Testing Gravity with Atom Interferometry

39th SLAC Summer Institute

Jason Hogan
Stanford University
August 3, 2011

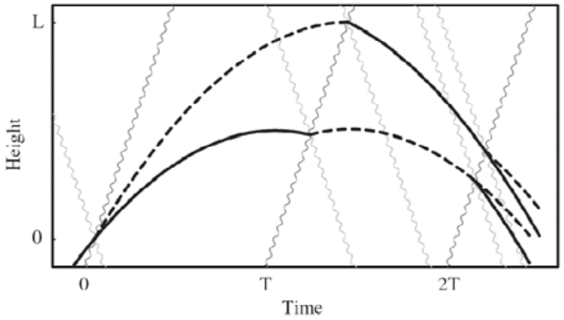
Precision Gravimetry

Stanford 10 m Equivalence Principle test

General Relativistic effects in the lab

Velocity dependent forces

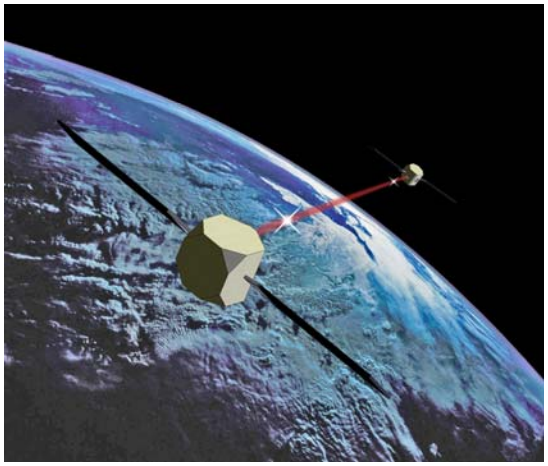
Nonlinear gravity



Gravitational wave detection

AGIS: Terrestrial GW detection

AGIS-LEO: Space GW detection



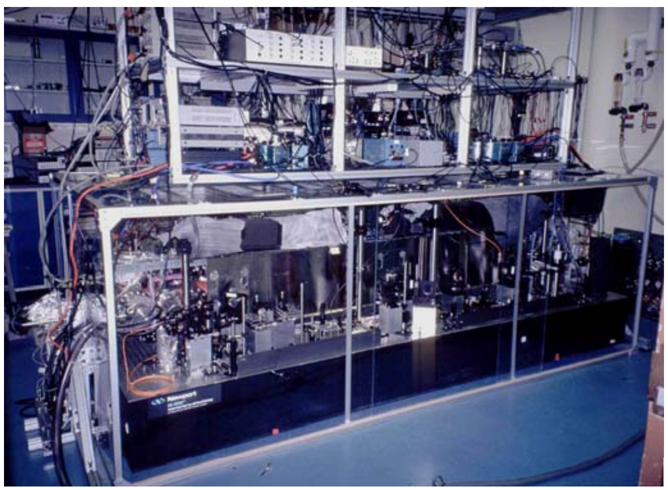
Cold Atom Inertial Sensors

Cold atom sensors:

- Laser cooling; ~10^8 atoms, ~10 uK (no cryogenics)
- Atom is freely falling (inertial test mass)
- Lasers measure motion of atom relative to sensor case

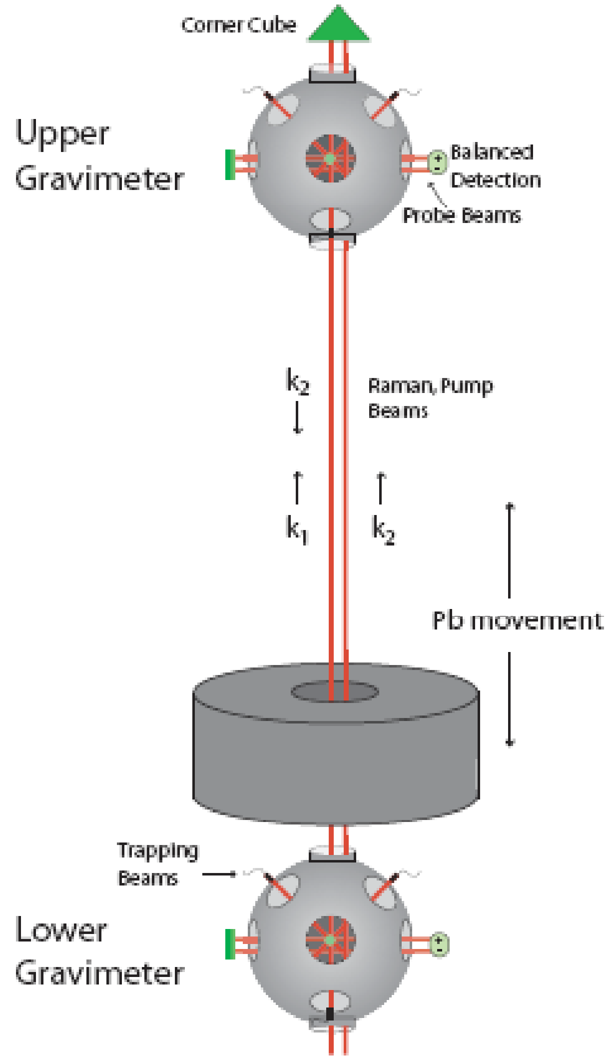
Some applications of atom interferometry (AI):

- Accelerometers (precision gravimetry)
- Gyroscopes
- Gradiometers (measure Newton's *G*, inertial guidance)



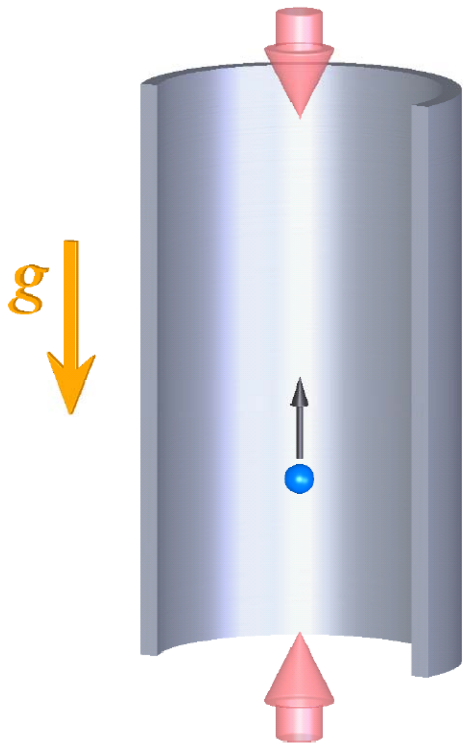
AI gyroscope (1997)

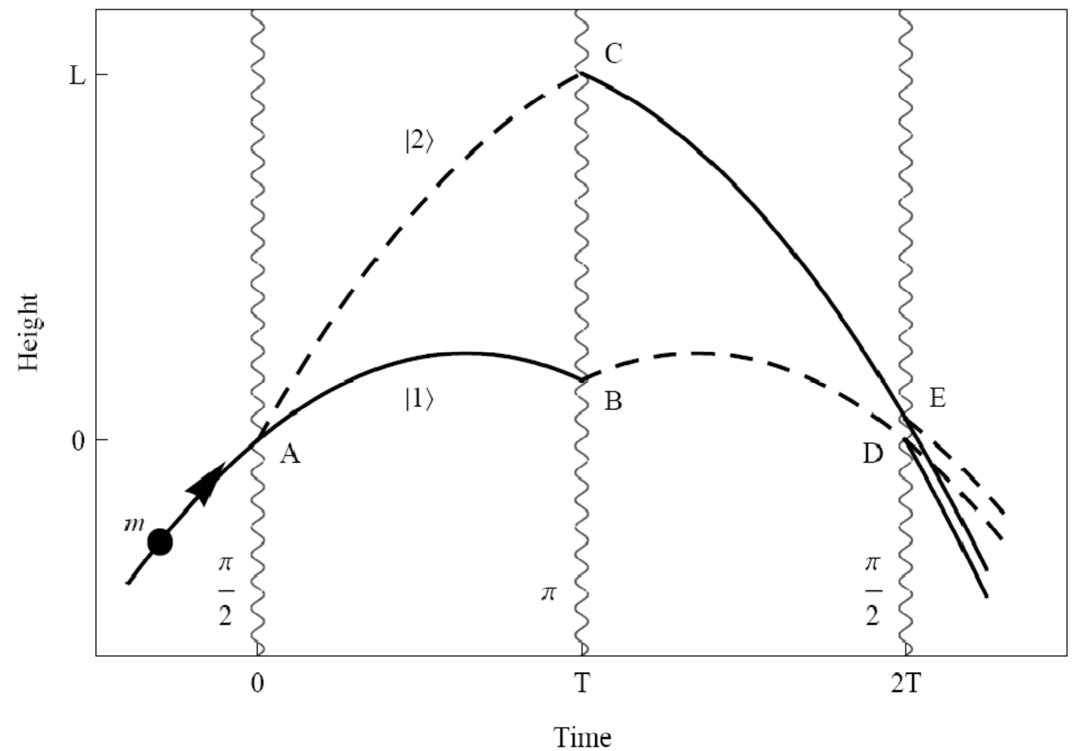
AI compact gyroscope (2008)



AI gradiometer (2003)

Light Pulse Atom Interferometry





- Vertical atomic fountain
- Atom is freely falling
- Lasers pulses are atom beamsplitters & mirrors (Raman or Bragg atom optics)
- $\frac{\pi}{2} - \pi - \frac{\pi}{2}$ pulse sequence

Light Pulse Atom Interferometry

$$|\text{atom}\rangle = |p\rangle$$

π/2 pulse — "beamsplitter"

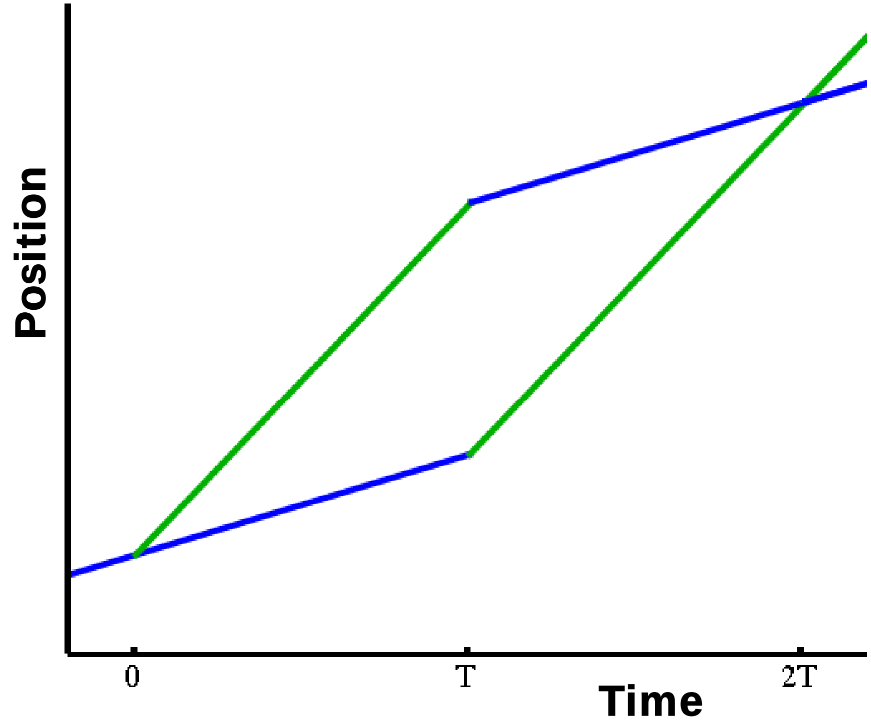
$$\frac{1}{\sqrt{2}}(|p\rangle + e^{i\Delta\phi}|p+k\rangle)$$

π pulse — "mirror"

$$\frac{1}{\sqrt{2}}(|p+k\rangle + e^{i\Delta\phi}|p\rangle)$$

π/2 pulse — "beamsplitter"

$$\frac{1}{2}((1+e^{i\Delta\phi})|p\rangle + ((1-e^{i\Delta\phi}))|p+k\rangle)$$



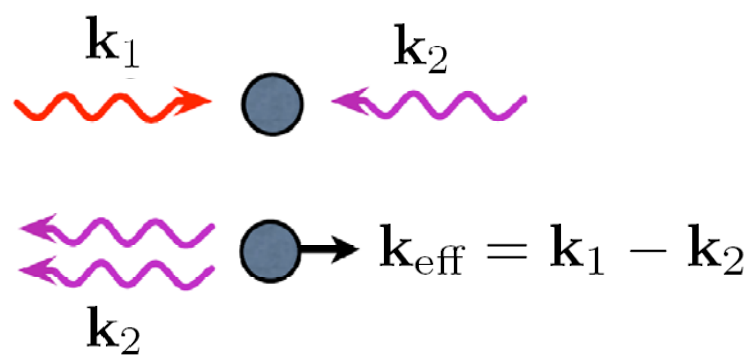
Measure the number of atoms in each final state →

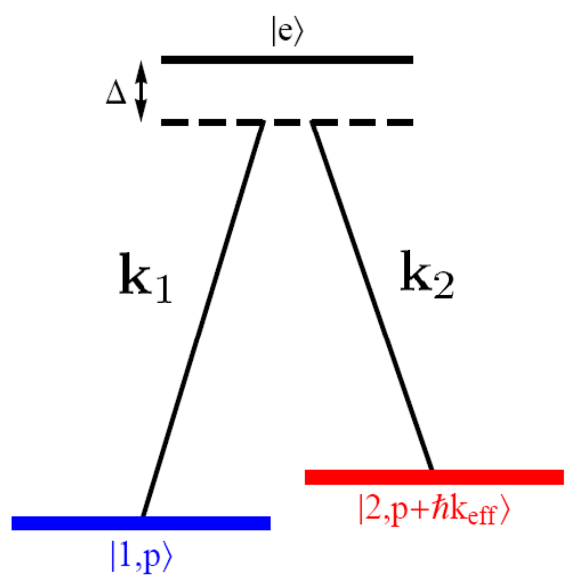
$$\text{Probability in State } |p\rangle = \cos^2\left(\frac{\Delta\phi}{2}\right)$$

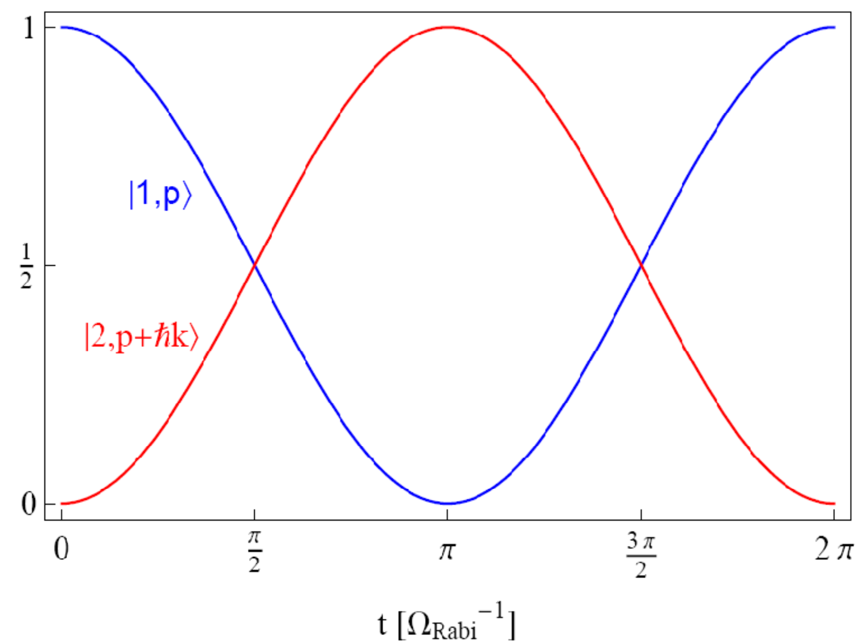
$$\text{Probability in State } |p+k\rangle = \sin^2\left(\frac{\Delta\phi}{2}\right)$$

Atom Optics

- Stimulated two photon process from far detuned excited state
- Effective two level system exhibits Rabi flopping
- Beamsplitter ($\pi/2$) and mirror (π) pulses possible







Understanding the Inertial Force Sensitivity

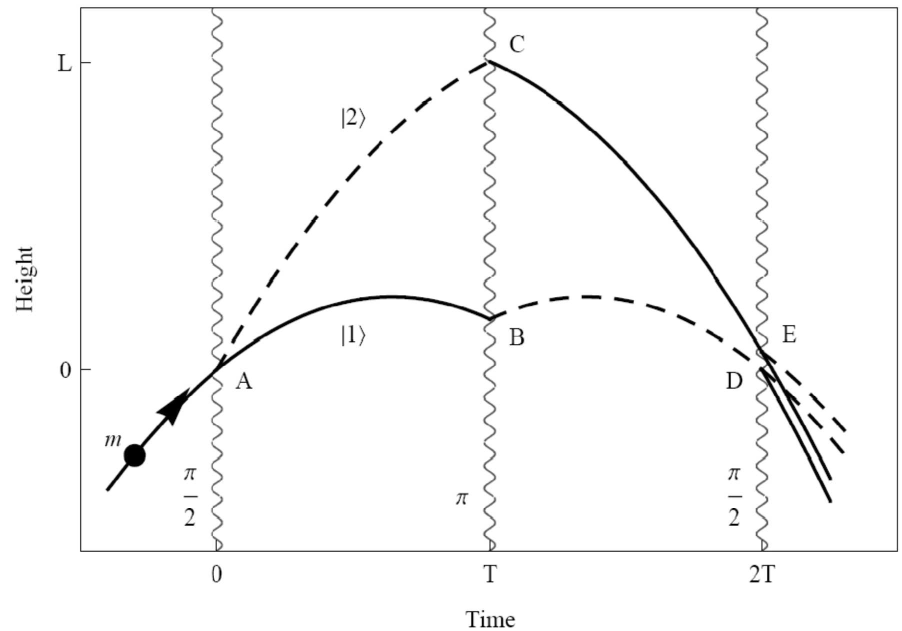
Atom-Light interaction:

The local phase of the laser is imprinted on the atom at each interaction point.

$$\phi_L \equiv \mathbf{k}\cdot\mathbf{x}_c(t_0) - \omega t_0$$

- Laser phase encodes the atom's position as a function of time

- Motion of the atom is measured w.r.t. a wavelength-scale "Laser-ruler" (~0.5 micron)



Example: Free-fall gravitational acceleration, (π/2 – π – π/2) sequence

$$\Delta\phi = (\phi_D - \phi_B) - (\phi_C - \phi_A)$$

$$\Delta\phi = k_{\text{eff}}\, g T^2 \sim g T^2 / \lambda$$

Accelerometer Sensitivity

$$\frac{\delta g}{g} \sim \frac{\delta \phi}{k_{\text{eff}} g T^2}$$

$$\Delta\phi = k_{\text{eff}} g T^2 \approx 3\times10^8 \,\text{rad}$$

(T ~ 1.3 s, $k_{\text{eff}} = 2k$)

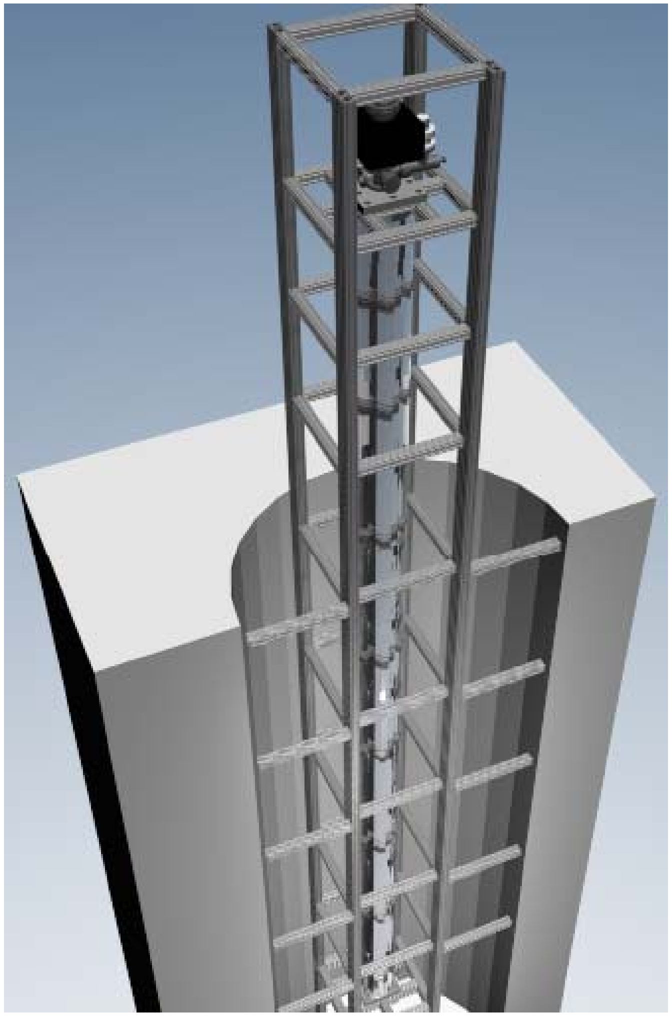
Shot noise limited detection @ 10^7 atoms per shot:

$$\delta\phi \sim \frac{1}{\sqrt{N}} \sim 3\times10^{-7}\,\text{rad}$$

(~ 1 month)

$$\delta g < 10^{-15} g$$

10 m atom drop tower



Exciting possibility for improvement:

LMT beamsplitters with $\hbar k_{\text{eff}} > 100\hbar k$ [1,2]

[1] H. Müller *et al.*, Phys. Rev. Lett. **100**, 180405 (2008)

[2] J. M. McGuirk et al., Phys. Rev. Lett. **85**, 4498 - 4501 (2000)

Testing the Equivalence Principle

Equivalence Principle Test

$$m_I a = m_G g$$

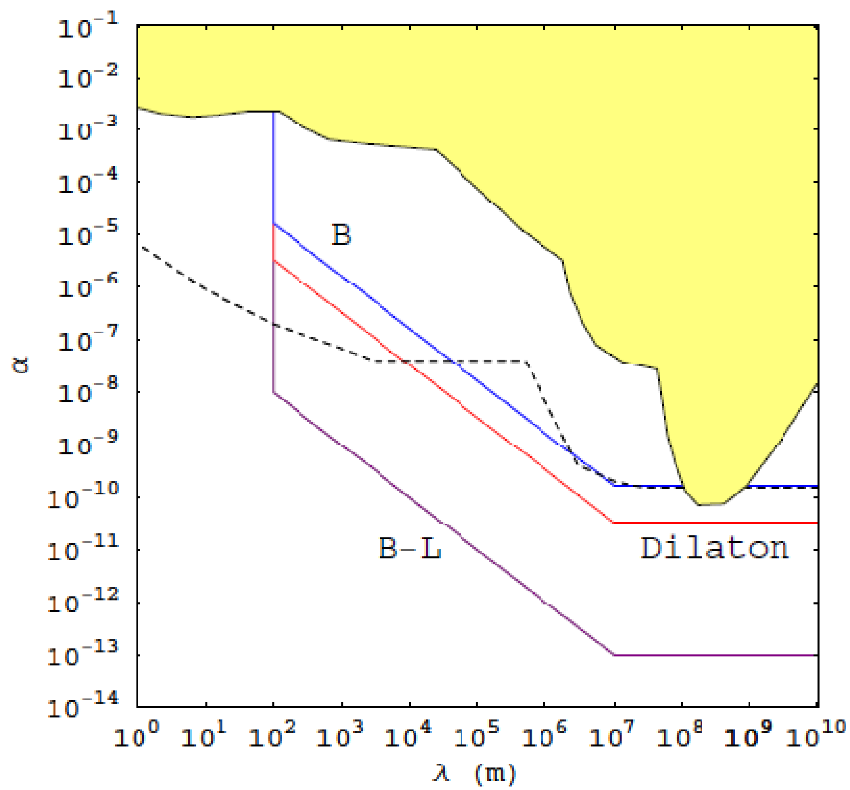
- Bodies fall (locally) at the same rate, independent of composition
- Gravity = Geometry

Why test the EP?

- Foundation of General Relativity
- Quantum theory of gravity (?)
  "Fifth forces"

$$V(r) = -\frac{GM_1M_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

"Yukawa type"



- EP test are sensitive to "charge" differences of new forces

Equivalence Principle Test

Use atom interferometric differential accelerometer to test EP

Co-falling ^{85}Rb and ^{87}Rb ensembles

Evaporatively cool to < 1 μK to enforce tight control over kinematic degrees of freedom

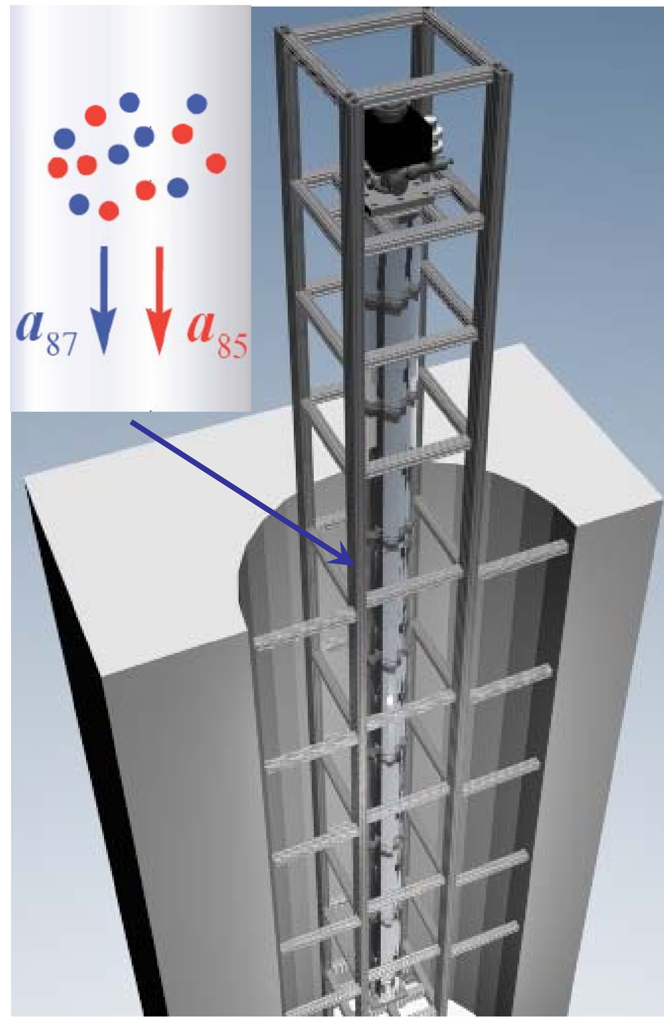
Statistical sensitivity

$\delta g \sim 10^{-15}$ g with 1 month data collection

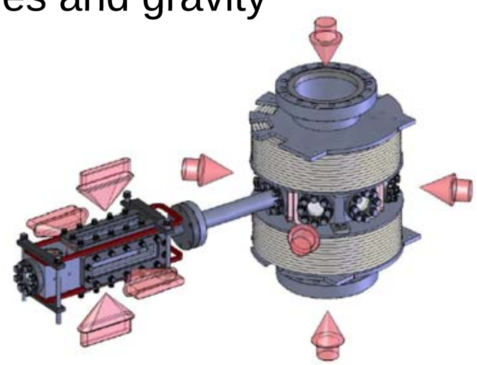
Systematic uncertainty

$\delta g < 10^{-15}$ g limited by magnetic field inhomogeneities and gravity anomalies.

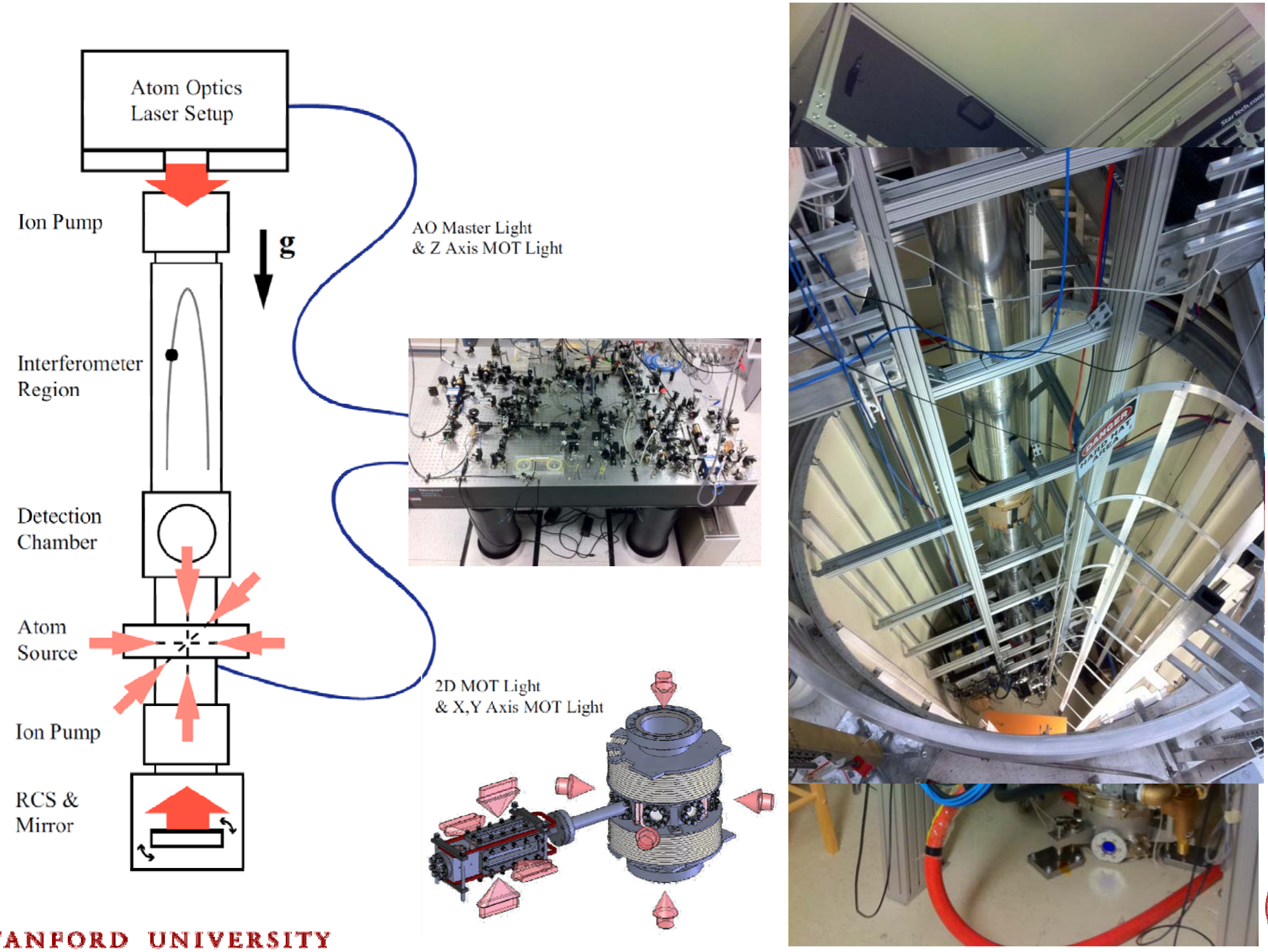
10 m atom drop tower



Atomic source



Stanford Atom Drop Tower Apparatus



Differential Measurement

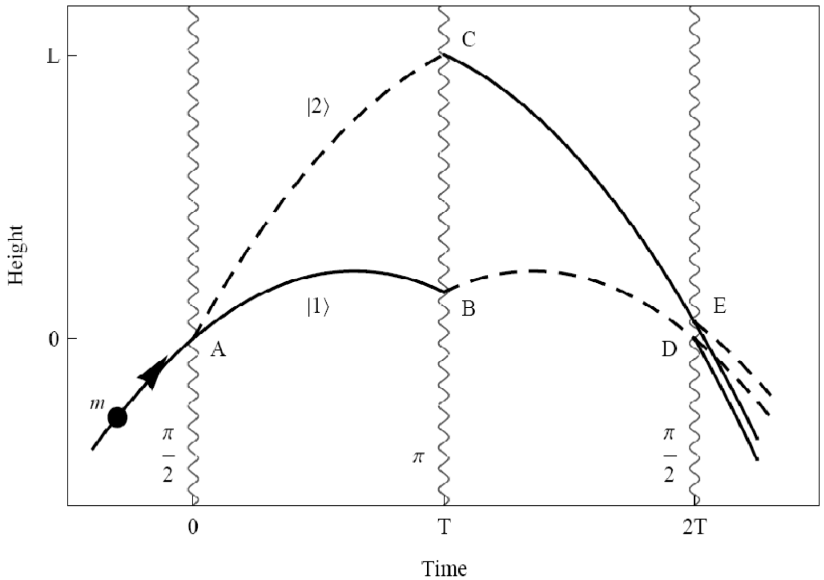
- Atom shot noise limit? What about *technical noise?*
  - → Seismic vibration
  - → Laser phase noise

- Rely on common mode suppression
  - → Both isotopes are manipulated with the same laser
  - → Final phase shifts are subtracted
  - → Suppresses many systematic errors as well

Semi-classical phase shift analysis

Three contributions:

- Laser phase at each node
- Propagation phase along each path
- Separation phase at end of interferometer



Include all relevant forces in the classical Lagrangian:

$$L = \frac{1}{2}m(\dot{\mathbf{r}} + \mathbf{\Omega} \times (\mathbf{r} + \mathbf{R}_e))^2 - m\phi(\mathbf{r} + \mathbf{R}_e) - \frac{1}{2}\alpha\mathbf{B}(\mathbf{r})^2$$

Rotation of Earth — Gravity gradients, etc. — Magnetic field shifts

EP Systematic Analysis

Use standard semi-classical methods to analyze spurious phase shifts from uncontrolled:

- Earth's Rotation
- Gravity anomalies/gradients
- Magnetic fields
- Proof-mass overlap
- Misalignments
- Finite pulse effects

Known systematic effects appear controllable at the $\delta g < 10^{-15}$ g level.

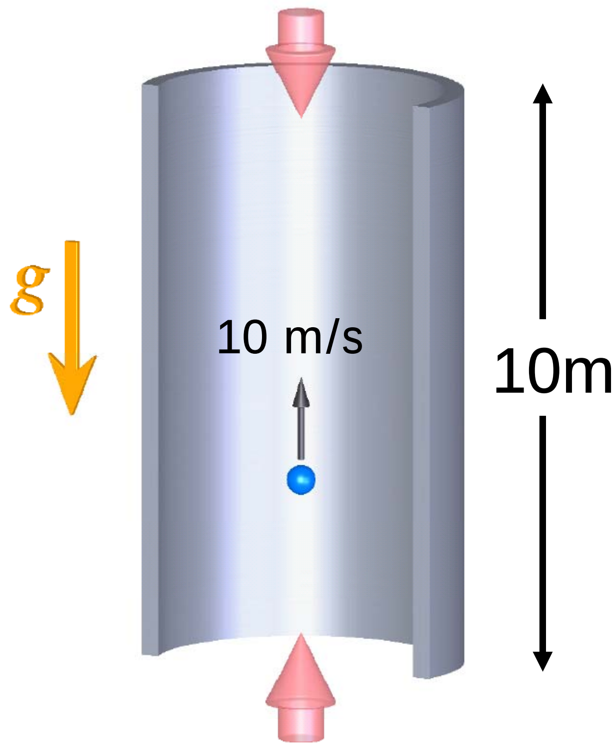
- Common mode cancellation between species is critical

Phase shift	Size (rad)	Fractional size
$-k_{\text{eff}} g T^2$	-2.85×10^{8}	1.00
$k_{\text{eff}} R_e \Omega_y^2 T^2$	6.18×10^{5}	2.17×10^{-3}
$-k_{\text{eff}} T_{zz} v_z T^3$	1.58×10^{3}	5.54×10^{-6}
$\frac{7}{12} k_{\text{eff}} g T_{zz} T^4$	-9.21×10^{2}	3.23×10^{-6}
$-3 k_{\text{eff}} v_z \Omega_y^2 T^3$	-5.14	1.80×10^{-8}
$2 k_{\text{eff}} v_x \Omega_y T^2$	3.35	1.18×10^{-8}
$\frac{7}{4} k_{\text{eff}} g \Omega_y^2 T^4$	3.00	1.05×10^{-8}
$-\frac{7}{12} k_{\text{eff}} R_e T_{zz} \Omega_y^2 T^4$	2.00	7.01×10^{-9}
$-\frac{\hbar k_{\text{eff}}^2}{2m} T_{zz} T^3$	7.05×10^{-1}	2.48×10^{-9}
$\frac{3}{4} k_{\text{eff}} g Q_{zzz} v_z T^5$	9.84×10^{-3}	3.46×10^{-11}
$-\frac{7}{12} k_{\text{eff}} Q_{zzz} v_z^2 T^4$	-7.66×10^{-3}	2.69×10^{-11}
$-\frac{7}{4} k_{\text{eff}} R_e \Omega_y^4 T^4$	-6.50×10^{-3}	2.28×10^{-11}
$-\frac{7}{4} k_{\text{eff}} R_e \Omega_y^2 \Omega_z^2 T^4$	-3.81×10^{-3}	1.34×10^{-11}
$-\frac{31}{120} k_{\text{eff}} g^2 Q_{zzz} T^6$	-3.39×10^{-3}	1.19×10^{-11}
$-\frac{3\hbar k_{\text{eff}}^2}{2m} \Omega_y^2 T^3$	-2.30×10^{-3}	8.06×10^{-12}
$\frac{1}{4} k_{\text{eff}} T_{zz}^2 v_z T^5$	2.19×10^{-3}	7.68×10^{-12}
$-\frac{31}{360} k_{\text{eff}} g T_{zz}^2 T^6$	-7.53×10^{-4}	2.65×10^{-12}
$3 k_{\text{eff}} v_y \Omega_y \Omega_z T^3$	2.98×10^{-4}	1.05×10^{-12}
$-k_{\text{eff}} \Omega_y \Omega_z y_0 T^2$	-7.41×10^{-5}	2.60×10^{-13}
$-\frac{3}{4} k_{\text{eff}} R_e Q_{zzz} v_z \Omega_y^2 T^5$	-2.14×10^{-5}	7.50×10^{-14}
$\frac{31}{60} k_{\text{eff}} g R_e Q_{zzz} \Omega_y^2 T^6$	1.47×10^{-5}	5.17×10^{-14}
$\frac{3}{2} k_{\text{eff}} T_{zz} v_z \Omega_y^2 T^5$	-1.42×10^{-5}	5.00×10^{-14}
$-\frac{7}{6} k_{\text{eff}} T_{zz} v_x \Omega_y T^4$	1.08×10^{-5}	3.81×10^{-14}
$-2 k_{\text{eff}} T_{xx} \Omega_y x_0 T^3$	-6.92×10^{-6}	2.43×10^{-14}
$-\frac{7\hbar k_{\text{eff}}^2}{12m} Q_{zzz} v_z T^4$	-6.84×10^{-6}	2.40×10^{-14}
$-\frac{7}{6} k_{\text{eff}} T_{xx} v_x \Omega_y T^4$	-5.42×10^{-6}	1.90×10^{-14}
$-\frac{31}{60} k_{\text{eff}} g T_{zz} \Omega_y^2 T^6$	4.90×10^{-6}	1.72×10^{-14}
$k_{\text{eff}} T_{xx} v_z \Omega_y^2 T^5$	4.75×10^{-6}	1.67×10^{-14}
$\frac{3\hbar k_{\text{eff}}^2}{8m} g Q_{zzz} T^5$	4.40×10^{-6}	1.55×10^{-14}
$\frac{31}{360} k_{\text{eff}} R_e T_{zz}^2 \Omega_y^2 T^6$	1.63×10^{-6}	5.74×10^{-15}
$-\frac{31}{90} k_{\text{eff}} g T_{xx} \Omega_y^2 T^6$	-1.63×10^{-6}	5.74×10^{-15}
$\frac{\hbar k_{\text{eff}}^2}{8m} T_{zz}^2 T^5$	9.78×10^{-7}	3.43×10^{-15}
$-\frac{\hbar k_{\text{eff}} \alpha B_0 (\partial_z B) T^2}{m}$	-7.67×10^{-8}	2.69×10^{-16}
$\frac{31}{60} k_{\text{eff}} g S_{zzzz} v_z^2 T^6$	-7.52×10^{-8}	2.64×10^{-16}
$-\frac{1}{4} k_{\text{eff}} S_{zzzz} v_z^3 T^5$	3.64×10^{-8}	1.28×10^{-16}
$\frac{31}{72} k_{\text{eff}} T_{zz} Q_{zzz} v_z^2 T^6$	-3.13×10^{-8}	1.10×10^{-16}

General Relativistic Effects

GR Back of the Envelope

$$\frac{\delta\phi}{\phi} \approx 10^{-15}$$



With this much sensitivity, does relativity start to affect results?

- Gravitational red shift of light:

$$\frac{\delta\nu}{\nu} = \frac{gL}{c^2} \approx 10^{-15}$$

- Special relativistic corrections:

$$\frac{v^2}{c^2} \approx 10^{-15}$$

General Relativity Effects

High precision motivates GR phase shift calculation

Consider Schwarzschild metric in the PPN expansion:

$$ds^2 = (1 + 2\phi + 2\beta\phi^2)dt^2 - (1 - 2\gamma\phi)dr^2 - r^2 d\Omega^2 \qquad \left(\phi = -\frac{GM}{r}\right)$$

$$\frac{d\vec{v}}{dt} = -\vec{\nabla}[\phi + (\beta + \gamma)\phi^2] + \gamma[3(\vec{v}\cdot\hat{r})^2 - 2\vec{v}^2]\vec{\nabla}\phi + 2\vec{v}(\vec{v}\cdot\vec{\nabla}\phi)$$

Newton's Gravity	Gravity Gravitates	Kinetic Energy Gravitates
$\frac{d\vec{v}}{dt} = -\nabla\phi$	$-\nabla\phi^2$	$-\vec{v}^2\nabla\phi$

$$\phi g \sim 10^{-9} g$$

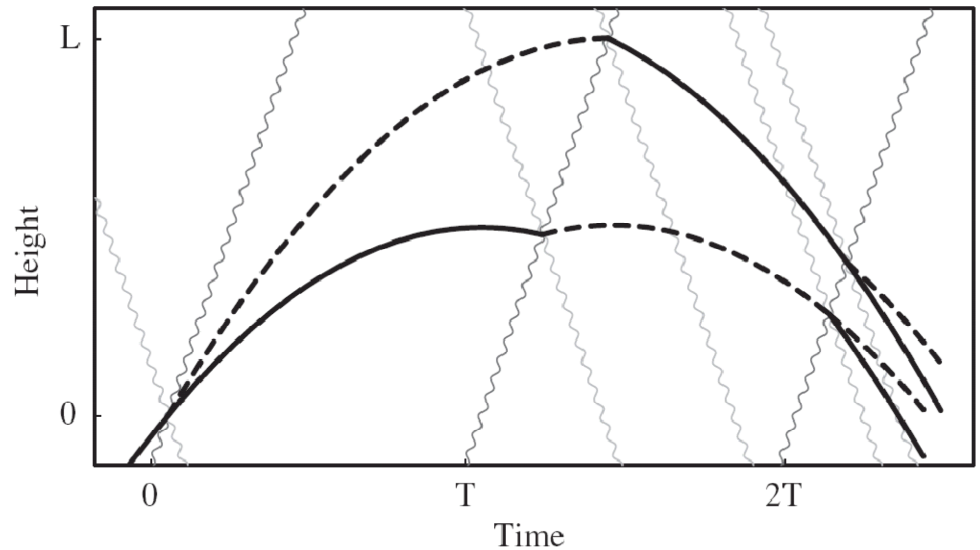
$$v^2 g \sim 10^{-15} g$$

(can only measure changes on the interferometer's scale)

$$10^{-9} g \left(\frac{10\ \mathrm{m}}{R_{\mathrm{Earth}}}\right) \sim 10^{-15} g$$

GR Phase Shift Calculation

- Geodesics for atoms and photons
- Propagation phase is the proper time (action) along geodesics
- Non-relativistic atom-light interaction in the LLF at each intersection point



Projected Experimental Limits:

Tested Effect	current limit	AI initial	AI upgrade	AI future	AI far future
PoE	3×10^{-13}	10^{-15}	10^{-16}	10^{-17}	10^{-19}
PPN (β, γ)	10^{-4}-10^{-5}	10^{-1}	10^{-2}	10^{-4}	10^{-6}

	Phase shift	Size (rad)	Interpretation
(1)	$-k_{\text{eff}}gT^2$	3×10^8	gravity
(2)	$-k_{\text{eff}}(\partial_r g)T^3 v_L$	-2×10^3	1st gradient
(3)	$-3k_{\text{eff}}gT^2 v_L$	4×10^1	Doppler shift
(4)	$(2-2\beta-\gamma)k_{\text{eff}}g\phi T^2$	2×10^{-1}	GR
(5)	$-\frac{7}{12}k_{\text{eff}}(\partial_r^2 g)T^4 v_L^2$	8×10^{-3}	2nd gradient
(6)	$-5k_{\text{eff}}gT^2 v_L^2$	3×10^{-6}	GR
(7)	$(2-2\beta-\gamma)k_{\text{eff}}\partial_r(g\phi)T^3 v_L$	2×10^{-6}	GR 1st grad
(8)	$-12k_{\text{eff}}g^2T^3 v_L$	-6×10^{-7}	GR

Improvements:

- Longer drop times
- LMT atom optics

Measurement Strategies

Can these effects be distinguished from backgrounds?

1. Velocity dependent gravity (Kinetic Energy Gravitates)

→ Phase shift $5k_{\text{eff}}gT^2v_L^2$ has unique scaling with v_L , T

→ Compare simultaneous interferometers with different v_L

2. Non-linear gravity (Gravity Gravitates)

$$\frac{d\vec{v}}{dt} = -\vec{\nabla}[\phi + (\beta + \gamma)\phi^2]$$

"Newtonian" mass density

Gravitational field energy!

Divergence: $$\nabla \cdot \frac{d\mathbf{v}}{dt} = -\nabla^2\phi - 2\,(\beta+\gamma)\,\nabla\cdot(\phi\mathbf{g}) \approx -4\pi G\rho - 2\,(\beta+\gamma)\,\mathbf{g}^2$$

So, in GR in vacuum: $$\nabla \cdot \frac{d\mathbf{v}}{dt} \neq 0$$

Can discriminate from Newtonian gravity using three axis measurement

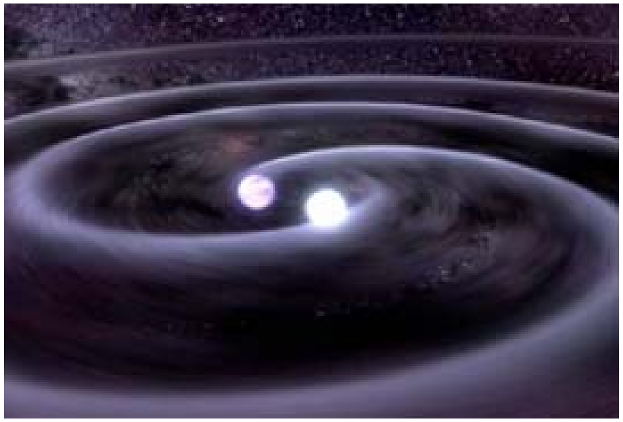
Gravitational Wave Detection

Some Gravitational Wave Sources

Atom sensor frequency band: 50 mHz – 10 Hz

White dwarf binaries

- Solar mass (< 100 kpc)
- Typically < 1 Hz
- Long lifetime in AGIS-LEO band



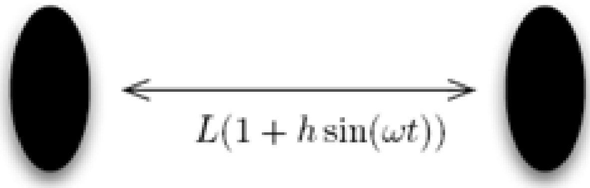
Black hole binaries

- Solar mass into intermediate mass (< 1 Mpc)
- Merger rate is uncertain (hard to see)
- Standard siren?

Cosmological

- Reheating
- Phase transition in early universe (*e.g.,* RS1)
- Cosmic string network $G\mu \sim 10^{-8}$

Gravitational Wave Phase Shift Signal



$$ds^2 = dt^2 - (1 + h\sin(\omega(t - z)))dx^2 - (1 - h\sin(\omega(t - z)))dy^2 - dz^2$$

Laser ranging an atom (or mirror) that is a distance *L* away:

Position → $x \sim L(1 + h\sin(\omega t))$

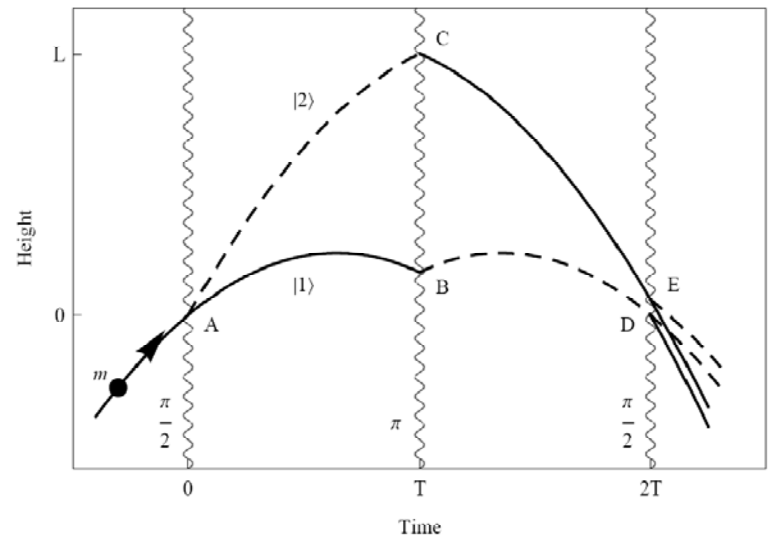
Acceleration → $a \sim hL\omega^2 \sin(\omega t)$

Phase Shift: $$\Delta\phi = kaT^2 \sim khL\omega^2 \sin(\omega t)T^2$$

Relativistic Calculation: $$\Delta\phi_{\text{tot}} = 2hk_{\text{eff}} \sin^2\left(\frac{\omega T}{2}\right) \frac{\sin(\omega L)}{\omega} \sin(\omega t)$$

Vibrations and Seismic Noise

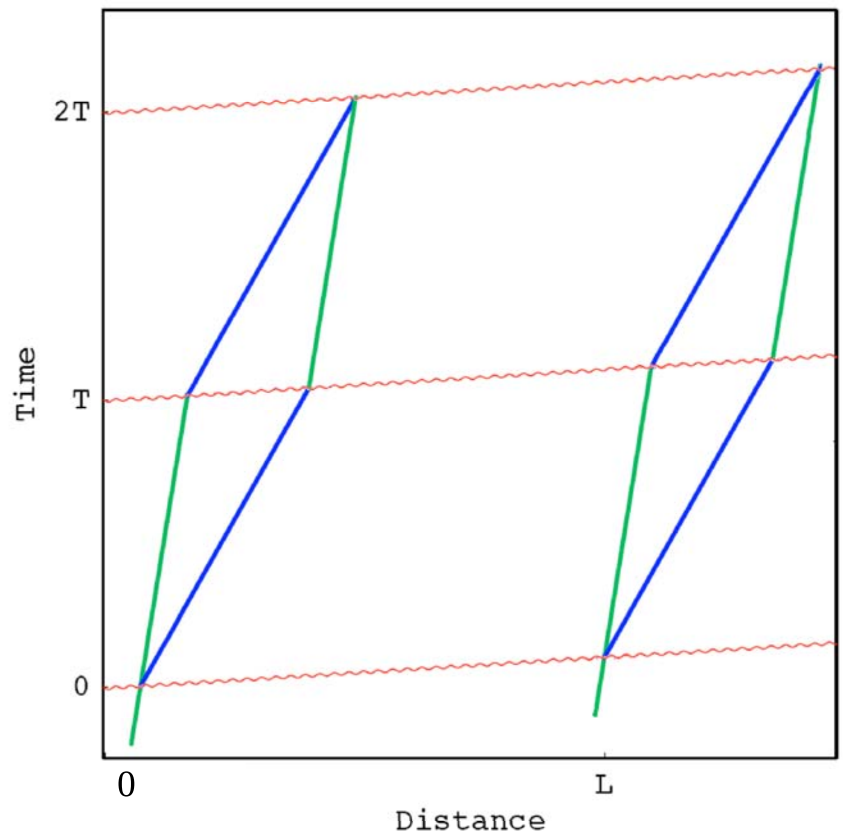
- Atom test mass is inertially decoupled (freely falling); insensitive to vibration
- Atoms analogous to LIGOs mirrors
- However, the lasers vibrate
- Laser has phase noise



Laser vibration and intrinsic phase noise are transferred to the atom's phase via the light pulses.

Differential Measurement

Run two, widely separated atom interferometers using common lasers.



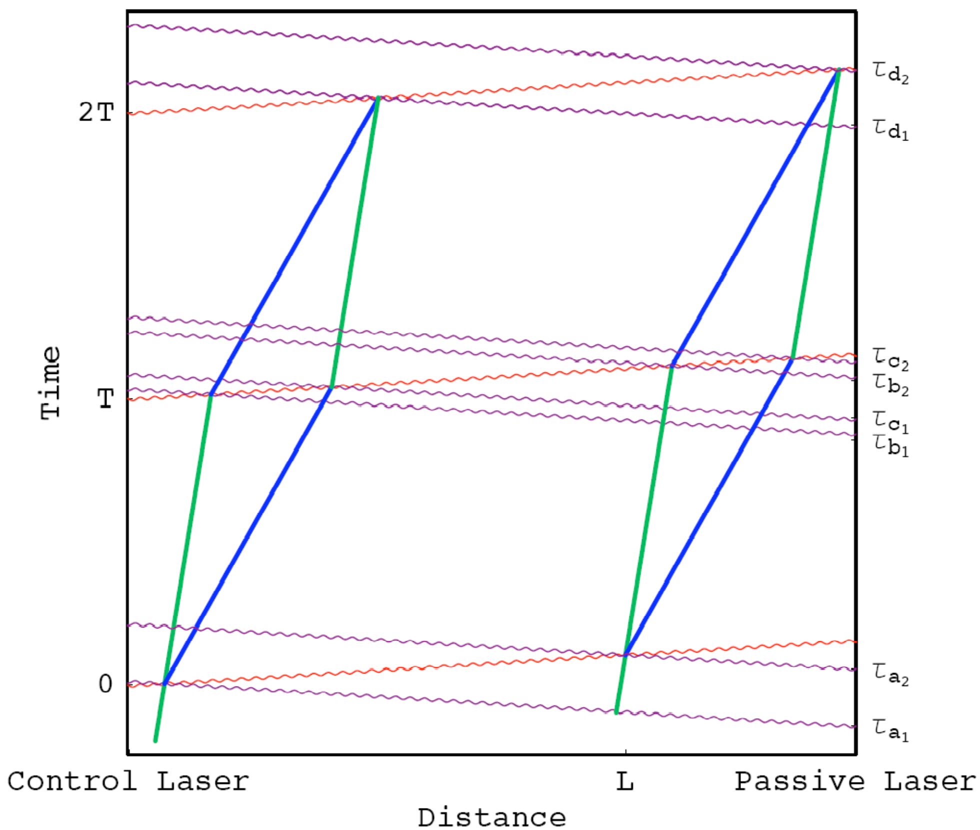
Measure differential phase shift between the two interferometers.

Gravitational wave signal is retained in the differential phase shift ~ $k_{\mathrm{eff}}\, hL$

Laser vibration and phase noise cancels (up to finite light travel time effects).

Differential Measurement

Run two, widely separated atom interferometers using common lasers.



Light from the second laser is not exactly common

→ Light travel time delay is a source of noise ($f > c/L$)

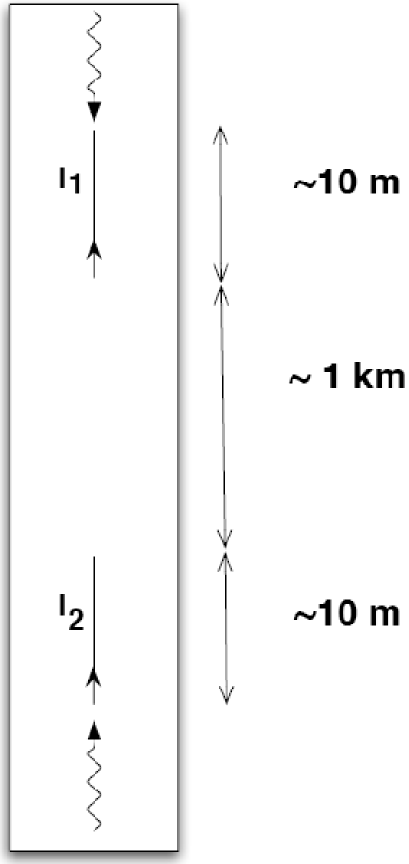
Proposed Configuration

- Run two, widely separated interferometers using common lasers
- Measure the differential phase shift

Benefits:

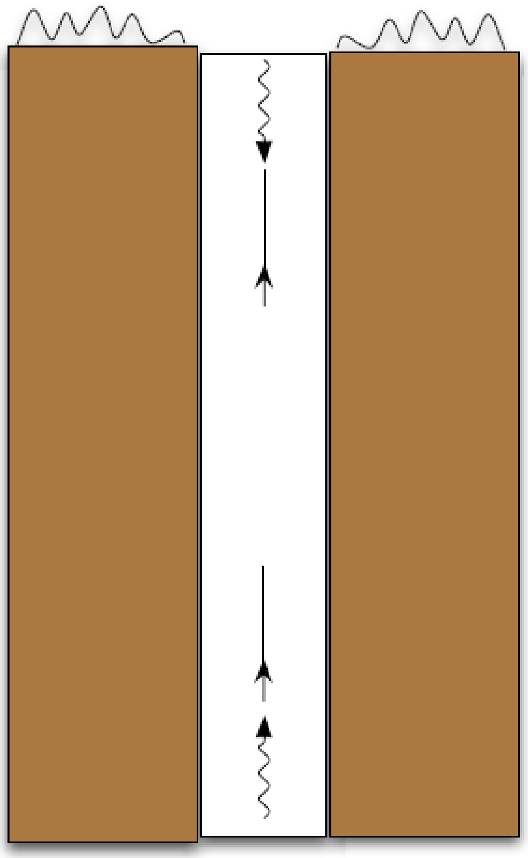
1. Signal scales with length ***L* ~ 1 km** between interferometers (easily increased)
2. Common-mode rejection of seismic & phase noise

Allows for a free fall time ***T* ~ 1 s.**
(Maximally sensitive in the **~1 Hz** band)



(Vertical mine shaft)

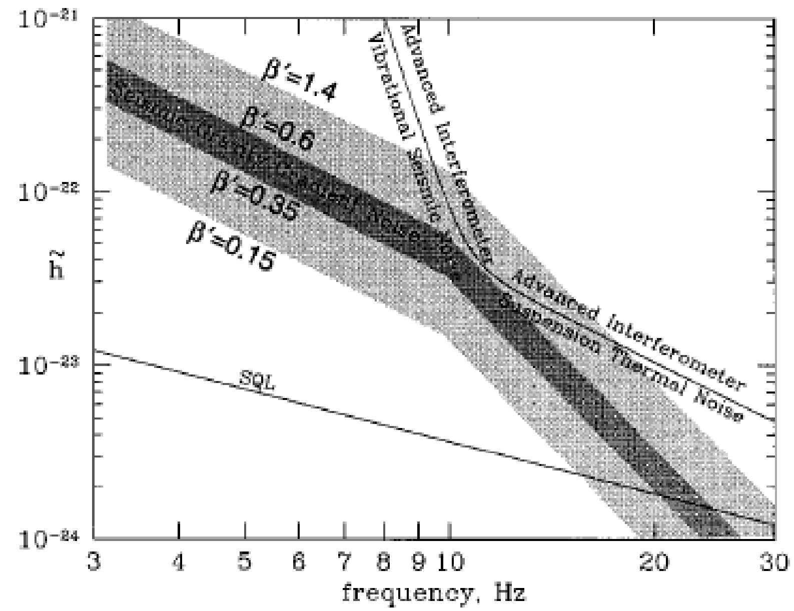
Gravity Gradient Noise Limit



Seismic fluctuations give rise to Newtonian gravity gradients which can not be shielded.

Seismic noise induced strain analysis for LIGO (Thorne and Hughes, PRD **58**)

.

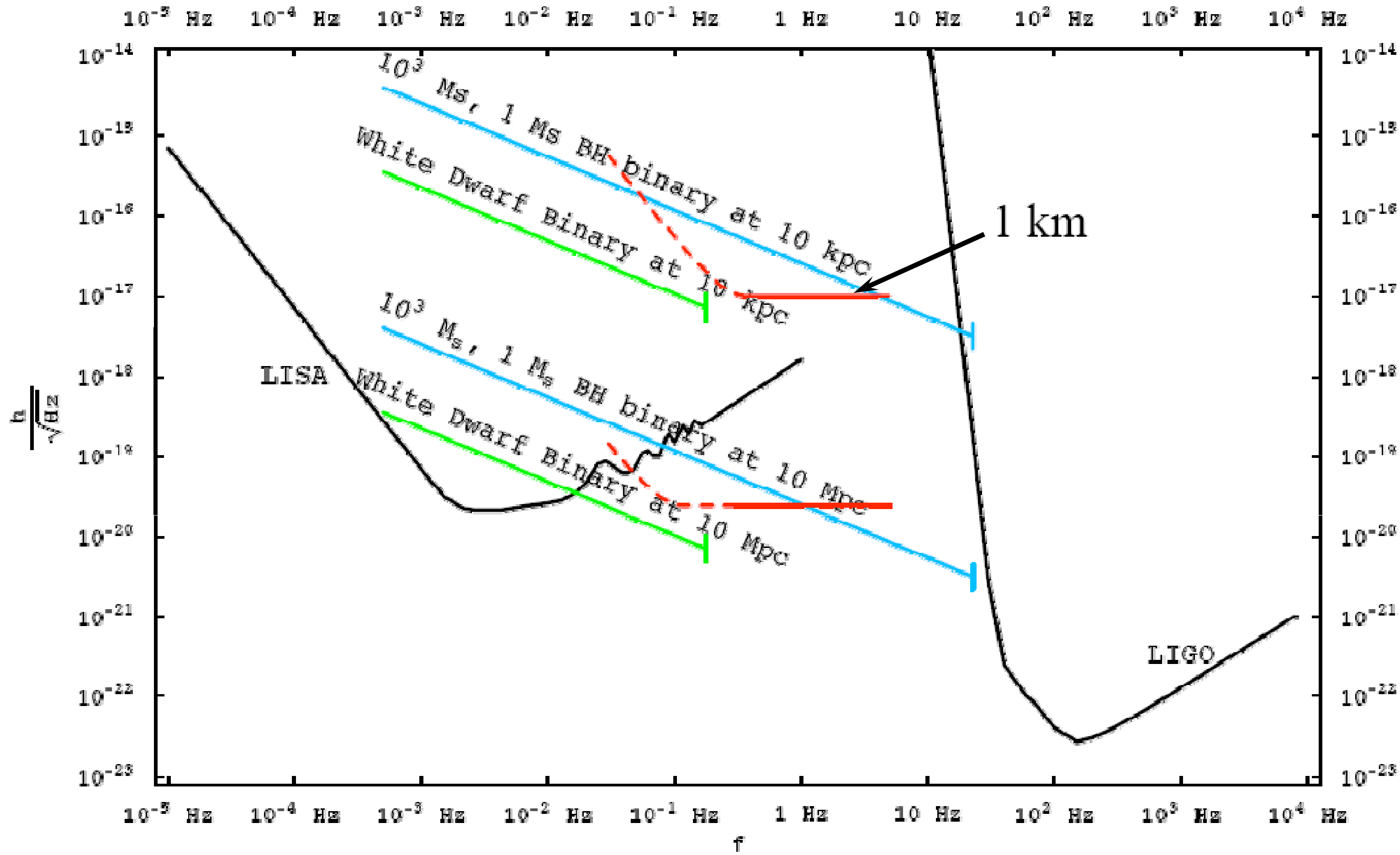


Allows for terrestrial gravitational wave detection down to

~ 0.3 Hz

Projected Terrestrial GW Sensitivity (AGIS)

AGIS: Atomic gravitational wave interferometric sensor



Setup	L	k_{eff}	T	I_L	Phase Sensitivity	f_d
Terrestrial 1	1 km	$1.6 \times 10^9\ \text{m}^{-1}$	1.4 s	10 m	10^{-4} rad	10 Hz
Terrestrial 2	4 km	$1.6 \times 10^{10}\ \text{m}^{-1}$	4.5 s	100 m	10^{-5} rad	10 Hz

Motivation to Operate in Space

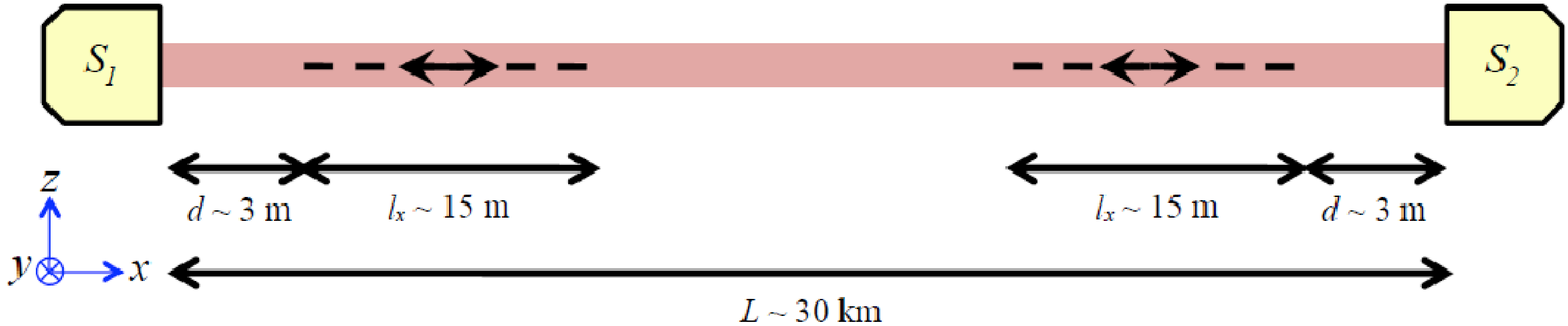
- Longer baselines
- No gravity bias (lower frequencies accessible)
- Gravity gradient noise

What constrains the sensitivity in space?

- LMT beamsplitters: $v_r \sim 1 \text{ m/s } (200\hbar k)$
- Low frequencies: $T > 1 \text{ s}$
- The satellite must be at least $v_r T > 1 \text{ m}$
- Gravity gradient noise from the satellite

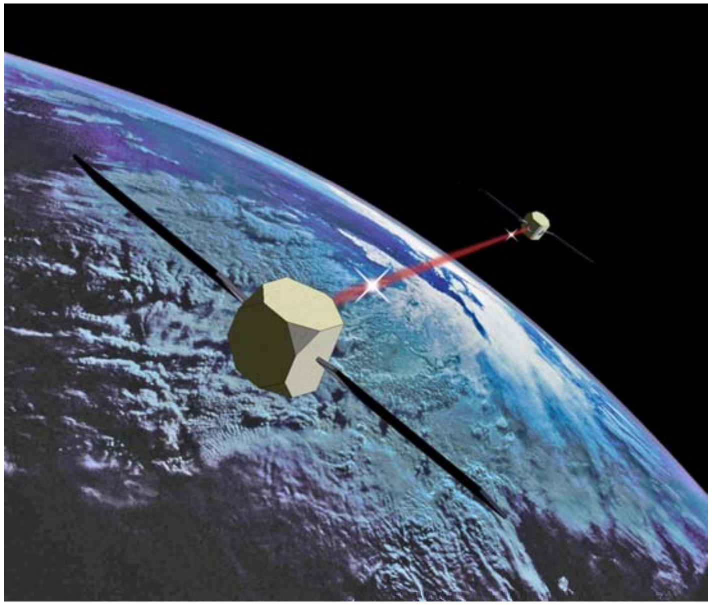
Move outside and away from the satellite.

AGIS-LEO Satellite Configuration



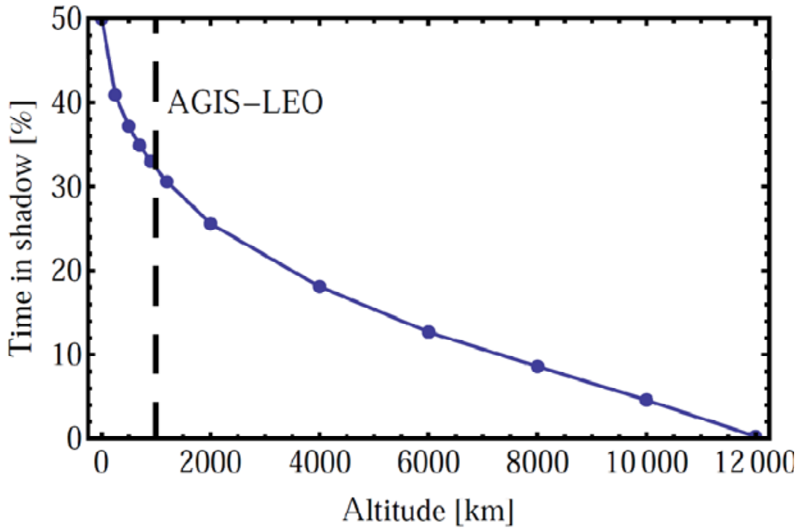
- Two satellites
- AI near each satellite
- Common interferometer laser
- Atoms shuttled into place with optical lattice
- Florescence imaging
- Baseline $L \sim 30$ km

Low Earth Orbit



LEO Incentives:

- Nearby: minimize mission cost
- Operate in Earth's shadow
- Earth science opportunities (?): magnetic fields and gravity gradients.



LEO Challenges:

- Vacuum
- Earth magnetic field, gravity gradients
- Large rotation bias; Coriolis deflections

1000 km orbit is favorable

LEO Rotation Bias

Coriolis deflections can prevent interference: $\Delta x \simeq \hbar/\Delta p \sim 10\ \mu\mathrm{m}\sqrt{\frac{100\ \mathrm{pK}}{\tau}}$ (coherence length)

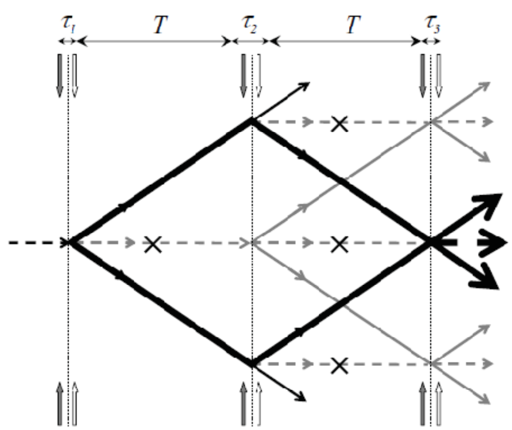
With AGIS-LEO parameters, the transverse displacement is:

$$z_C = \frac{1}{2}\left|2\boldsymbol{\Omega}\times\mathbf{v}\right|T^2 = \Omega_{\mathrm{or}}(k_{\mathrm{eff}}/m)T^2 \sim 2\ \mathrm{cm}$$

Standard three pulse ($\pi/2 - \pi - \pi/2$) sequence **does not close.**

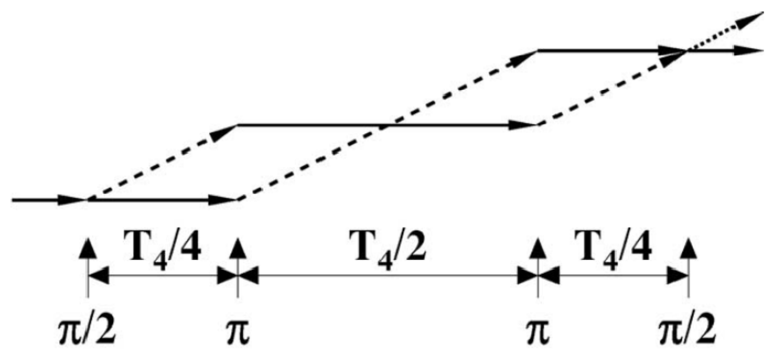
---

1. Double diffraction beamsplitters



N. Malossi et al., Phys. Rev. A 81, 013617 (2010).

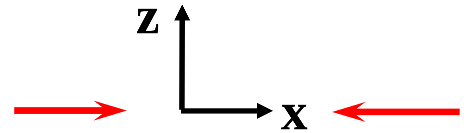
2. Multiple pulse sequences

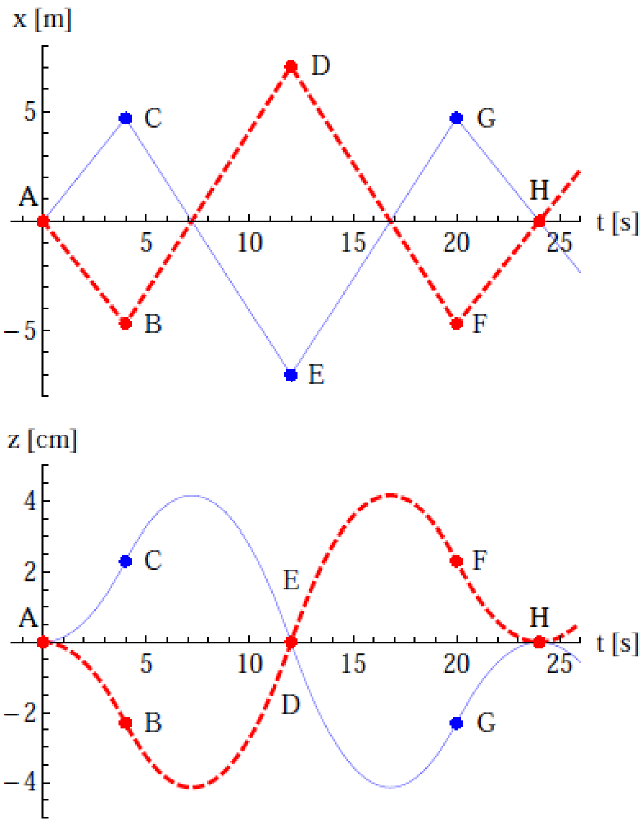


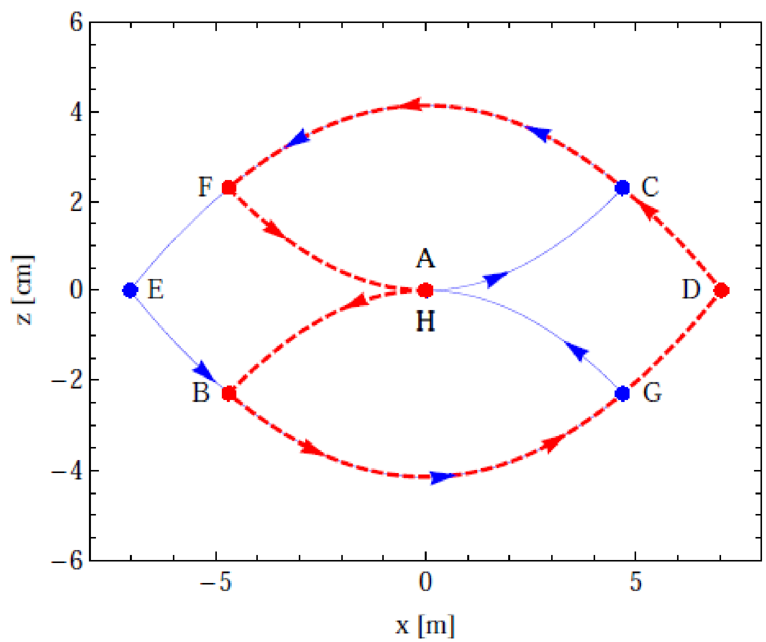
B. Dubetsky and M. A. Kasevich, Phys. Rev. A 74, 023615 (2006)

AI Geometry with Large Rotation Bias

Five pulse sequence: $(\pi/2 - \pi - \pi - \pi - \pi/2)$







$$\mathbf{k}_i = \kappa_i k_{\text{eff}} \hat{\mathbf{x}}$$

t	κ
0	1
T	9/4
3T	5/2
5T	9/4
6T	2

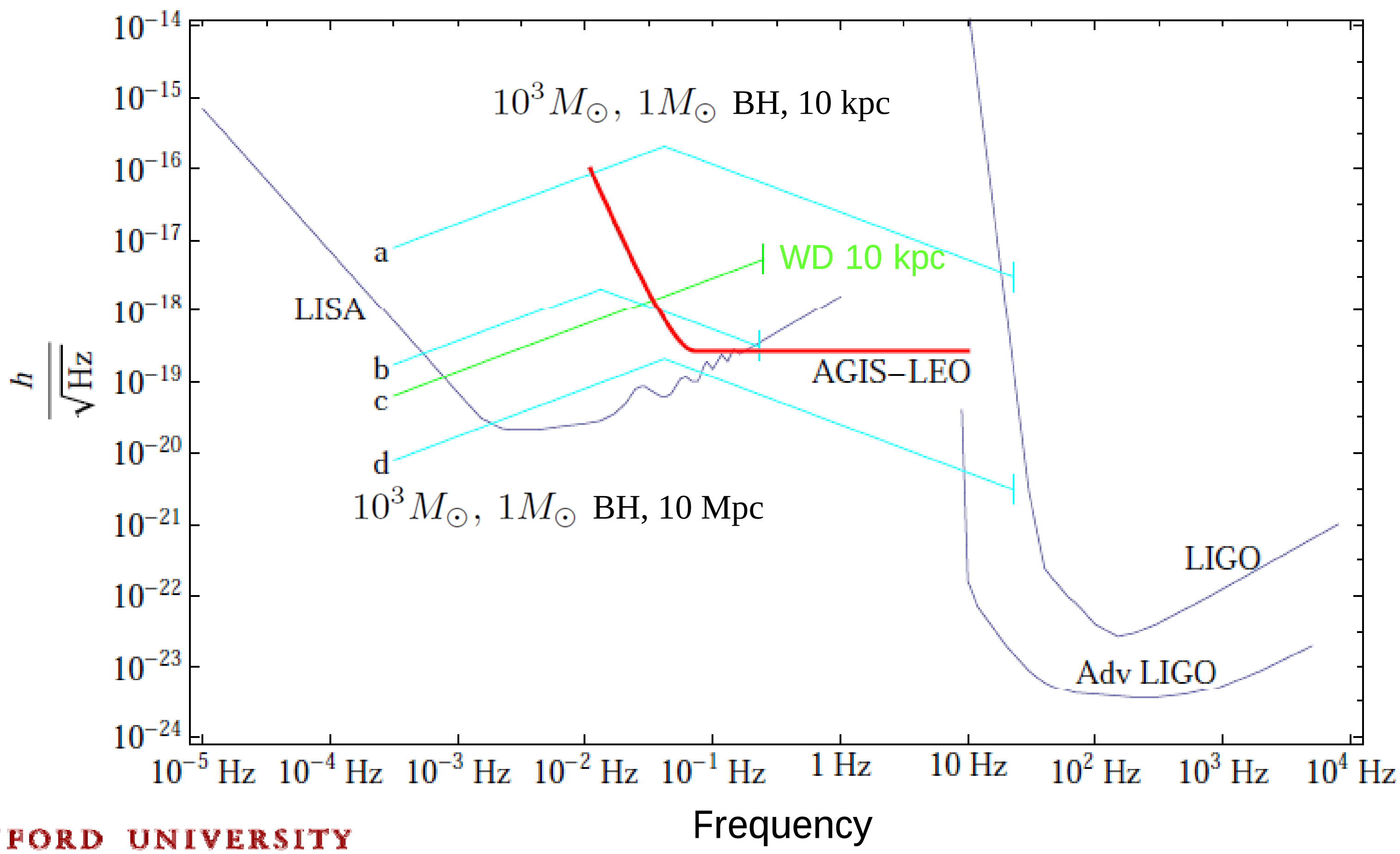
- Beamsplitter momenta chosen to give symmetry and closure
- Insensitive to acceleration + gravity gradients

AGIS-LEO Sensitivity

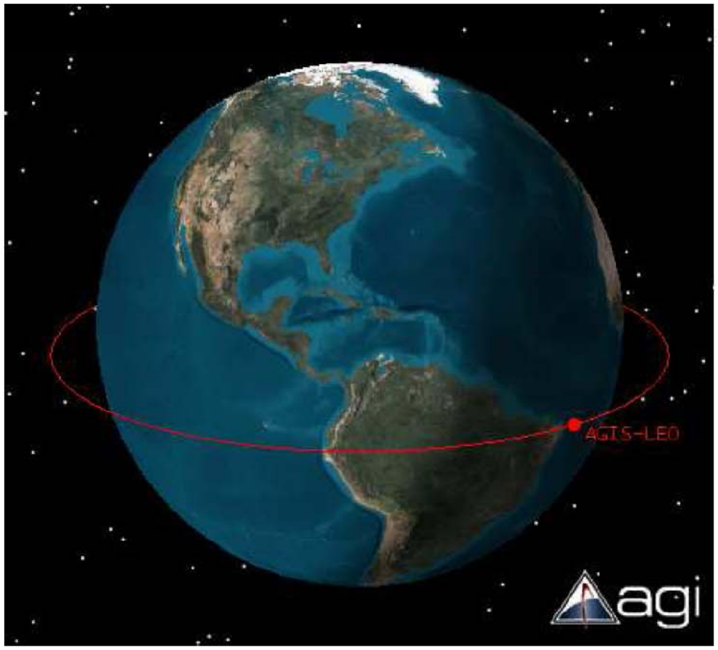
Five pulse sequence; shot noise strain sensitivity

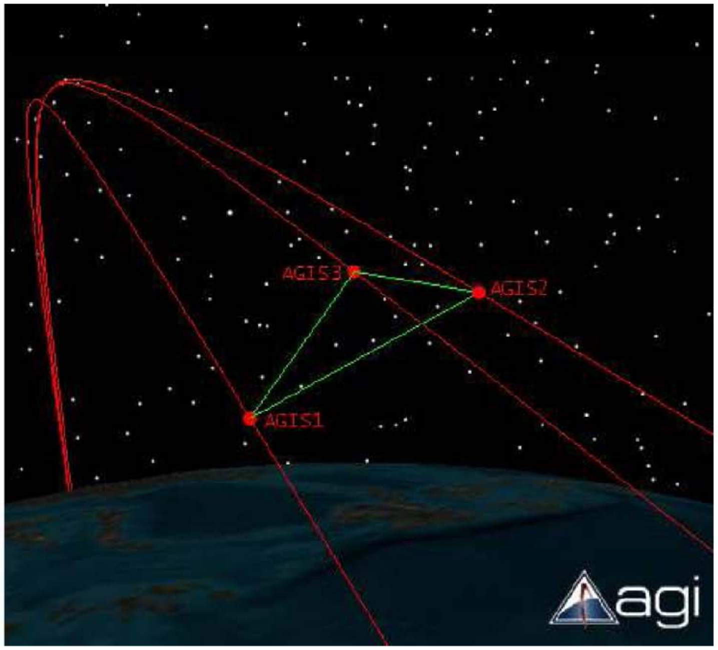
$$\Delta\phi_{\mathrm{GW}} = 8k_{\mathrm{eff}}hL\sin^4(\omega T/2)\left(\frac{7+8\cos\omega T}{2}\right)\sin\theta_{\mathrm{GW}}$$

$$\hbar k_{\mathrm{eff}} = 200\hbar k$$
$$\delta\phi = 10^{-4}\ \mathrm{rad}/\sqrt{\mathrm{Hz}}$$
$$T = 4\ \mathrm{s}$$
$$L = 30\ \mathrm{km}$$



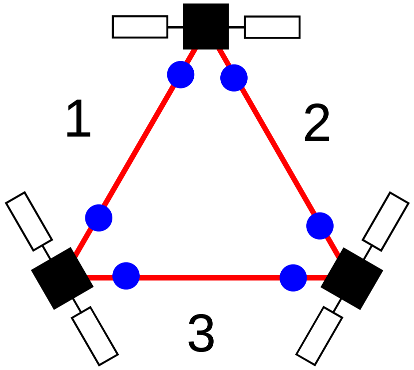
Three Satellite Configuration



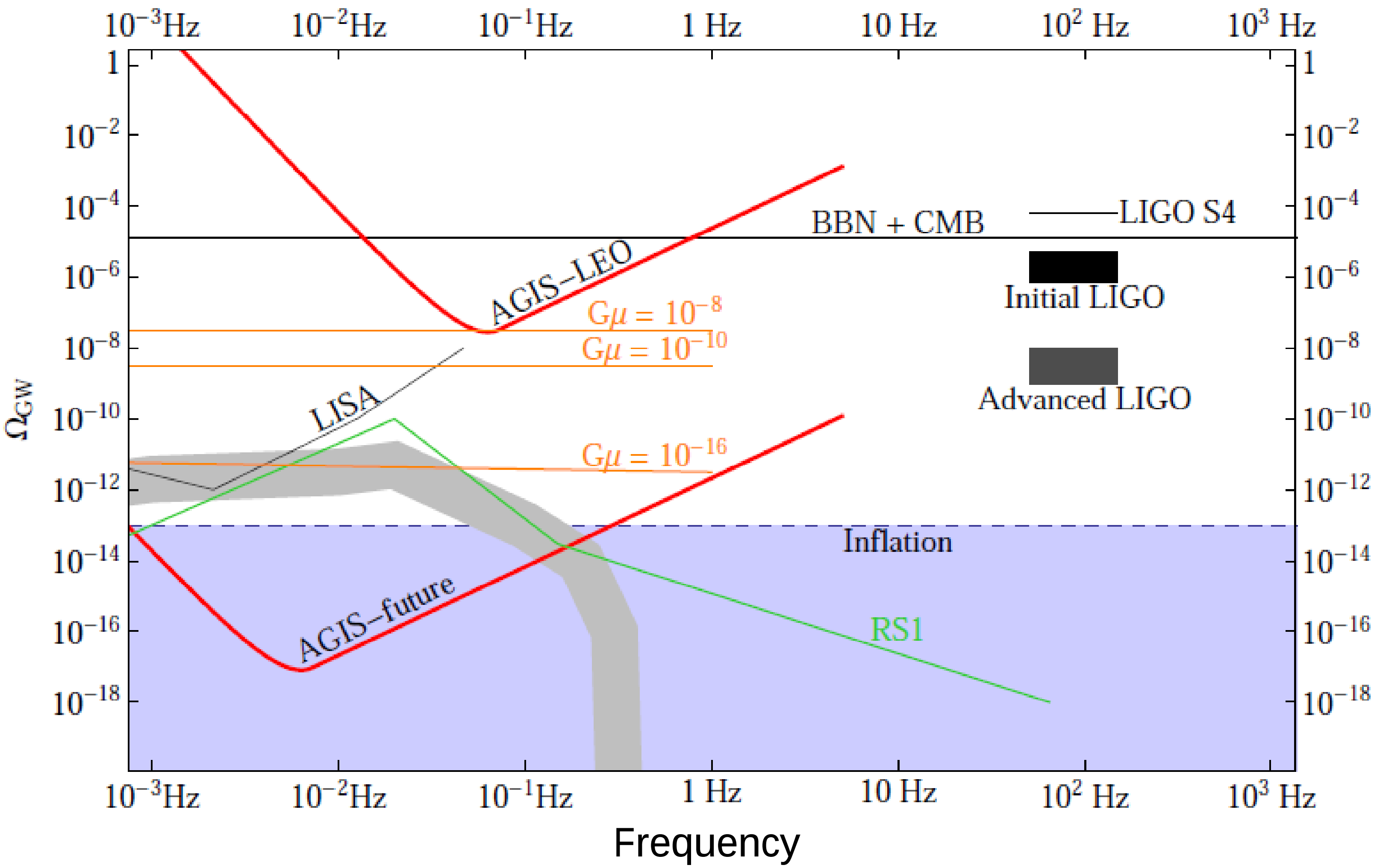


Observation of **stochastic sources** require correlation among multiple detectors.

Three satellites with an AI pair between each.
(3 independent detectors)



AGIS Stochastic Sensitivity



Boom Configuration

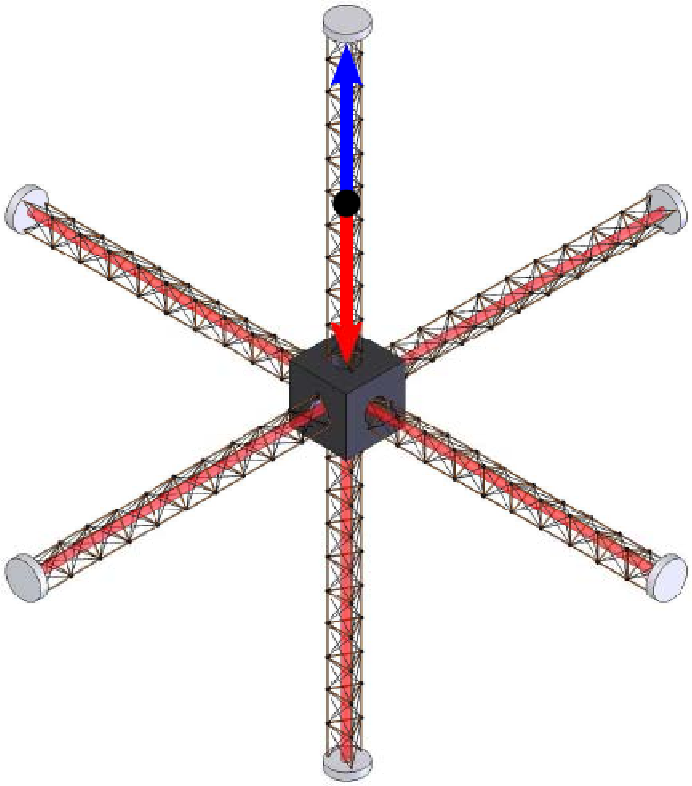
- Single-satellite design using light, self-deploying booms (> 100 m, commercially available)

- Boom can be covered in thin, non-structural wall to provide protection from Sun + improved vacuum

- Advanced atom optics (LMT, etc.) needed to compensate for shorter baseline

- Scientifically interesting sensitivities possible

$10^{-16}/\mathrm{Hz}^{1/2} – 10^{-18}/\mathrm{Hz}^{1/2}$ (10 mHz – 1 Hz)



Conceptual three axis detector

Image source: ABLE Coilable booms (http://www.aec-able.com/Booms/coilboom.html)

Collaborators

Stanford University

Mark Kasevich (PI)
David Johnson
Susannah Dickerson
Tim Kovachy
Alex Sugarbaker
Sheng-wey Chiow
Peter Graham
Savas Dimopoulos
Surjeet Rajendran

Space Telescope Science Institute

Babak Saif

Laboratoire Charles Fabry de l'Institut d'Optique

Philippe Bouyer

NASA Goddard Space Flight Center

Bernard D. Seery
Lee Feinberg
Ritva Keski-Kuha