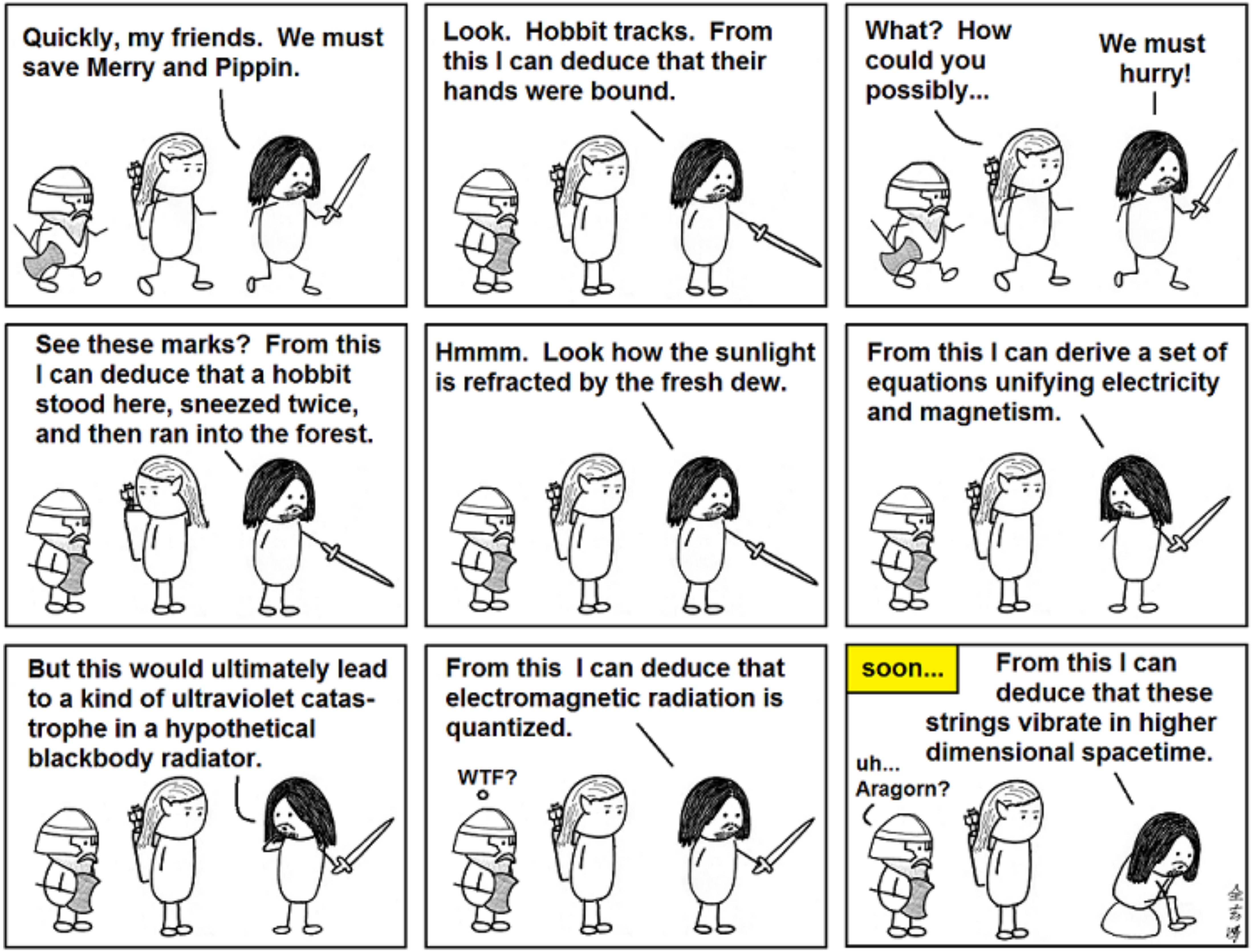

Quickly, my friends. We must save Merry and Pippin.
Look. Hobbit tracks. From this I can deduce that their hands were bound.
What? How could you possibly...
We must hurry!
See these marks? From this I can deduce that a hobbit stood here, sneezed twice, and then ran into the forest.
Hmmm. Look how the sunlight is refracted by the fresh dew.
From this I can derive a set of equations unifying electricity and magnetism.
But this would ultimately lead to a kind of ultraviolet catas-trophe in a hypothetical blackbody radiator.
From this I can deduce that electromagnetic radiation is quantized.
WTF?
soon...
From this I can deduce that these strings vibrate in higher dimensional spacetime.
uh... Aragorn?

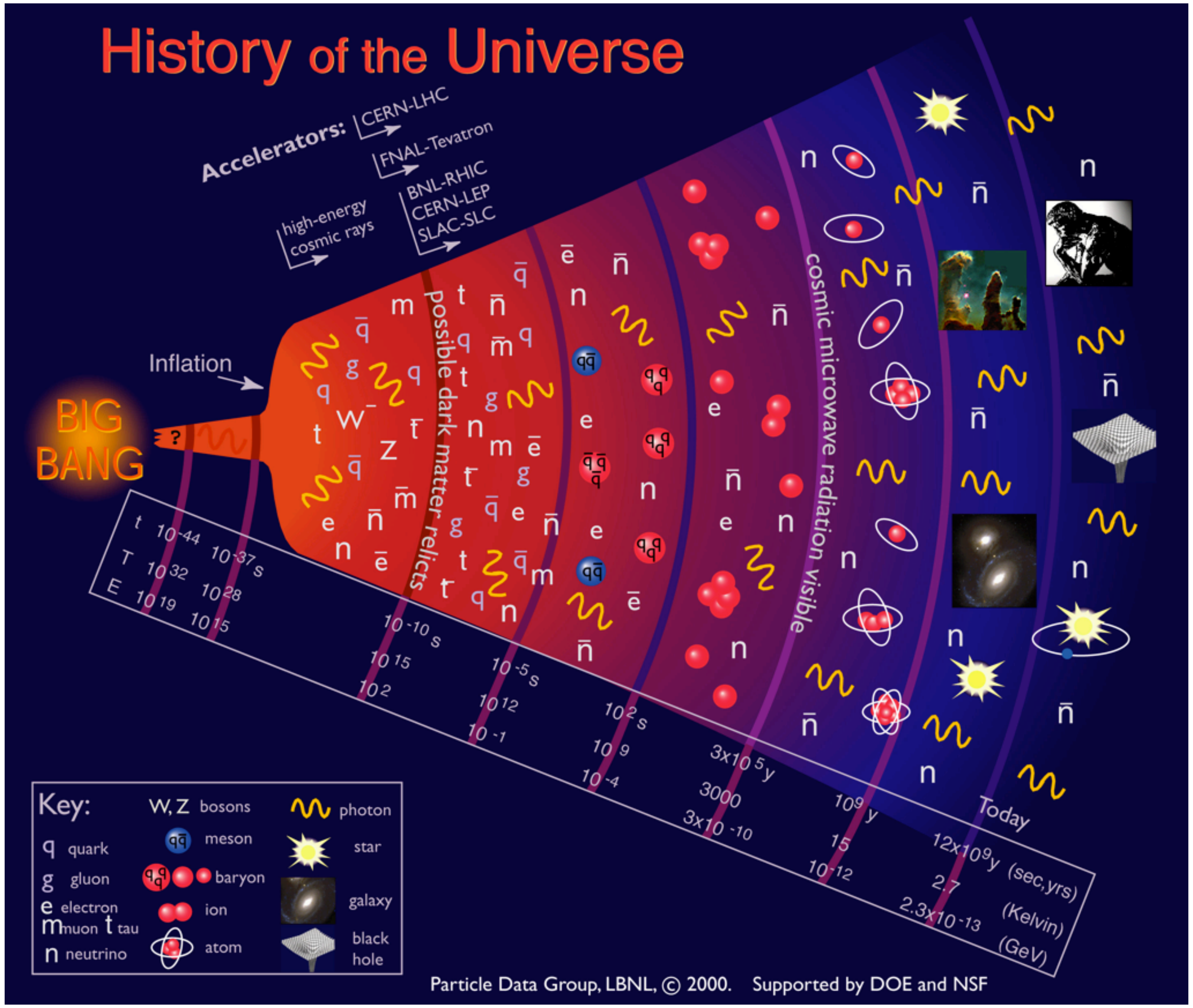
A highly idiosyncratic

"Summary of the Universe"

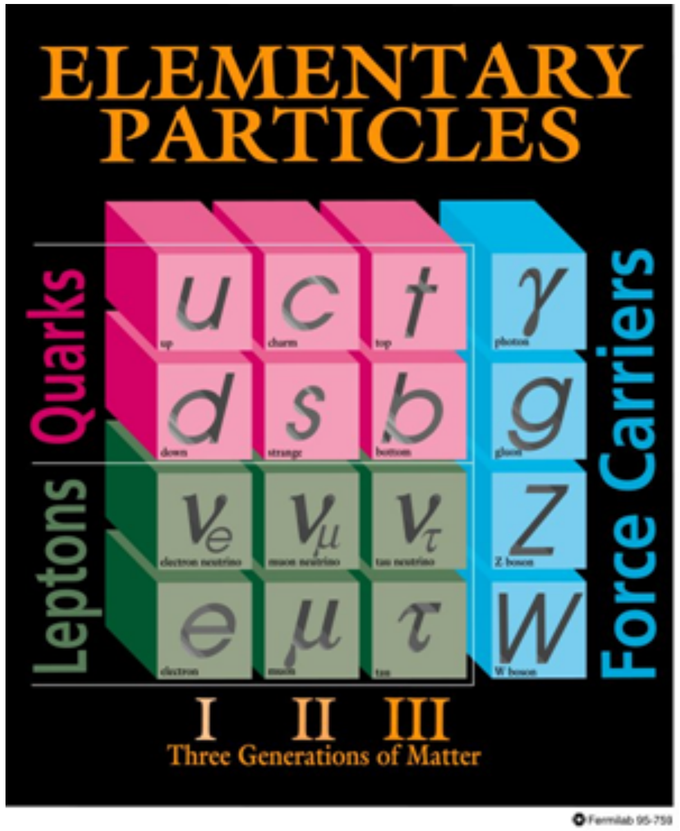
Shamit Kachru (Stanford & SLAC)
SLAC Summer Institute, 2011

1. Introduction

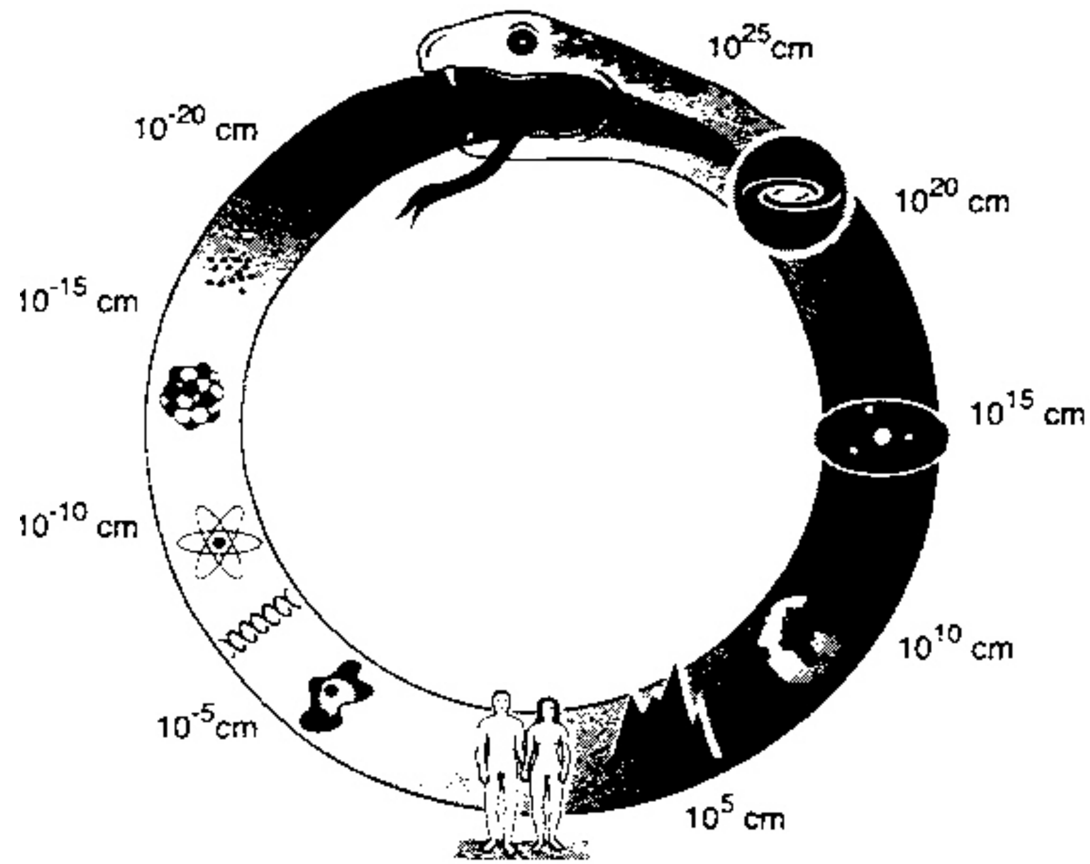
Over the course of the past ten days, you've heard a lot about the frontiers of our knowledge of cosmological history:



and the microphysical participants that help us to understand it:



The fact that the two subjects go hand-in-hand is no longer surprising to us; it has been clear at least since the renaissance of the 1970s that particle physics has a lot to teach us about cosmology, and vice versa.



I am going to spend the bulk of my talk describing my, likely highly idiosyncratic, view of where some of the major questions that will drive theoretical paradigm shifts in the future now lie. This will drive me to distances and energies that lie beyond (perhaps, well beyond) the scales visible in the Ouroboros above.

My basic organisation will orbit around three areas of basic ignorance in our current understanding, so I will really be summarizing our ignorance of the Universe:

1. We do not understand how our Universe originated (where are we coming from?).

2. We do not understand how to describe our Universe today, theoretically (where are we?).

3. We do not understand what will happen to our Universe in the deep future (where are we going?).

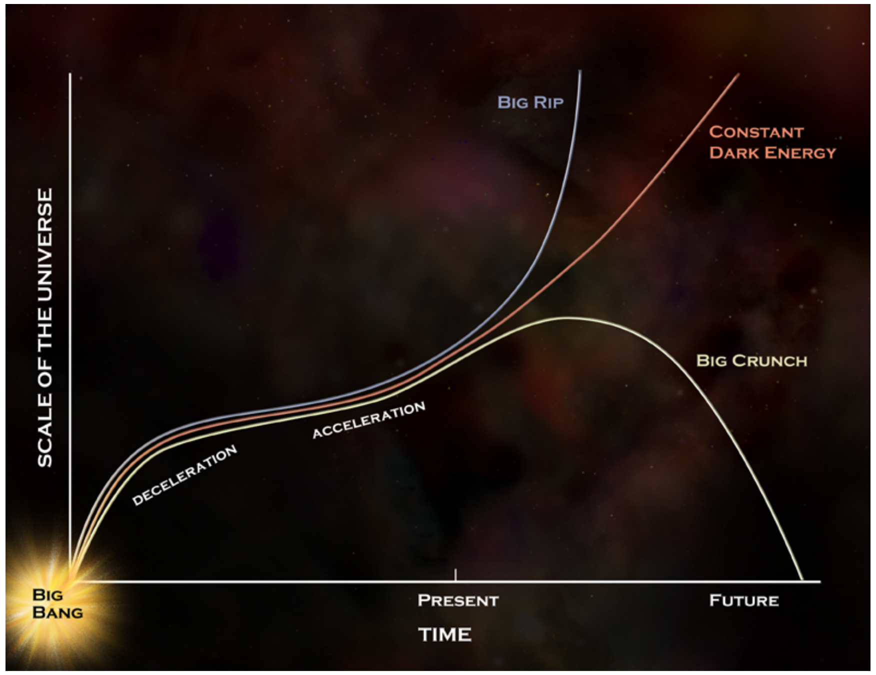
I will discuss, broadly speaking, the (theoretical) problems and opportunities in each area. But I will start with 2, and work forwards (and backwards) to 3 (and 1).

II. Where are we today?

Since the discovery of the Hubble expansion, it has been useful to model our Universe in terms of FLRW metrics:

$$ds^2 = -dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2(\theta)d\phi^2)\right)$$

The great discovery of the 1990s, which you heard a lot about yesterday (and throughout), is that currently the expansion of the Universe is accelerating:



The most reasonable explanation is that we basically live in de Sitter space, the maximally symmetric solution of Einstein's equations with positive cosmological constant.

$$ds^2 = -dt^2 + e^{2Ht}(dx^2 + dy^2 + dz^2)$$

This immediately raises two questions. We have convincing answers for neither.

A. How does one formulate a theory of gravity in de Sitter space?

B. What explains the existence and magnitude of the Hubble constant governing accelerated expansion?

Let us discuss these issues in turn.

A. How does one formulate gravity in de Sitter space?

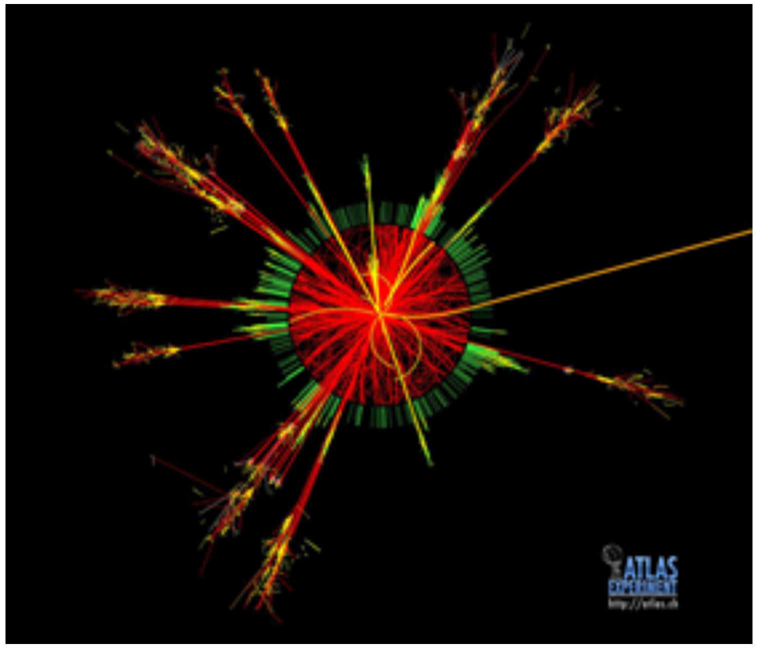
This may sound like a confusing question at first. After all, we know that the correct theory of gravity for all practical purposes is Einstein's general theory of relativity:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

Einstein's equation comes from a well-defined variational problem, so can't we just do standard Lagrangian field theory with it?

The difficulties with this are well known. Some are deeper than others.

Minor problem: The theory is non-renormalizable. It has poor behavior in high-energy scattering.



$$P_{\text{gravitational scattering,i}} \sim G_N E^2$$

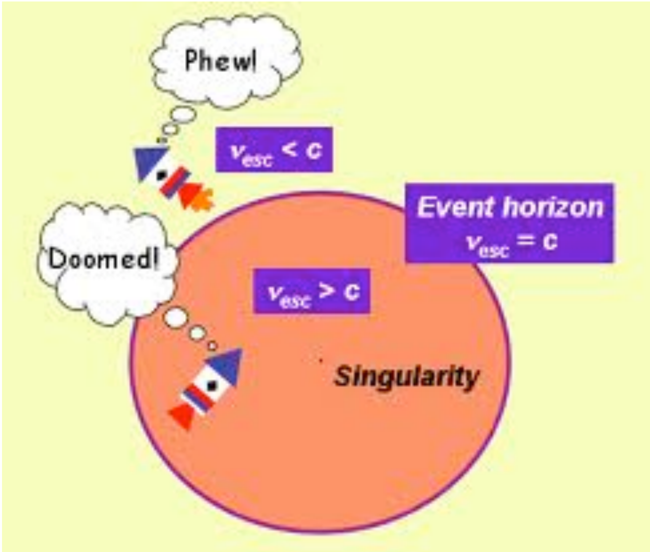
Deeper issue: There is very strong evidence that gravitational theories are fundamentally not like local Lagrangian field theories.

The evidence for the second, more startling problem grew out of classic studies of black holes in the 1970s (carried out by numerous relativists).

The simplest solution of Einstein's theory of relativity is the Schwarzschild black hole:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)(cdt)^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Something funny happens here at r=2GM : there is an event horizon.



Studies of black holes by Bekenstein, Hawking and others in the 1970s, yielded some striking analogies between these strange objects and more familiar physical systems.

For instance, a charged, rotating black hole was found to be governed by the "equation of state" :

$$dM = \kappa dA + \Omega dJ + \Phi dQ$$

"surface gravity" of the black hole

Other theorems showed that:

* The horizon area A of a black hole increases or stays constant, but never decreases, as a function of time.

* The surface gravity is constant over the event horizon

Now, recall the thermodynamic identity:

$$dE = TdS + ...$$

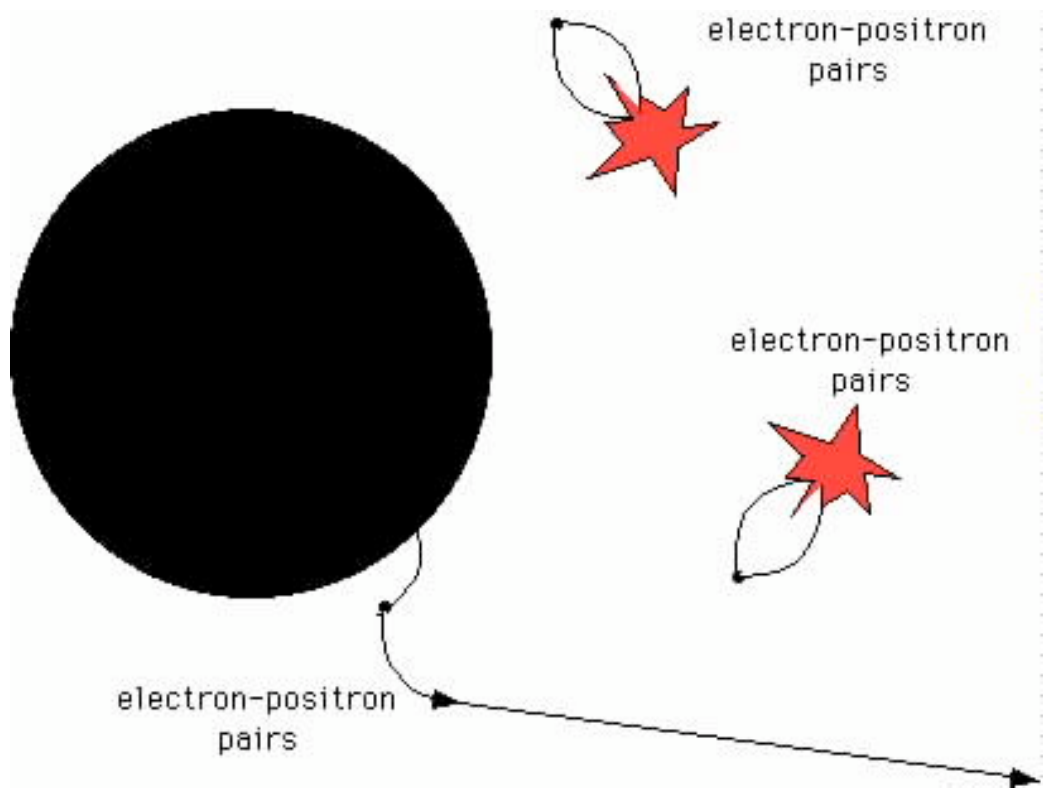
This suggests an analogy:

Mass of black hole = Energy

Area of event horizon = Entropy

Surface gravity = Temperature

The discovery of Hawking radiation in 1974 gave strong evidence that this analogy should be taken seriously.



But the fact that the entropy scales like the area of the horizon, and isn't extensive in the volume, is confusing.

After all, the generic fate of a distribution of matter, due to the Jeans instability, is to eventually collapse into a big black hole:

For matter in a volume V, the resulting event horizon will have area less than the area of the surface bounding V.

In standard thermodynamics, entropy is extensive, because you can excite local bits of stuff at each point in space. These results suggest that gravity theories are not like this.

In fact, it was proposed (somewhat vaguely) by ‘t Hooft and Susskind, that in any theory of gravity, one can “holographically” formulate the physics on a surface of co-dimension one in the larger space-time.

This statement has been made more precise, with overwhelming evidence in some cases, in the framework of string theory.

The poster-child for successful implementation of holography is anti-de Sitter space. This is the evil twin of de Sitter space; it is the maximally symmetric solution of gravity with a negative cosmological constant. It is easy to get AdS spaces (in less than ten dimensions) out of string theory constructions.

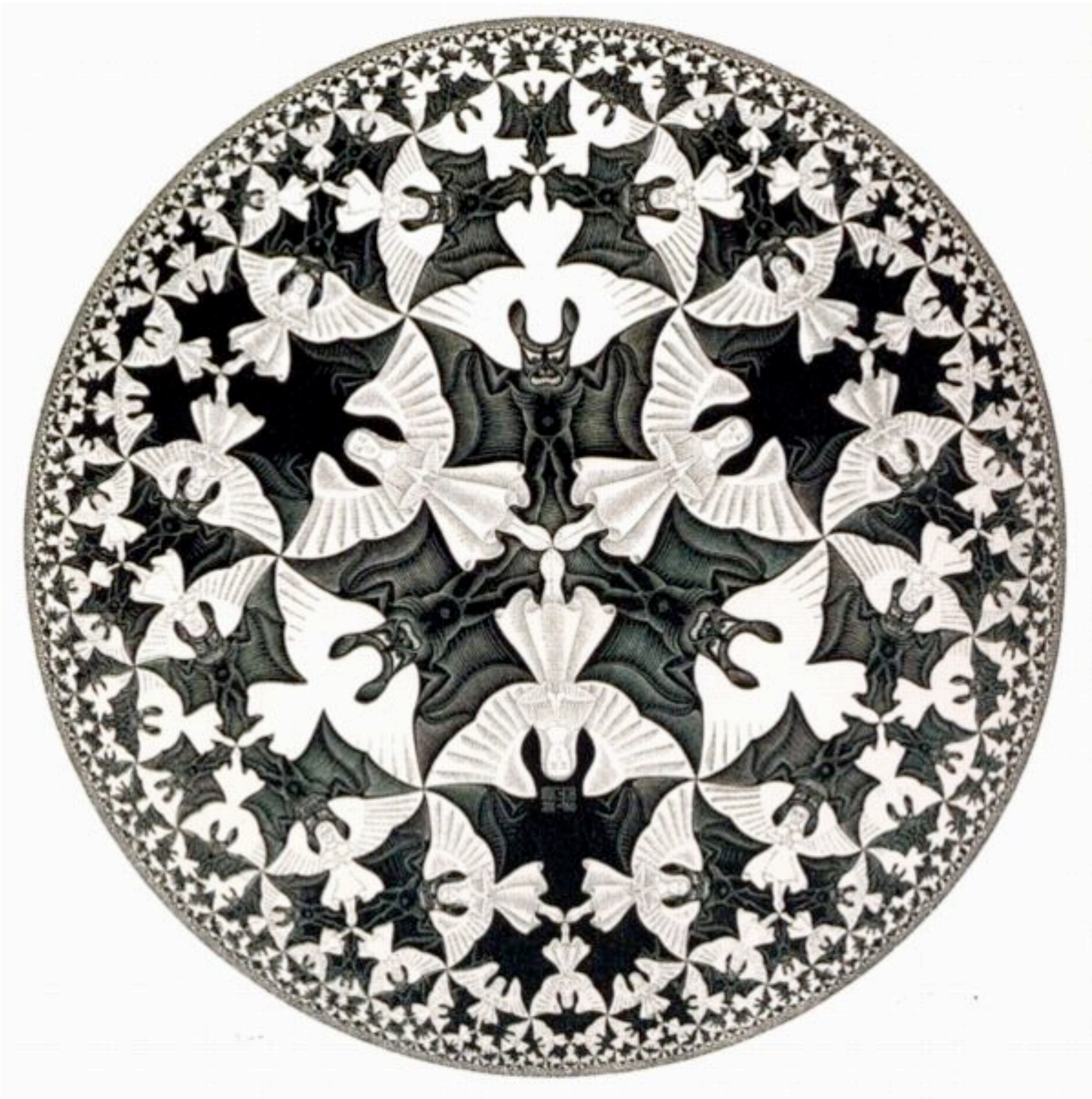
* The metric of 4d AdS space is given by:

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + r^2(dx^2 + dy^2) \ .$$

* This metric has a manifest scaling symmetry:

$$(t, x, y) \to \lambda(t, x, y), \ \ r \to \frac{r}{\lambda}$$

A pictorial representation of the scale invariance:



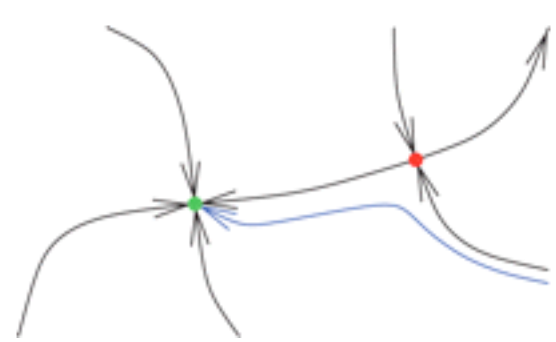
Escher

* In fact, deeper investigation will reveal that the metric enjoys the full symmetries of the so-called "conformal group" SO(3,2). (This can be made manifest by writing AdS space as a hyperboloid in the 5d flat space with signature (3,2)).

* An amusing connection: this is the same symmetry that characterises field theories at fixed points of the renormalisation group!

$$\frac{d\lambda_i}{dt} = \beta_i(\lambda)$$

$$\beta_i(\lambda = \lambda_*) = 0$$



These are called "conformal field theories."

This is not a coincidence.

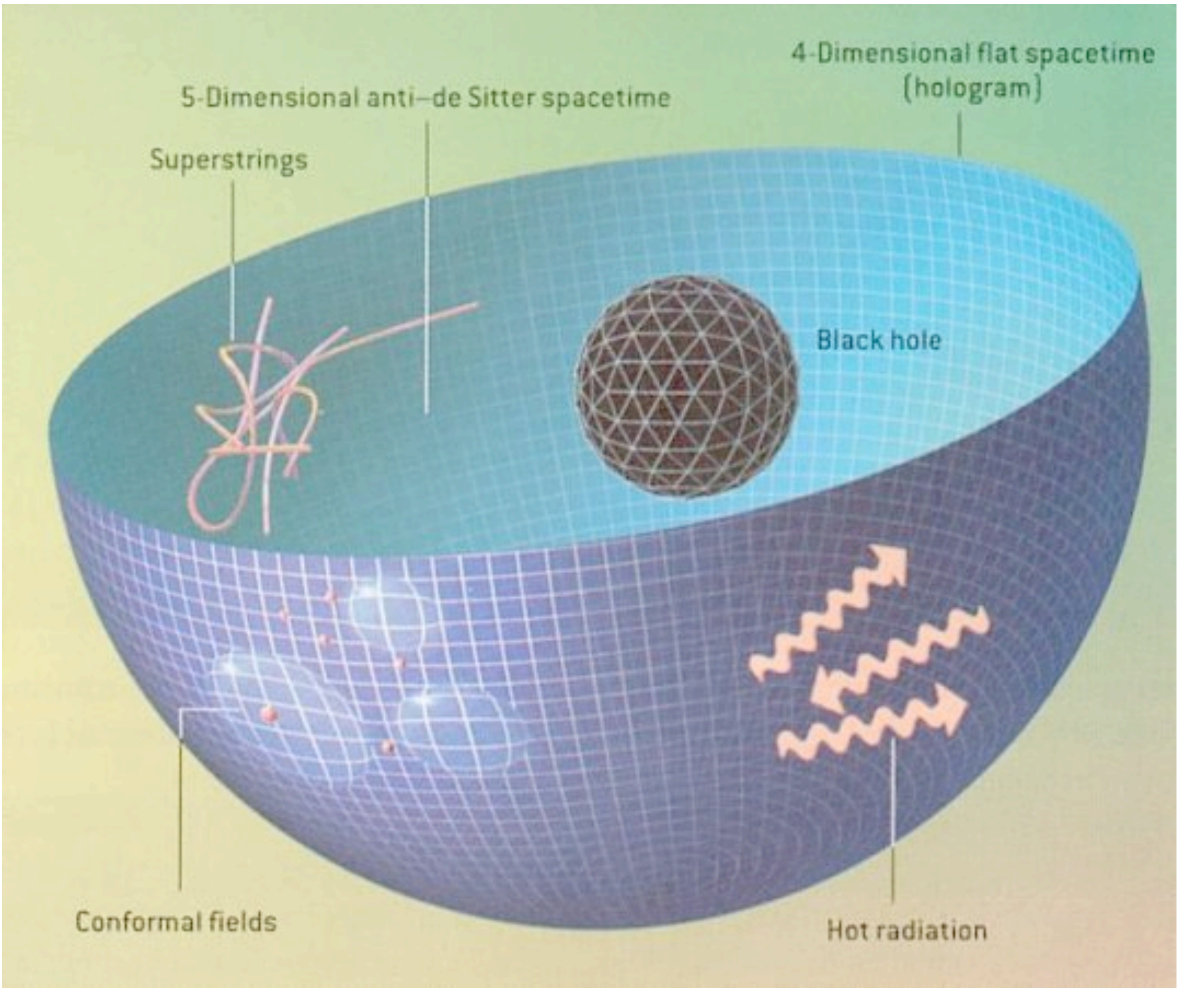
Maldacena

There are infinite classes of string constructions of AdS solutions, for which a precise "dual" conformal field theory is known.

* The energy scale in the dual field theory is geometrized in the AdS metric by the extra coordinate r; a d-dimensional field theory is dual to d+1 dimensional AdS gravity.

* This is a precise realization of holography! A higher-dimensional gravity theory is completely equivalent to a non-gravitational theory (with extensive entropy) one dimension down.

Globally, AdS looks like a can, where photons can reach the edge in finite time.



Relativists would say such a space is "not globally hyperbolic"; in addition to initial conditions, one must specify boundary data at the edge of the can. This is the data which specifies the field theory Lagrangian

One can obtain flat space as a subtle limit of AdS, and the flat-space S-matrix as a suitable limit of correlation functions in the dual quantum field theory.

This gives us a precise formulation of quantum gravity in (asymptotically) AdS spaces, and flat space as a limit.

Now, finally, let me get to problem A: how to formulate gravity in de Sitter space. We have just discussed our great success with AdS, and by a natural limit, Minkowski space. But we seemingly live in something more like dS. Do we get hints about how to make sense of this theory?

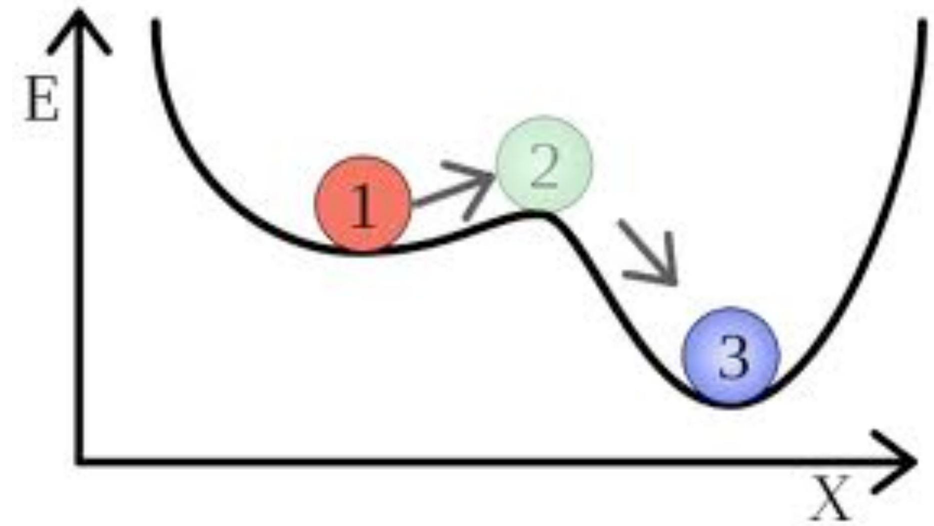
In holography, de Sitter space is still the odd man out! Research on dS holography is a fascinating and confusing subject of intense current interest:

* The space-time does not have a natural spatial infinity like the boundary of the can in AdS; there are only past and future infinities in time.

* Even these aren't robust; generic initial data in the far past causes a crunch before reaching future infinity!

* The causal structure is confusing. There are cosmological horizons which separate different observers; should we even be able to formulate a global theory of quantum gravity in dS space?

* In the context of a larger framework like string theory, it seems likely that dS solutions are only metastable at best.

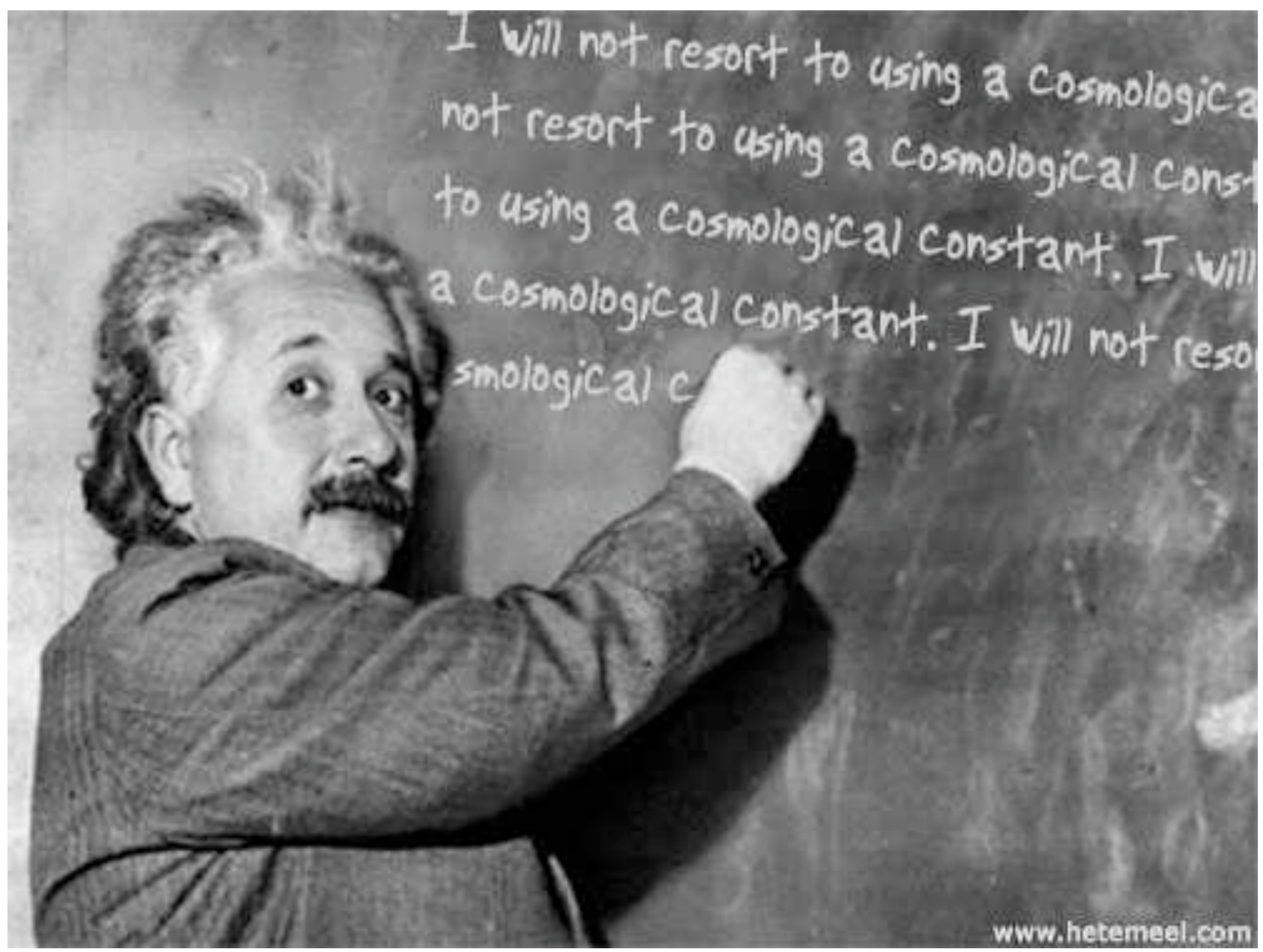


Many leading researchers today would agree that it is at least plausible, and maybe highly likely, that de Sitter quantum gravity is simply not a well-defined theory in and of itself. This is in stark contrast with our current view of AdS and Minkowski gravity.

So, it is very interesting, to say the least, that we find ourselves (approximately) inhabiting the de Sitter version of a maximally symmetric space-time!

B. Can we understand the dark energy?

I don't think modified gravity theories or quintessence add anything to the mix; I will assume we are trying to explain the value of a hard cosmological constant.



Is there any sense in which today's value, of roughly $10^{-120} M_{PL}^4$, is natural?

* As 't Hooft taught us, small dimensionless ratios of scales can be "natural" if a symmetry is restored when the ratio is precisely zero.

* The only known symmetry that can forbid a vacuum energy is the combination of supersymmetry and an unbroken global R-symmetry.

* This is completely unrealistic. (Next time someone tells you the only problem is that the vacuum energy isn't zero, while zero would have been natural, you should SMACK THEM).

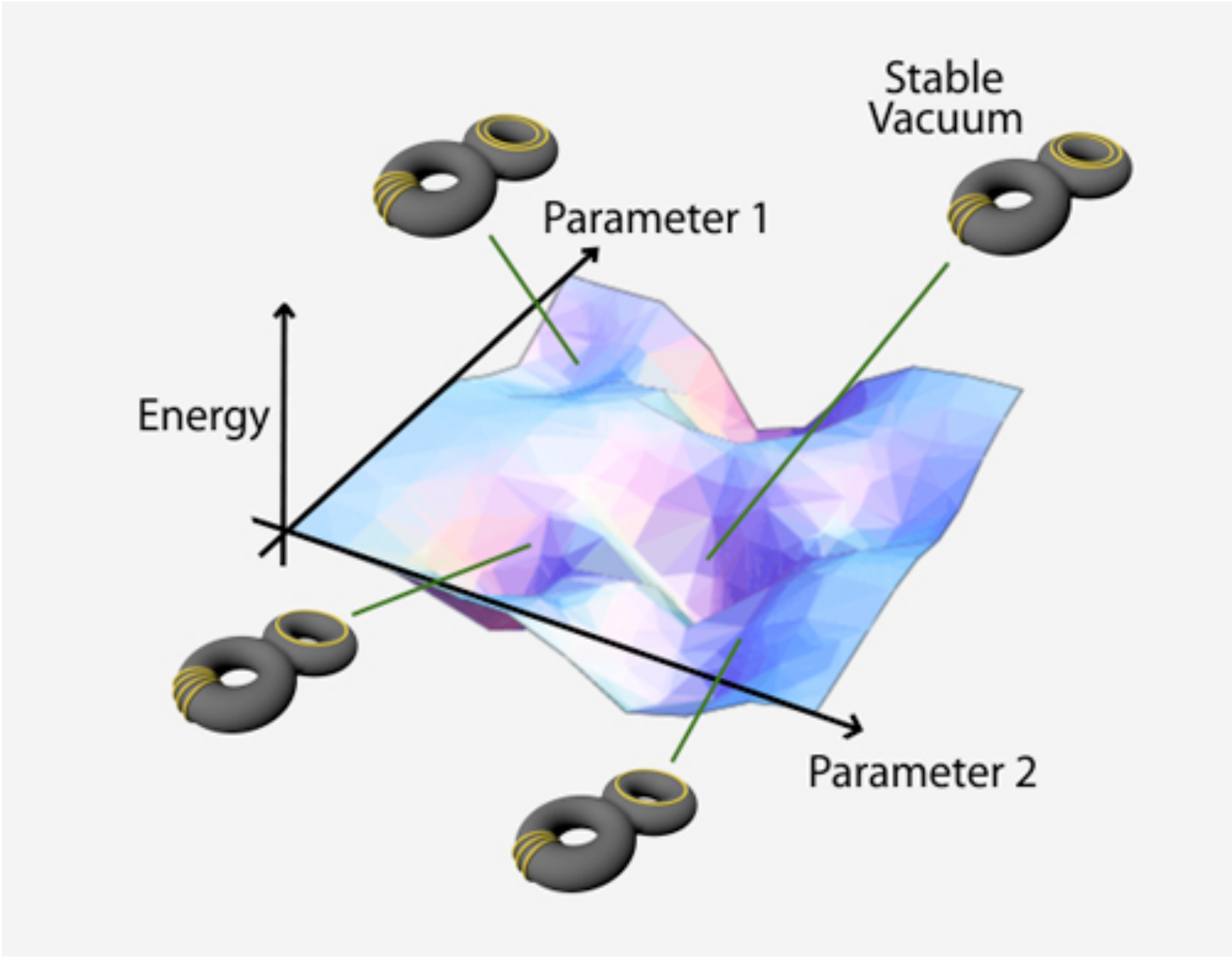
Instead, the existence of a variety of scales in Nature:

High Energy Theory	E_0
1. Weinberg-Salam	$M_W \sim 80$ GeV
2. grand unified theory	$M_{\rm GUT} \sim 10^{16}$ GeV
3. QCD	$M_\rho \sim .8$ GeV
4. lattice field theory	–
5. string theory	$M_{\rm string} \sim 10^{18}$ GeV

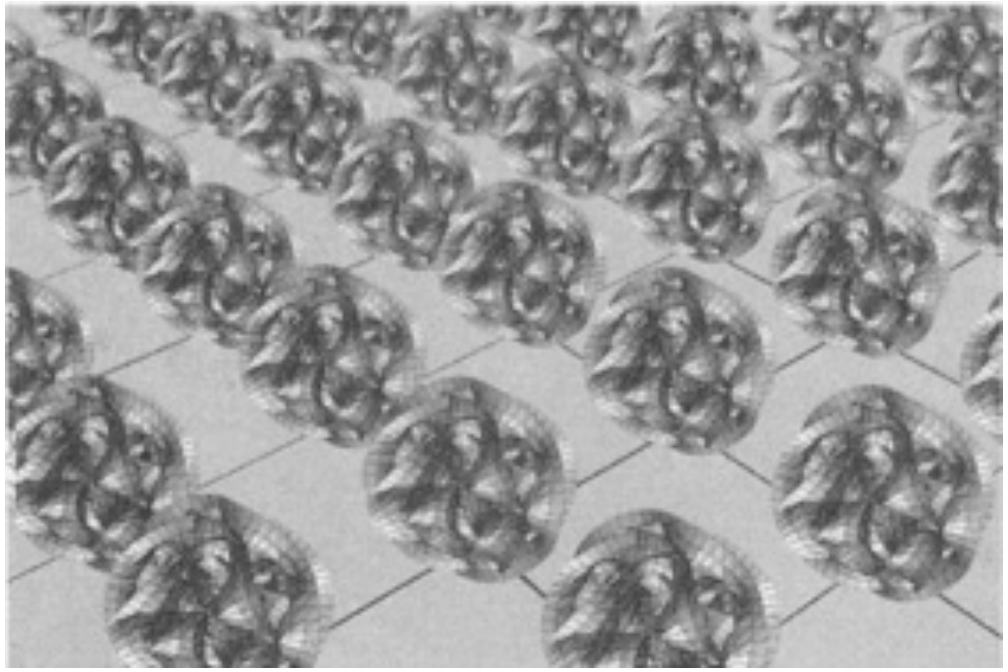
gives rich opportunities to generate contributions to the vacuum energy at all scales (both through radiative corrections, and through free energies generated in phase transitions).

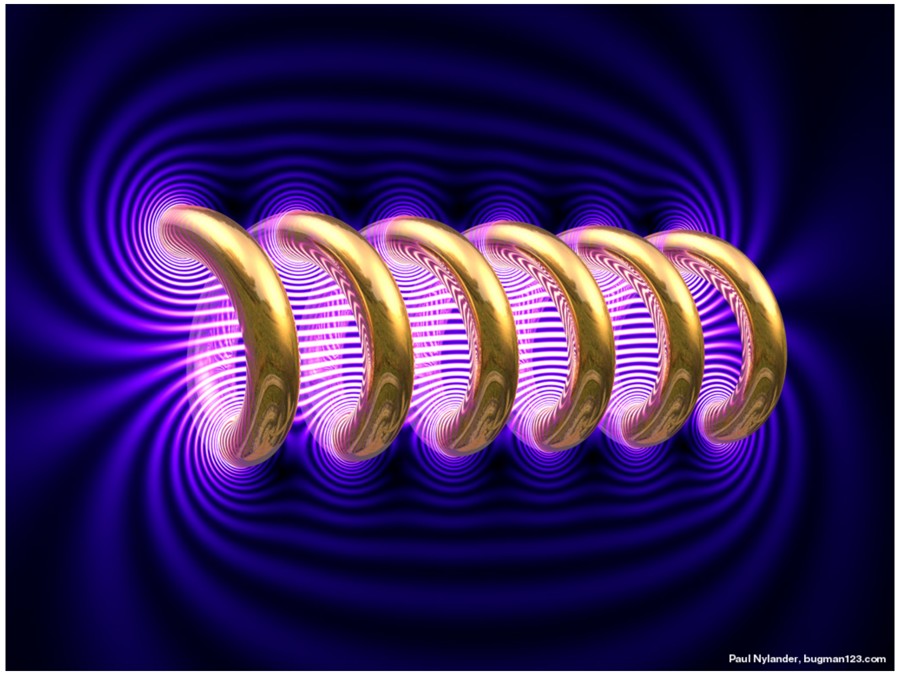
So, how are we to understand the presence of a small but non-zero vacuum energy?

An alternative which has been discussed very extensively, and on which I'll just spend a brief moment now, is to postulate that in fact the vacuum energy "scans" over many values in a large set of vacua. Such a set of vacua seems to be realised in string theory (where the parameters involve the size and shape of the extra dimensions this theory requires):



A rough understanding of where the large vacuum degeneracy comes from is easy to attain. Maximal symmetry of our observed four dimensions does not constrain what goes on in the "extra" 6 dimensions of string compactifications. And generically, they are threaded by analogues of magnetic flux (both due to consistency conditions, and to help stabilize them to some preferred shape and avoid unwanted massless shape moduli):





If we imagine there are K "cycles" in the manifold that can be threaded by flux, and each can be threaded by say anywhere from 1-10 units, we would end up with

$$N_{\text{choices}} \sim 10^K \qquad K_{\text{typical}} \sim 100s$$

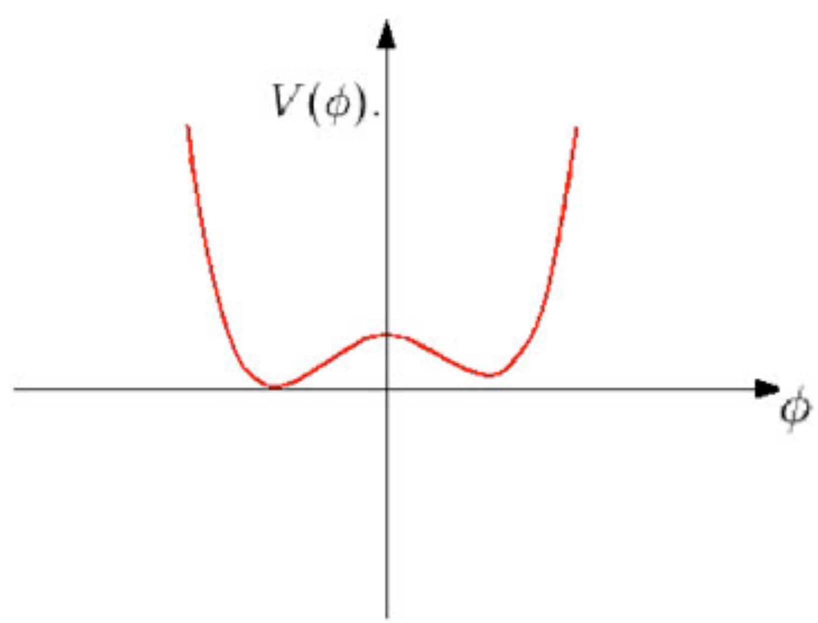
In examples we can analyze, there is a stable vacuum for a large fraction of these possible flux choices. This gives rise to a large vacuum degeneracy.

If we follow Weinberg's old idea that the distribution of cosmological constants should be flat around zero, and require that galaxy formation should be possible, we'd conclude that the observed value of the vacuum energy is typical. If this is the correct explanation, it furthers the Copernican revolution one more step.

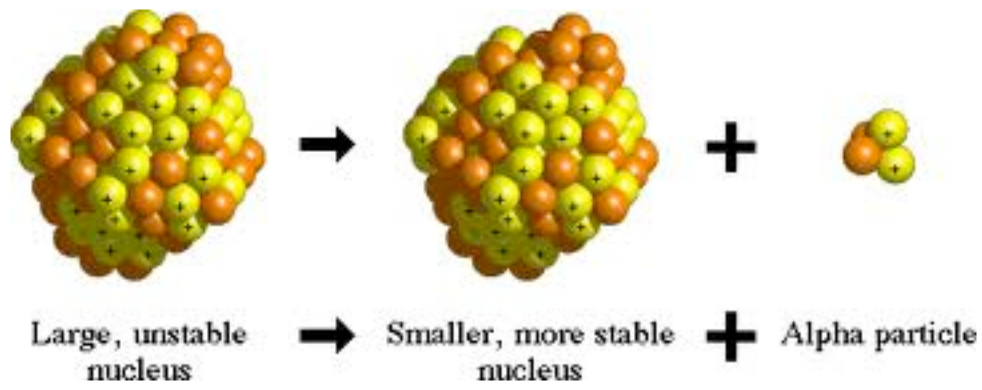
3. Where are we going?

I’ve now built up enough ingredients that I can explain some of the most “radically conservative” ideas about what our future holds.

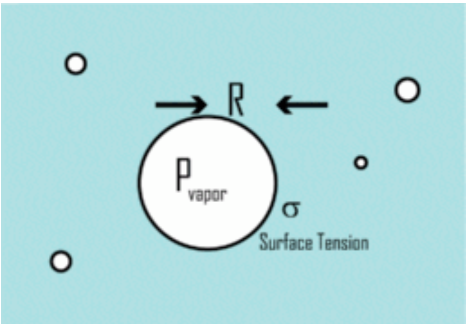
In any picture like that suggested by current incarnations of string theory, it seems likely that we inhabit one of many possible vacua. Lets imagine there were just two:



In quantum mechanics, e.g. of alpha decay, we know following Gamow that tunneling can occur through a potential barrier:

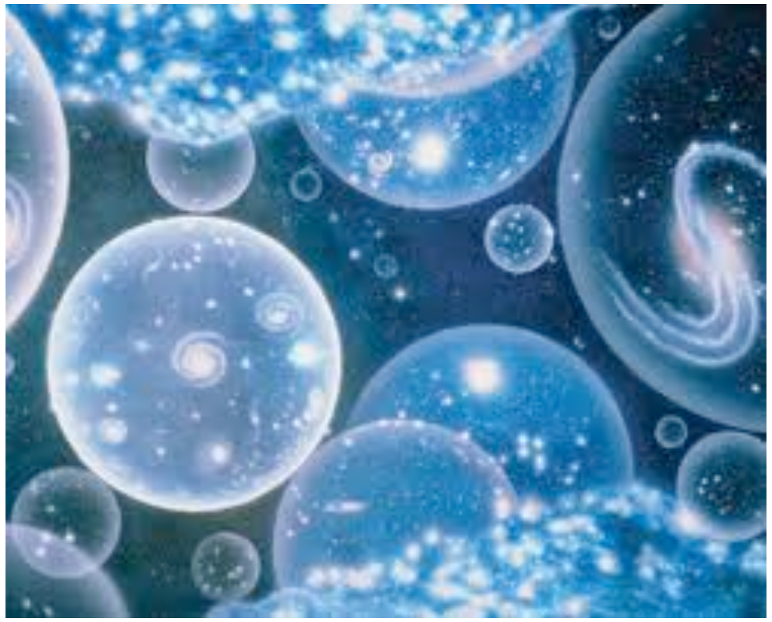


The same is true for vacua in quantum field theory. As demonstrated elegantly by Coleman, the fate of a false vacuum (after, usually, an exponentially long period of time) is to suffer from nucleation of bubbles of "truer" vacuum:



On the other hand, a false vacuum with positive vacuum energy is exponentially expanding.

So while bubbles of truer vacua can nucleate and expand inside of it, generically, the volume of space in false vacuum can continue to grow as well!



The result (the "Lindeverse") is a complicated, eternally inflating set of nested bubbles, each in principle containing distinct laws of low-energy physics.

Can one ever test this idea?

If one is very lucky, perhaps yes:

First Observational Tests of Eternal Inflation: Analysis Methods and WMAP 7-Year Results

Stephen M. Feeney,[1,*] Matthew C. Johnson,[2,3,†] Daniel J. Mortlock,[4,‡] and Hiranya V. Peiris[1,5,§]

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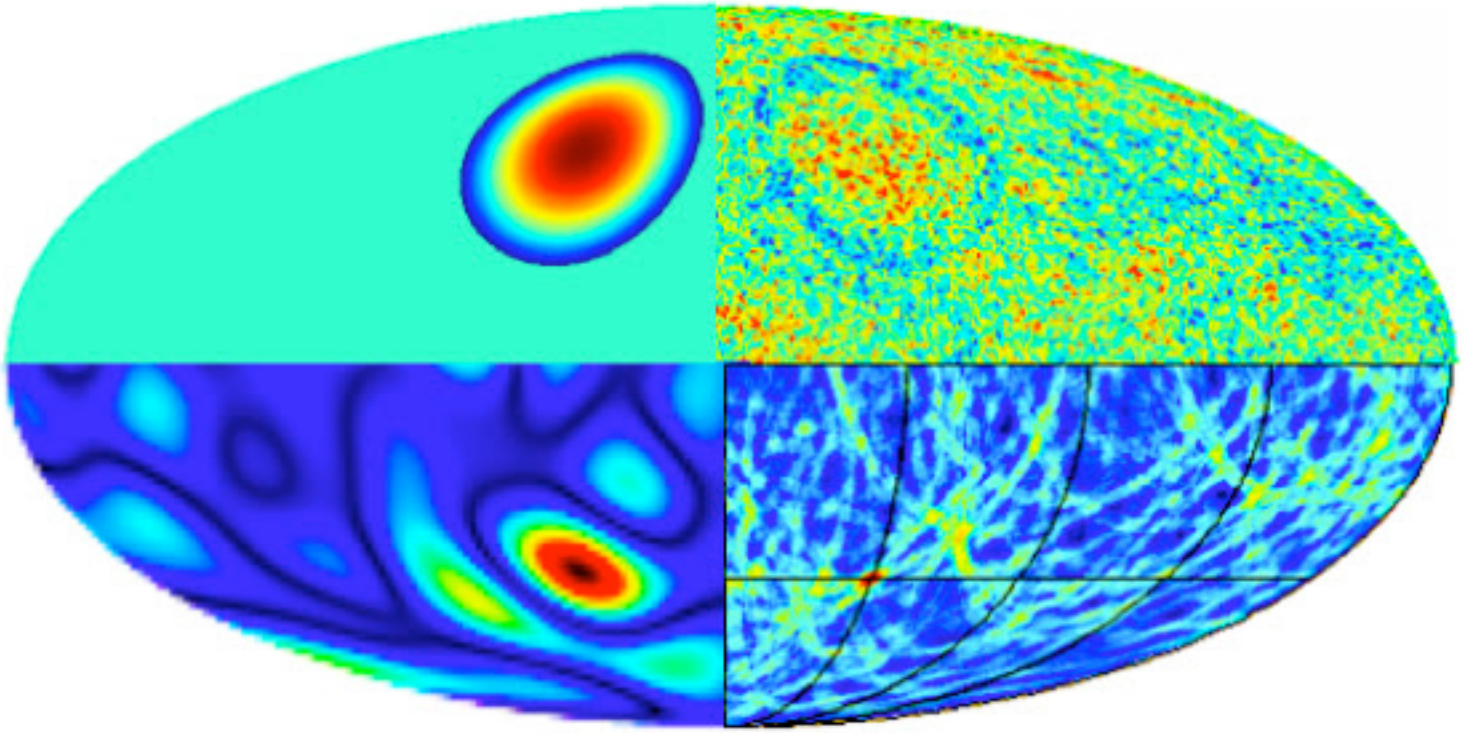
(Dated: July 13, 2011)

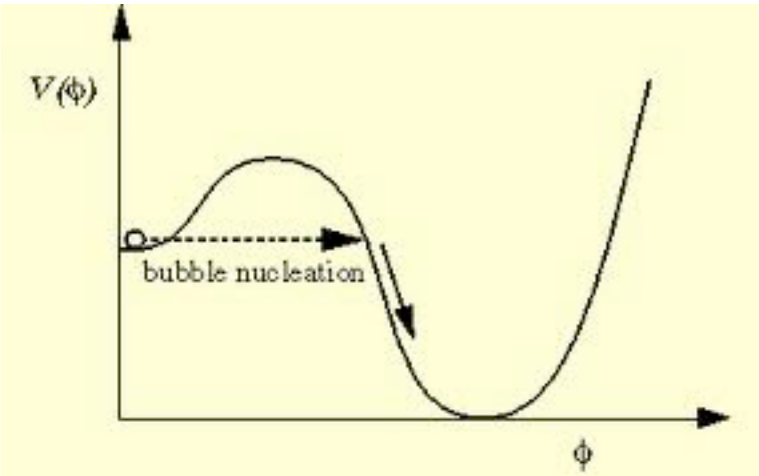
In the picture of eternal inflation, our observable universe resides inside a single bubble nucleated from an inflating false vacuum. Many of the theories giving rise to eternal inflation predict that we have causal access to collisions with other bubble universes, providing an opportunity to confront these theories with observation. We present the results from the first observational search for the effects of bubble collisions, using cosmic microwave background data from the WMAP satellite. Our search targets a generic set of properties associated with a bubble collision spacetime, which we describe in detail. We use a modular algorithm that is designed to avoid *a posteriori* selection effects, automatically picking out the most promising signals, performing a search for causal boundaries, and conducting a full Bayesian parameter estimation and model selection analysis. We outline each component of this algorithm, describing its response to simulated CMB skies with and without bubble collisions. Comparing the results for simulated bubble collisions to the results from an analysis of the WMAP 7-year data, we rule out bubble collisions over a range of parameter space. Our model selection results based on WMAP 7-year data do not warrant augmenting ΛCDM with bubble collisions. Data from the *Planck* satellite can be used to more definitively test the bubble collision hypothesis.



Imprints of collisions of "other bubbles" with our own, could conceivably be hidden in the CMB.

But even if these ideas are right, it seems a priori highly unlikely that such signatures would be there; slow-roll inflation hides them, and bubble nucleation times are typically so large as to lead to well-separated bubbles.

In any case, if such a picture is true, our long term fate is pretty clear:



We are going to decay (perhaps to a ten-dimensional non-compact situation, perhaps to something less striking).

Since these ideas may seem pretty outlandish (and who among you is confident that string theory is right anyhow?), I would like to emphasize one thing:

This picture of the large-scale structure of space-time does not depend on string theory. It depends on only three assumptions:

1. The correct microphysical theory has multiple vacua (true of all high-energy theories I've encountered).

2. Quantum tunneling between distinct vacua is allowed.

3. Positive vacuum energy (present in some of these vacua) causes exponentially quick expansion.

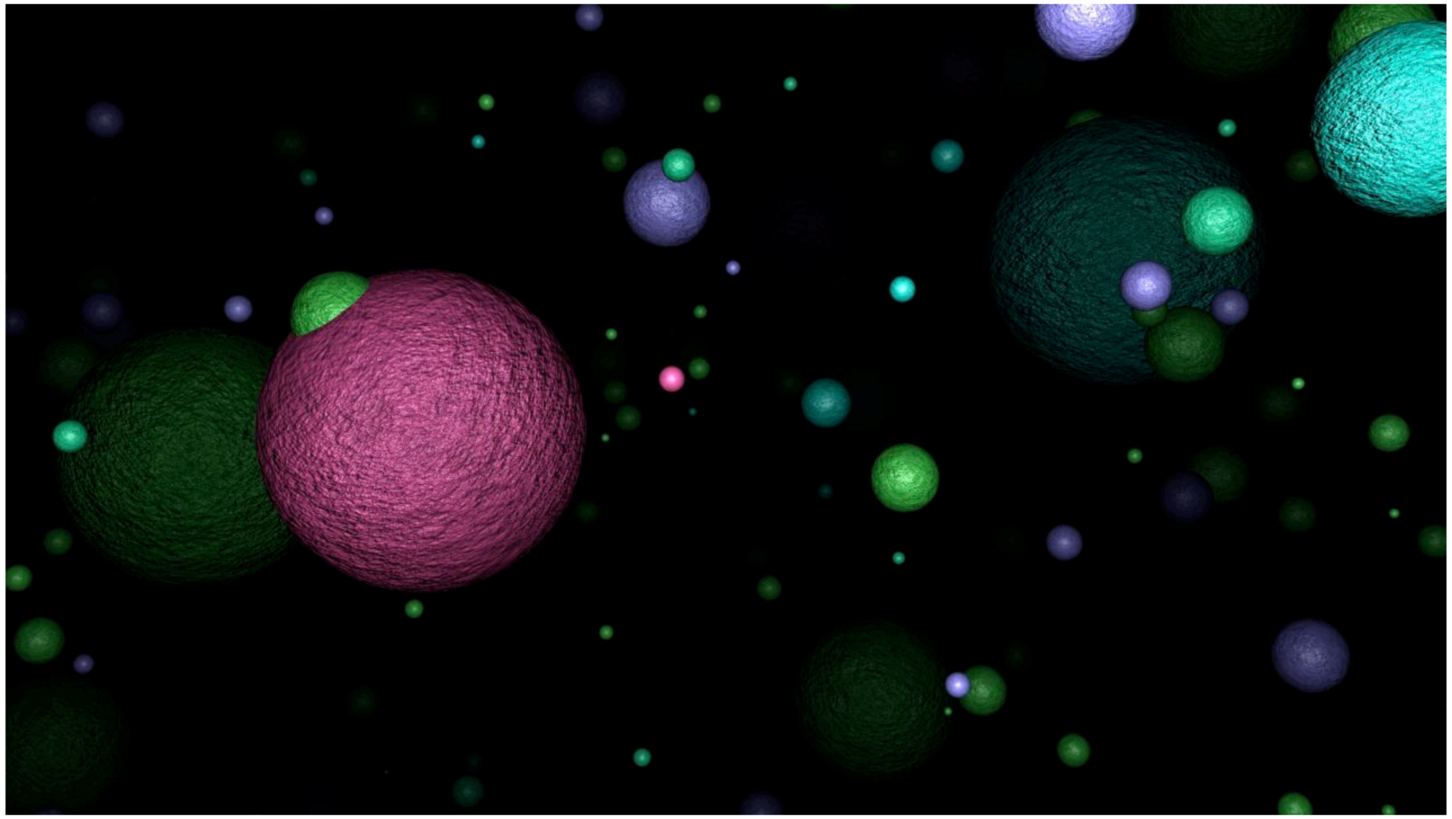
Therefore, I believe this picture is radically conservative in the sense of Wheeler. It takes simple ideas to their logical extreme; if it is wrong, one of these simple ideas must fall.

4. Origins

We all know that the "singularity theorems" of Penrose and Hawking, guarantee that the Universe began with a singularity.

I will disagree with this momentarily, but let me first discuss how it fits in with my previous claims.

A natural thought is that if the Universe is undergoing "eternal inflation," perhaps there is no singularity? Perhaps it has always been inflating, with bubbles nucleating here and there?



Nice thought, but it ain't so:

Inflationary spacetimes are not past-complete

Arvind Borde,[1,2] Alan H. Guth,[1,3] and Alexander Vilenkin[1]

Like all of the singularity theorems, their theorem shows under certain assumptions that the space-time is past geodesically incomplete.

This does NOT necessarily mean that there is a curvature singularity. But it does mean that there exist "observers" who could reach, in finite affine time, a region where space-time has an "edge." There, a priori unknown boundary conditions would be required to give a complete theory.

This is not so satisfactory. But I would like to point out now that the power of the singularity theorems has been greatly overstated, and there are many issues there still to investigate as we try to unravel our distant past.

Consider, for simplicity, the FLRW cosmologies:

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2(\theta) d\phi^2) \right) .$$

To prove the singularity theorems, one is required to assume an energy condition. That is, one is required to assume that:

$$T_{\mu\nu} v^\mu v^\nu \geq 0$$

for some class of vector fields v.

Now for Universes with k=-1 or 0, one is just required to assume the "Null energy condition" - that is, v above should be any future-pointing null vector field. This condition is in agreement with everything we know about macroscopic sources in our Universe.

In terms of equations of state for perfect fluids, for instance, this boils down to the condition:

$$p = w\rho$$

$$w \geq -1$$

No problem.

For k=+1, things are a bit more confusing. The singularity theorems require one to assume the so-called "strong-energy condition." In terms of w, this is basically requiring that w should satisfy $w \geq -\frac{1}{3}$.

We know, essentially for sure, that the strong energy condition is violated in our Universe, and by many reasonable toy physical models as well.

This means that as of today, we have no singularity theorem that gives strong evidence for a breakdown of general relativity in our distant past. In fact, one can investigate two inter-related questions:

i) Can one build singularity-free, eternal cosmologies?

ii) Can one build cosmologies that go through one or more cycles of "big bangs" and "crunches," where the scale factor varies between a large maximal and small minimal size (but stays within the regime of control of GR)?

For observational reasons I believe in any realistic context such solutions would need to be matched onto inflation; but I think these are interesting and foundational questions.

We've been exploring these issues with several collaborators at SLAC & Stanford (Graham, Horn, Rajendran, and Torroba).

We find that one can make very simple classical oscillating solutions of general relativity, which enjoy an infinite number of crunches and bangs, using just a cosmological constant and one matter source obeying all reasonable energy conditions.

* If the ratio of maximal to minimal scale factor is very large, we find that there are classical instabilities; one can (and must) tune to obtain a large number of cycles.

* For mild ratios of maximal to minimal size, there seem to be completely stable classical oscillating eternal cosmologies.

The simplest such solution arises for a matter source with w=-2/3, which is attained in certain topological defect model, for instance. The equations boil down to those governing a simple harmonic oscillator:

$$\ddot{a} + \frac{|\Lambda|}{3} a = \frac{4\pi}{3} G_N c$$

$$\Rightarrow \quad a(t) \quad = \quad \frac{1}{\sqrt{\gamma}\omega}\left(1 + \sqrt{1-\gamma}\cos(\omega t)\right)$$

$$\omega \quad \equiv \quad \sqrt{\frac{|\Lambda|}{3}}\,, \ \gamma \equiv \frac{3|\Lambda|}{(4\pi G_N c)^2}$$

The parameter gamma roughly controls the ratio between the minimal and maximal value of the scale factor.

The simplest classical metric perturbations with various momenta behave as follows:

* They obey an equation that can be recast as a Klein-Gordon equation for a scalar field:

$$\phi'' + 2\frac{a'}{a}\phi' + k(k+2)\phi \;=\; 0$$

- The homogeneous mode exhibits linear growth,

$$\phi(\eta) \sim \frac{\eta}{\gamma^2}$$

- For intermediate momenta $k < 1/\sqrt{\gamma}$,

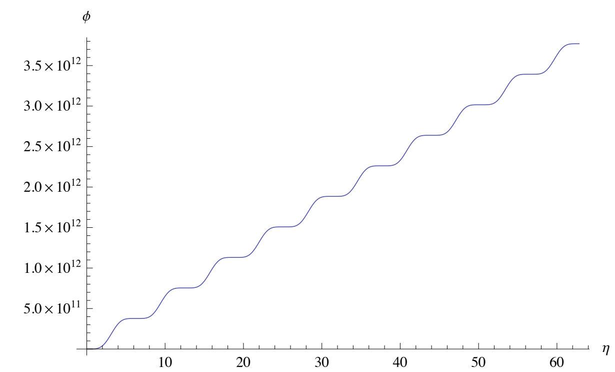
$$\phi(\eta) \sim \exp\left[\sqrt{1 - \frac{k^2}{k_c^2}}\,\eta\right] \quad , \quad k_c^2 \sim 1/\gamma$$

- At high momenta, modulated Minkowski modes,

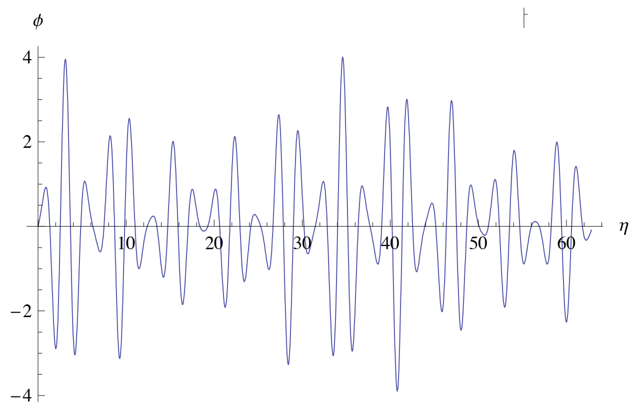
$$\phi(\eta) \sim (\sin\eta)\; e^{ik\eta}$$

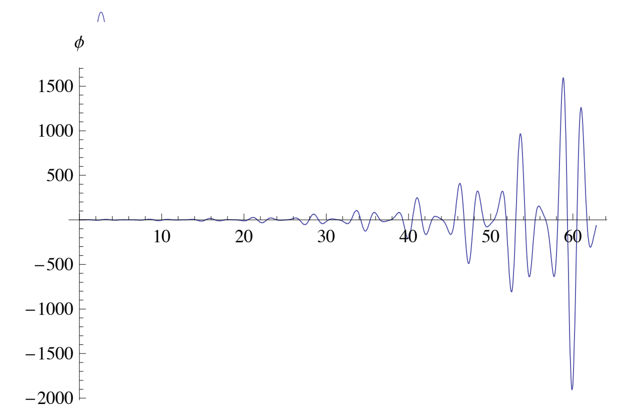
Instantiating plots:

Linear growth of homogeneous mode



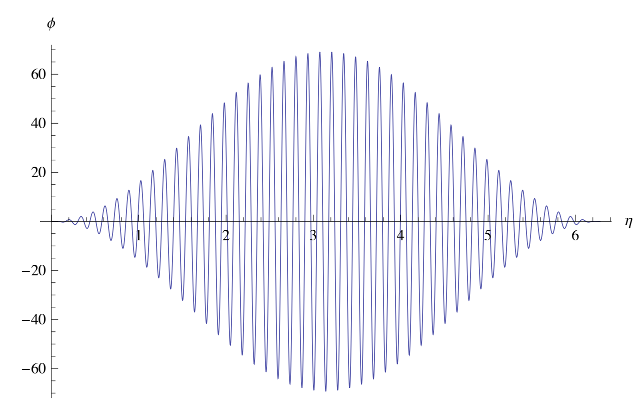
$\gamma \sim 1$





$\gamma \ll 1$

Oscillatory high-momentum modes



For mild values of gamma, no instabilities! (The homogeneous growing mode just represents the way two sinusoidal functions with slightly different frequencies grow apart in perturbation theory).

I believe that one might, however, be able to prove a quantum singularity theorem that eliminates many such possibilities. Intuitive argument:

* Any such cosmology contains, among other things, gravity.

* Time-dependence of the scale-factor will lead to graviton production.

*Once sufficiently many gravitons are present, the averaged stress-energy tensor will satisfy the strong energy condition, enabling one to prove a theorem from the Raychaudhuri equations.

In any case, it is clear from the fact that there is still active debate over how to past-complete eternal inflation; whether our Universe began at a finite time in the past or has been eternally present; and whether there may have even been earlier contracting phases in our Universe's evolution; that our cosmological origins remain shrouded in mist.

So I guess my summary of at least our knowledge of the Universe, would have to be: we don't yet know where we came from, we don't understand where we are, and we have only disturbing speculations about where we're going.

Thanks for your attention!