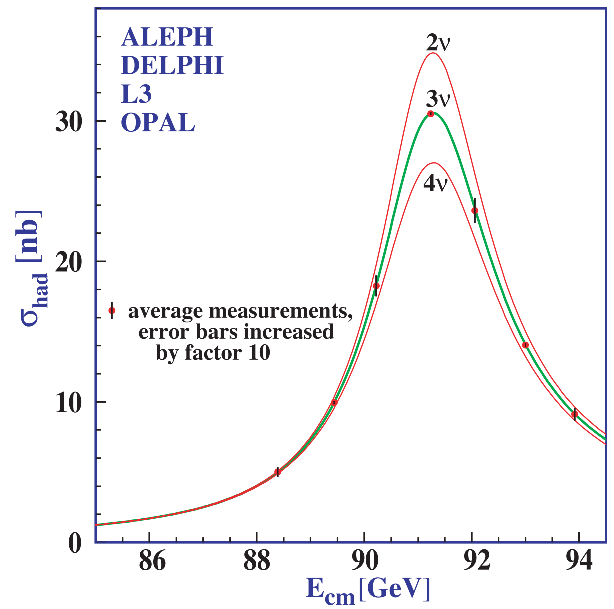


- **Neutrinos as a Probe**
- **Electroweak Interactions of Neutrinos**
- **Mass, Mixing, and Intrinsic Properties**
- **Neutrino Oscillations**
- **Models**
- **Outstanding Issues**

Reference: *The Standard Model and Beyond*, **CRC Press**

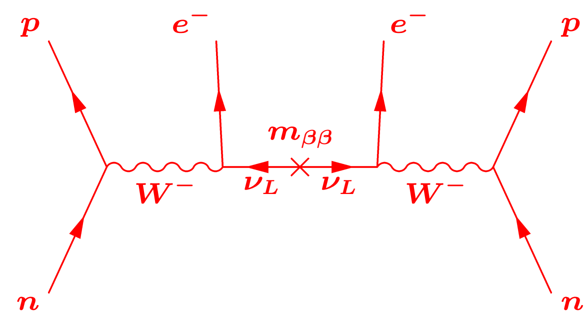
Neutrino Counting

- **Invisible Z width: $N_\nu = 3 + \Delta N_\nu$**
  - $\Delta N_\nu = -0.015(9)$ (also counts light $\tilde{\nu}$ $(1/2)$, triplet Majoron + scalar (2), etc.)
- **Cosmology: big bang nucleosynthesis, large scale structure**

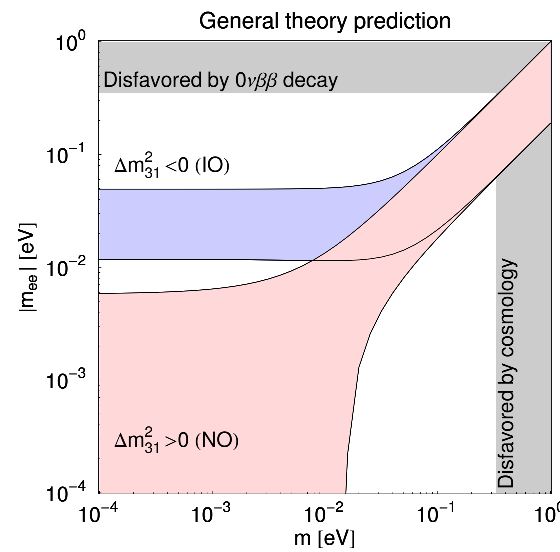


Dirac or Majorana

- $\beta\beta_{0\nu}$
  - $nn \to ppe^- e^-$ $(m_{\beta\beta} \equiv \sum_i U_{ei}^2 m_i)$



- Magnetic or electric dipole moments
  - Dirac: may have diagonal $(\bar{\nu}\sigma^{\alpha\beta}\nu F_{\alpha\beta})$ or transition $(\bar{\nu}_i\sigma^{\alpha\beta}\nu_j F_{\alpha\beta},\ i \neq j)$ moments
  - Majorana: only transition moments
    (diagonal Dirac from off-diagonal Majorana)
  - One loop (Dirac):
    $\mu_i \sim \frac{3eG_F m_i}{8\sqrt{2}\pi^2} \sim 3.2 \times 10^{-19} \left(\frac{m_i}{1\text{ eV}}\right) \mu_B$
    (much larger in some models)
  - Laboratory, astrophysical limits:
    $\mu < 10^{-10} - 10^{-12}\mu_B$



Winter, 1004.4160

Absolute Mass Scale

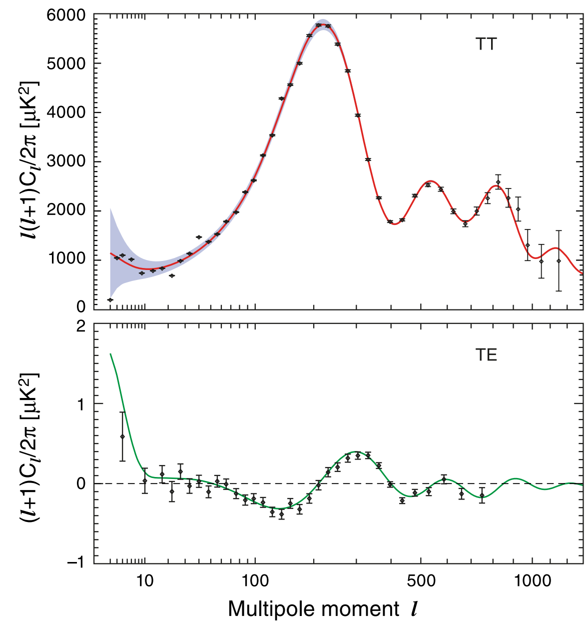
- **Tritium β spectrum (KATRIN)**

  $$m_{\nu_e} \equiv \left(\sum_i |U_{ei}^2| m_i^2\right)^{1/2} \lesssim 2 \text{ eV} \to 0.2 \text{ eV}$$

- **Large scale structure:**

  $$\Sigma \equiv \sum_i m_i \lesssim (0.5 - 1) \text{ eV} \to \mathcal{O}(0.05) \text{ eV}$$

- **If $\beta\beta_{0\nu}$ observed ($m_{\beta\beta} \gtrsim 0.01$ eV) $\to$ inverted or degenerate**



Neutrino Oscillations and Propagation

- **The two flavor case:**

$$\underbrace{|\nu_e\rangle}_{\text{weak}} = \underbrace{|\nu_1\rangle\cos\theta + |\nu_2\rangle\sin\theta}_{\text{mass}}, \qquad |\nu_\mu\rangle = -|\nu_1\rangle\sin\theta + |\nu_2\rangle\cos\theta$$

- $t = 0$: $\quad |\nu(0)\rangle = |\nu_\mu\rangle \qquad$ (from $\pi^+ \to \mu^+\nu_\mu$)

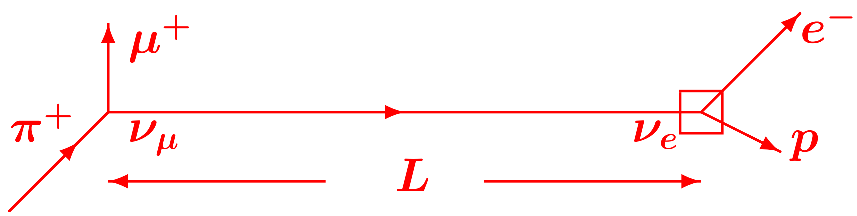
- $t > 0$:

$$|\nu(t)\rangle = -|\nu_1\rangle\sin\theta e^{-iE_1 t} + |\nu_2\rangle\cos\theta e^{-iE_2 t}$$
$$\sim \left[-|\nu_1\rangle\sin\theta e^{-i\frac{m_1^2 t}{2E}} + |\nu_2\rangle\cos\theta e^{-i\frac{m_2^2 t}{2E}}\right] e^{-iEt}$$

- **Probability of oscillating to ν_e** (appearance experiment)

$$P_{\nu_\mu \to \nu_e}(L) = |\langle \nu_e | \nu(t) \rangle|^2 = \sin^2\theta \cos^2\theta \left| -e^{-i\frac{m_1^2 t}{2E}t} + e^{-i\frac{m_2^2 t}{2E}t} \right|^2$$

$$= \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \xrightarrow[\Delta m^2 L/4E \text{ large}]{} \frac{1}{2}\sin^2 2\theta$$



- $\Delta m^2 = m_2^2 - m_1^2, \qquad L \sim t$

- **Oscillation length:** $L_{osc} = \frac{4\pi E}{\Delta m^2}$

- **Survival probability** (disappearance experiment):

$$P_{\nu_\mu \to \nu_\mu}(L) = 1 - P_{\nu_\mu \to \nu_e}(L)$$

Three or More Families

$$|\nu(0)\rangle = \underbrace{|\nu_a\rangle}_{\text{weak}} = \sum_i U_{ai}^* \underbrace{|\nu_i\rangle}_{\text{mass}} \Rightarrow \ |\nu(t)\rangle \sim \sum_i U_{ai}^* \, |\nu_i\rangle \, e^{-i\frac{m_i^2 t}{2E}}$$

$$\begin{aligned} P_{\nu_a \to \nu_b}(L) =& |\langle \nu_b | \nu(t) \rangle|^2 \\ =& \delta_{ab} - 4 \sum_{i<j} \Re e \left(U_{ai}^* U_{bi} U_{aj} U_{bj}^* \right) \sin^2 \left(\frac{\Delta_{ij} L}{4E} \right) \\ & + 2 \sum_{i<j} \underbrace{\Im m \left(U_{ai}^* U_{bi} U_{aj} U_{bj}^* \right)}_{CP \text{ violating}} \sin \left(\frac{\Delta_{ij} L}{2E} \right) \end{aligned}$$

- $\Delta_{ij} \equiv m_i^2 - m_j^2$
- Rephasing invariant

Antineutrinos

$$P_{\nu_{aL}\to\nu_{bL}}(L) = \delta_{ab} - 4\sum_{i<j}\Re e\left(U^*_{ai}U_{bi}U_{aj}U^*_{bj}\right)\sin^2\left(\frac{\Delta_{ij}L}{4E}\right)$$

$$+2\sum_{i<j}\underbrace{\Im m\left(U^*_{ai}U_{bi}U_{aj}U^*_{bj}\right)}_{CP \text{ violating}}\sin\left(\frac{\Delta_{ij}L}{2E}\right)$$

$$P_{\nu_{aL}\to\nu_{bL}}(L) \xrightarrow[U\to U^*]{} P_{\nu_{bL}\to\nu_{aL}}(L)$$

$$P_{\nu^c_{aR}\to\nu^c_{bR}}(L) = P_{\nu_{bL}\to\nu_{aL}}(L) \text{ (by } CPT\text{)}$$

$$= \delta_{ab} - 4\sum_{i<j}\Re e\left(U^*_{ai}U_{bi}U_{aj}U^*_{bj}\right)\sin^2\left(\frac{\Delta_{ij}L}{4E}\right)$$

$$-2\sum_{i<j}\Im m\left(U^*_{ai}U_{bi}U_{aj}U^*_{bj}\right)\sin\left(\frac{\Delta_{ij}L}{2E}\right)$$

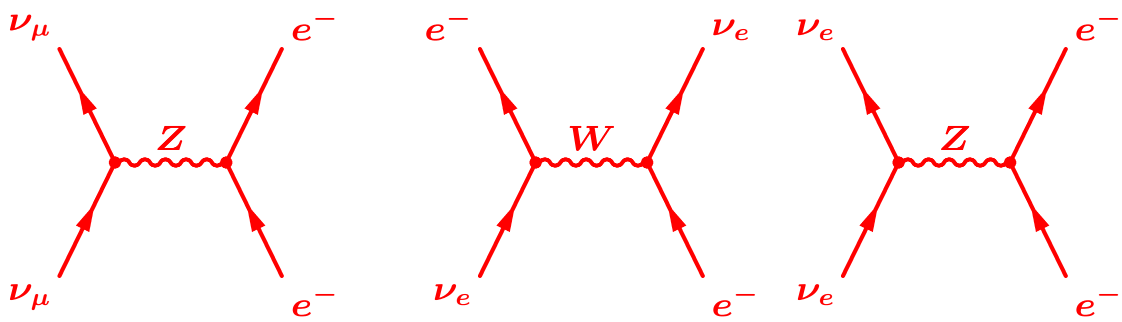
Propagation in Matter

- Coherent forward scattering in matter $\Rightarrow$ potential (cf optics)

- Two flavor oscillations in vacuum: $|\nu(t)\rangle = \sum_a c_a(t)|\nu_a\rangle$

$$i\frac{d}{dt}\begin{pmatrix} c_a(t) \\ c_b(t) \end{pmatrix} = \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}\begin{pmatrix} c_a(t) \\ c_b(t) \end{pmatrix}$$

- In matter (different interactions of ν_a and ν_b):



$$i\frac{d}{dt}\begin{pmatrix} c_a(t) \\ c_b(t) \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \frac{G_F}{\sqrt{2}}n & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta - \frac{G_F}{\sqrt{2}}n \end{pmatrix} \begin{pmatrix} c_a(t) \\ c_b(t) \end{pmatrix}$$

$$n = \begin{cases} n_e & \text{for} \quad \nu_{eL} \leftrightarrow \nu_{\mu L}, \nu_{\tau L} \\ 0 & \text{for} \quad \nu_{\mu L} \leftrightarrow \nu_{\tau L} \\ n_e - \frac{1}{2}n_n & \text{for} \quad \nu_{eL} \leftrightarrow \nu_L^c \\ -\frac{1}{2}n_n & \text{for} \quad \nu_{\mu L}, \nu_{\tau L} \leftrightarrow \nu_L^c \end{cases}$$

- Signs reversed for ν_R^c, ν_R
- Solar (MSW), atmospheric (not sterile; also WNC), long baseline (hierarchy), supernovae
- MINOS ν_L vs ν_R^c: new flavor-dependent matter interaction? (to mimic CPT violation)

Long Baseline (LBL) Oscillation Experiments

- 3 ν oscillations, small s_{13} and $\Delta m^2_\odot$ (Akhmedov et al, JHEP 04, 078):

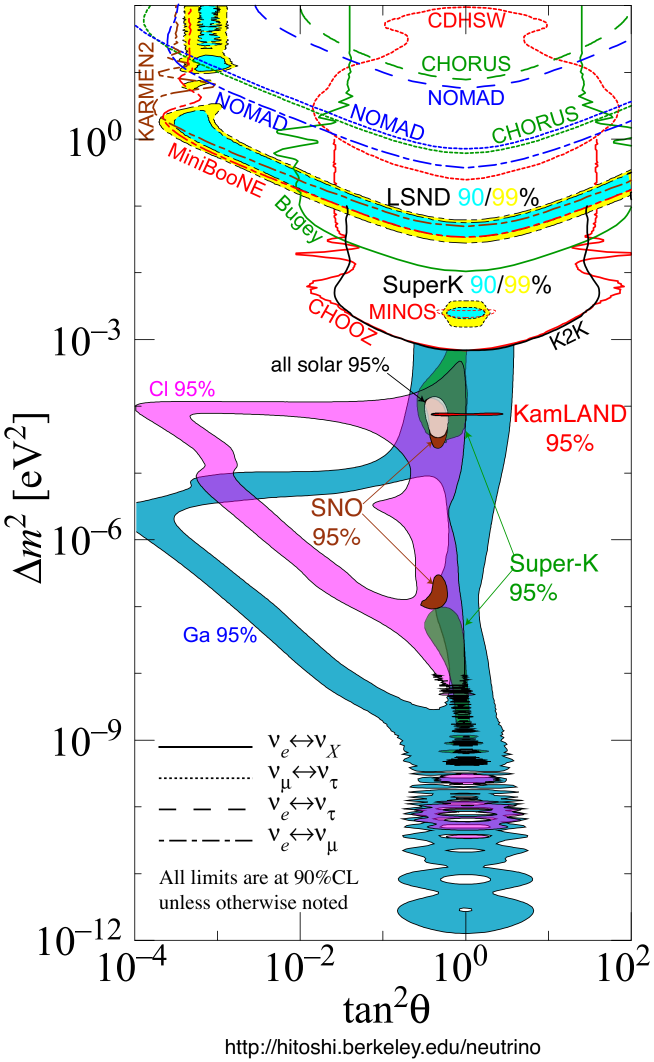
$$P_{\nu_\mu \to \nu_e} = \alpha^2 \sin^2 2\theta_{12}\, c_{23}^2 \frac{\sin^2 A\Delta}{A^2} + 4\, s_{13}^2\, s_{23}^2 \frac{\sin^2 (A-1)\Delta}{(A-1)^2}$$
$$+ 2\,\alpha\, s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23} \cos(\Delta + \delta)\, \frac{\sin A\Delta}{A} \frac{\sin(A-1)\Delta}{A-1}$$

where

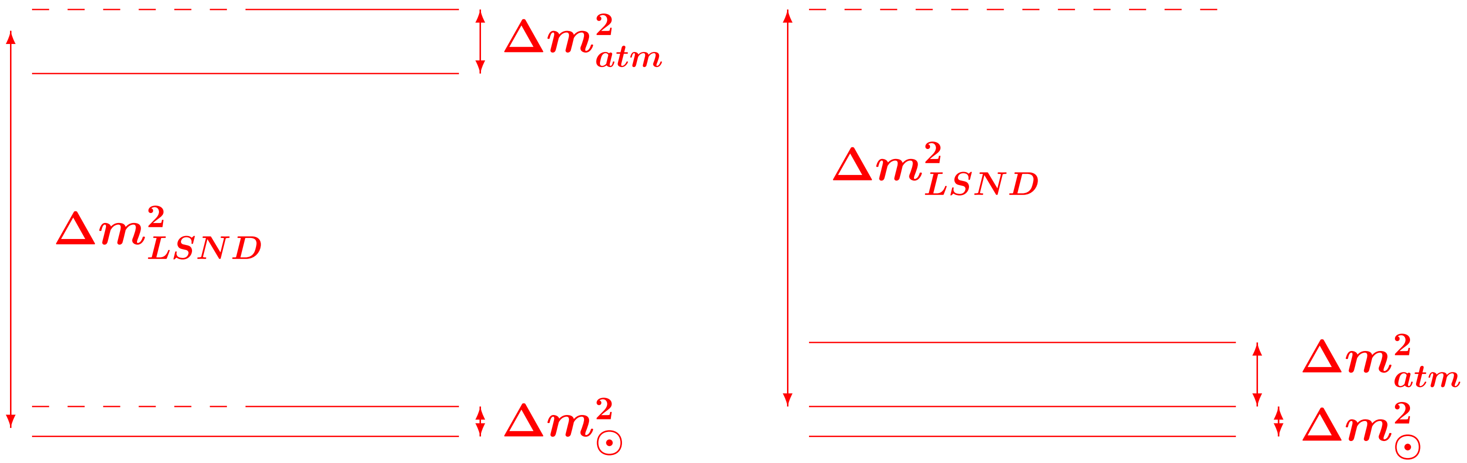
$$\alpha = \frac{\Delta m^2_\odot}{|\Delta m^2_{\rm Atm}|} \sim 0.03, \qquad \Delta = \frac{\Delta m^2_{\rm Atm} L}{4E}, \qquad \underbrace{A = \frac{2\sqrt{2} E G_F n_e}{\Delta m^2_{\rm Atm}}}_{\text{matter}}$$

- $\delta \to -\delta$ and $A \to -A$ for $P_{\bar\nu_\mu \to \bar\nu_e}$
- $\Delta, A > 0$ (normal), $\Delta, A < 0$ (inverted)
- In principle, determine s_{13}, δ, hierarchy (easier if s_{13} from reactor)

Sterile Neutrinos



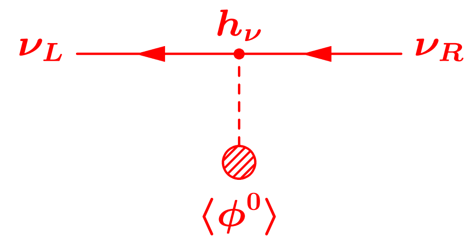
- **1^{st} class oscillations:**
  $\nu_a \to \nu_b$, both active
- **2^{nd} class:** active $\to$ sterile
- **LSND:** third Δ_{ij} $\Rightarrow$ sterile ν's
- **LSND + MiniBooNE:**
  sterile ν's and CP violation
  (but reactor, accelerator disappearance; other ν_e appearance; cosmology?)
- **$\nu_L \to \nu_L^c$ appearance**
  (need new interactions)
- **Non-orthogonal light neutrinos**
  (from mixing with heavy)

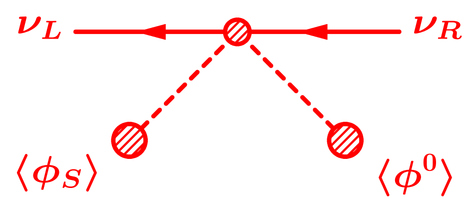

Δm^2_{atm}
Δm^2_{LSND}
$\Delta m^2_{\odot}$
Δm^2_{LSND}
Δm^2_{atm}
$\Delta m^2_{\odot}$

Models

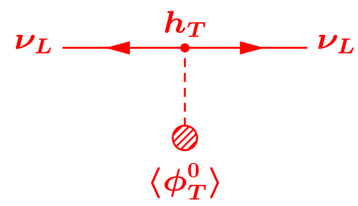
- No unbroken gauge symmetry forbids Majorana mass for active ν's
- Seesaw $\Rightarrow$ leptogenesis (heavy ν_{2M} decay in Type I)
- *But*
  - New TeV physics or string constraints may forbid seesaw
  - *Appropriate* masses not automatic (e.g., $SO(10) \Rightarrow$ Higgs 126 or HDO) so that Majorana may be negligible
  - Alternative baryogenesis (e.g., electroweak)
  - Other plausible mechanisms for both Majorana and Dirac (especially in strings)
- Type, mechanism, scale is open question

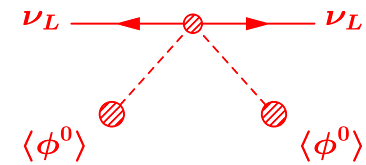
- **Small Dirac masses**
  - **Usually forbid bare mass**
    (string? $U(1)'$?)
  - **Higher dimensional operator**
    ($m_D \sim \nu\langle\varphi_S\rangle/M_P$)
  - **Large extra dimension**
    (wave function overlap)
  - **String instanton**
  - **Non-holomorphic soft**

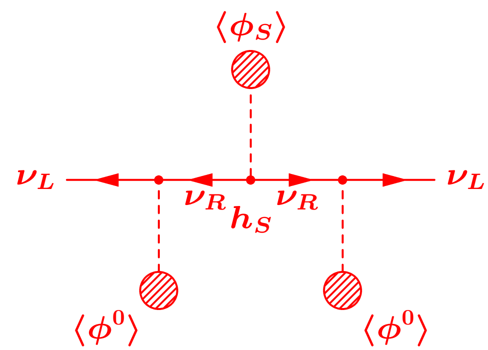


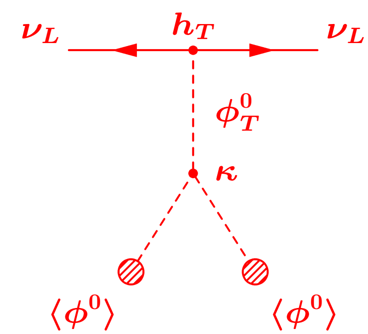


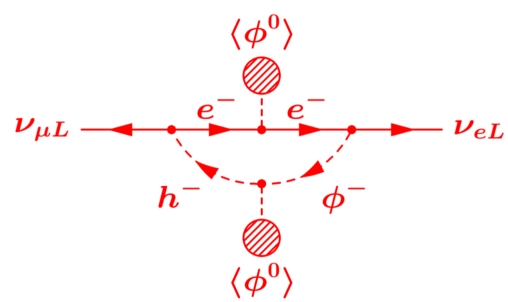
- **Small Majorana masses**
  - **Higgs triplet**
    (spontaneous $\not{L}$ excluded)
  - **Stringy Weinberg operator**
  - **Heavy ν_R** (Type I seesaw)
  - **Heavy Higgs triplet**
    (Type II seesaw)
  - **Loops** (new scalars)
  - **TeV** (extended) **seesaws**
  - **R_P violation**











Outstanding Issues (intrinsic properties)

- **Scale of underlying physics?** (string, GUT, TeV?)
- **Mechanism?** (seesaw, LED, HDO, stringy instanton?)
- **Hierarchy, U_{e3}, leptonic CP violation?** (mechanism, leptogenesis)
- **Absolute mass scale?** (cosmology)
- **Dirac or Majorana?** (mechanism, scale, leptogenesis)
- **Baryon asymmetry?** (leptogenesis, electroweak baryogenesis, other?)

Outstanding Issues (intrinsic properties)

- **Scale of underlying physics?** (string, GUT, TeV?) (LHC, flavor)
- **Mechanism?** (seesaw, LED, HDO, stringy instanton?)(indirect: LHC)
- **Hierarchy, U_{e3}, leptonic CP violation?** (mechanism, leptogenesis) (long baseline, reactor, $\beta\beta_{0\nu}$, supernova)
- **Absolute mass scale?** (cosmology) (β decay, cosmology, $\beta\beta_{0\nu}$, supernova)
- **Dirac or Majorana?** (mechanism, scale, leptogenesis) ($\beta\beta_{0\nu}$)
- **Baryon asymmetry?** (leptogenesis, electroweak baryogenesis, other?) (indirect: LHC)

Other Topics

- Puzzles/anomalies (LSND, NuTeV, MiniBooNE, GSI, MINOS)
- Quantum subtleties
- Sterile ν models/constraints
- ν decay
- Decoherence
- Non-standard interactions
- Heavy ν's
- CPT, Lorentz, equivalence violation
- FCNC (associated $\tilde{\nu}$, $\tilde{\ell}$)
- R_P violation
- Mass-varying ν's
- Time-varying ν's
- Correlated LHC physics
- Model approaches (GUTs, family symmetries, string instantons/HDOs, textures, anarchy, bimaximal, tri-bimaximal, discrete $S4/A4$)

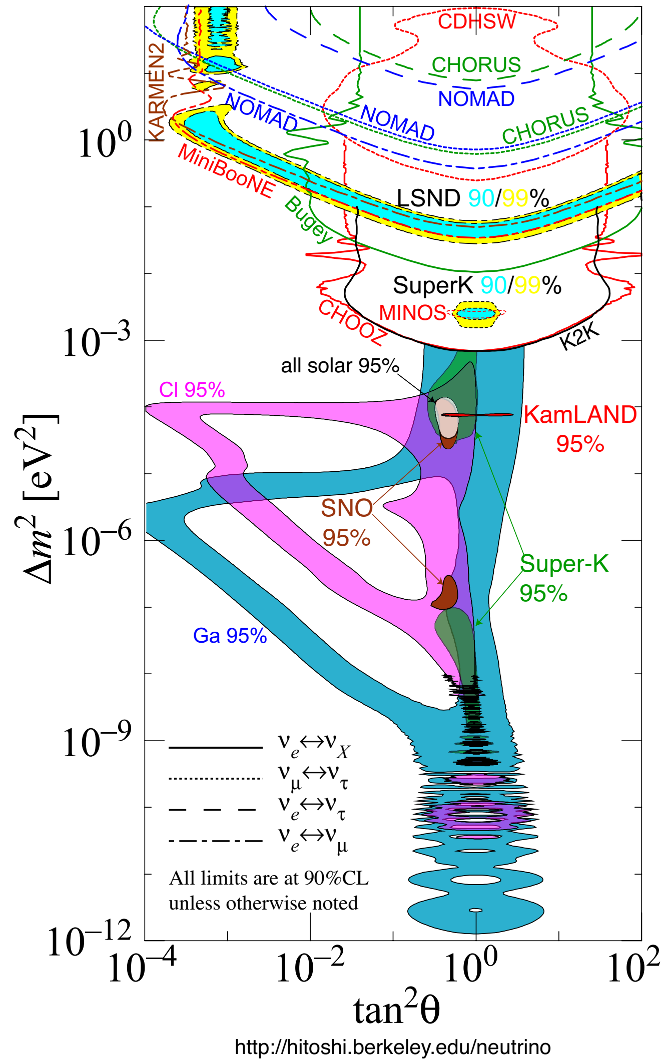
Neutrino Spectra

ν Oscillations

- $P_{\nu_a \to \nu_b} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$

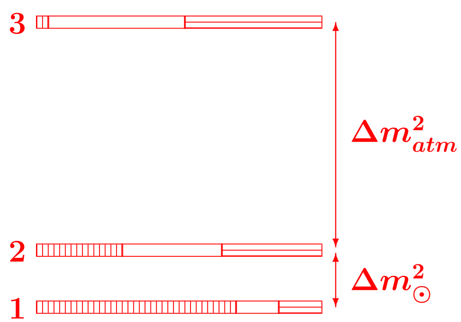
3 ν Patterns

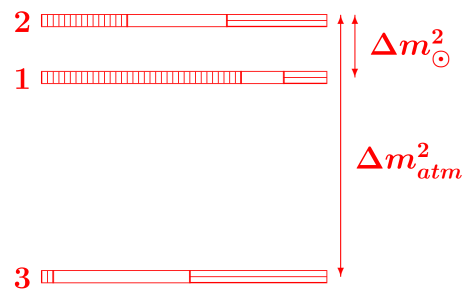
- **Solar: LMA**
  (SNO, KamLAND, Borexino)
- $\Delta m^2_\odot \sim 8 \times 10^{-5}$ **eV^2, mixing large but nonmaximal**
- **Atmospheric + K2K + MINOS:** $|\Delta m^2_{\rm Atm}| \sim 2.4 \times 10^{-3}$ **eV^2, near-maximal mixing**
- **Reactor:** U_{e3} **small** $(U \equiv V_\ell^\dagger)$



- Mixings: let $\nu_\pm \equiv \frac{1}{\sqrt{2}}(\nu_\mu \pm \nu_\tau)$:

$$\begin{aligned}
\nu_3 &\sim \nu_+ \\
\nu_2 &\sim \cos\theta_\odot\ \nu_- - \sin\theta_\odot\ \nu_e \\
\nu_1 &\sim \sin\theta_\odot\ \nu_- + \cos\theta_\odot\ \nu_e
\end{aligned}$$





- Normal hierarchy
  - Analogous to quarks, charged leptons
  - $\beta\beta_{0\nu}$ rate very small

- Inverted hierarchy
  - $\beta\beta_{0\nu}$ if Majorana

- Degenerate pattern for $|m| \gg \sqrt{|\Delta m^2|}$

Conclusions

- **Neutrino physics is extremely interesting**

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- **Neutrino physics is extremely difficult**