Electroweak Symmetry Breaking (EWSB): The Basics

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Lecture 1

SLAC NATIONAL ACCELERATOR LABORATORY

A brief bibliography

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2. H.E. Haber, "Lectures on Electroweak Symmetry Breaking," in *Testing the Standard Model*, Proceedings of the Theoretical Advanced Study Institute (TASI-90), edited by M. Cvetic and P. Langacker (World Scientific, Singapore, 1991), pp. 340–475.

3. M. Carena and H.E. Haber, "Higgs boson theory and phenomenology," Prog. Part. Nucl. Phys. **50**, 63 (2003) [arXiv:hep-ph/0208209].

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6. G.C. Branco, P.M. Ferreira, L. Lavoura, M.N. Rebelo, M. Sher and J.P. Silva, "Theory and phenomenology of two-Higgs-doublet models," Phys. Rept. **516**, 1 (2012) [arXiv:1106.0034 [hep-ph]].

Lecture I

Electroweak symmetry breaking before 4 July 2012

Outline

- The Standard Model before 4 July 2012—what was missing?
- mass generation and the Goldstone boson
- The significance of the TeV scale—Part 1
- Electroweak symmetry breaking dynamics of the Standard Model (SM)
- Constraining the Standard Model Higgs boson mass

Particle content of the Standard Model

FERMIONS matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

BOSONS force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Something is missing...

What was missing?

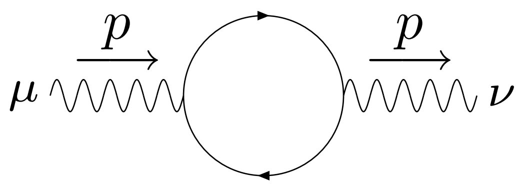
The theory of $W^\pm$ and Z gauge bosons must be *gauge invariant*; otherwise the theory is mathematically inconsistent. You may have heard that "gauge invariance implies that the gauge boson mass must be zero," since a mass term of the form $m^2 A^a_\mu A^{\mu a}$ is not gauge invariant.

So, what is the origin of the $W^\pm$ and Z boson masses? Gauge bosons are massless at tree-level, but perhaps a mass may be generated when quantum corrections are included. The tree-level gauge boson propagator $G^0_{\mu\nu}$ (in the Landau gauge) is:

$$G^0_{\mu\nu}(p) = \frac{-i}{p^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) .$$

The pole at $p^2 = 0$ indicates that the tree-level gauge boson mass is zero. Let's now include the radiative corrections.

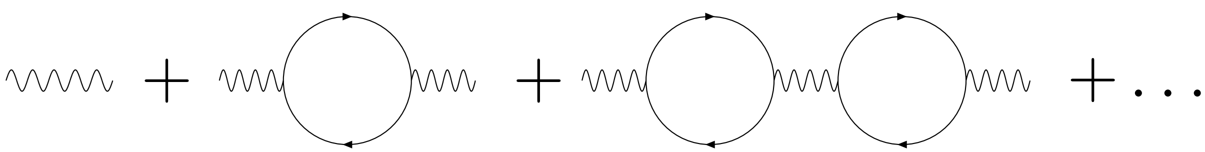
The polarization tensor $\Pi_{\mu\nu}(p)$ is defined as:



$$i\,\Pi_{\mu\nu}(p) \equiv i(p_\mu p_\nu - p^2 g_{\mu\nu})\Pi(p^2)$$

where the form of $\Pi_{\mu\nu}(p)$ is governed by covariance with respect to Lorentz transformations, and is constrained by gauge invariance, i.e. it satisfies $p^\mu \Pi_{\mu\nu}(p) = p^\nu \Pi_{\mu\nu}(p) = 0$.

The renormalized propagator is the sum of a geometric series



$$= \frac{-i(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})}{p^2[1+\Pi(p^2)]}$$

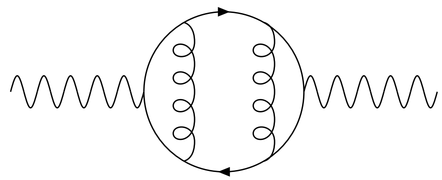
The pole at $p^2 = 0$ is shifted to a non-zero value if:

$$\Pi(p^2) \underset{p^2\to 0}{\simeq} \frac{-g^2 v^2}{p^2}\,.$$

Then $p^2[1+\Pi(p^2)] = p^2 - g^2 v^2$, yielding a gauge boson mass of gv.

Interpretation of the $p^2 = 0$ pole of $\Pi(p^2)$

The pole at $p^2 = 0$ corresponds to a propagating massless scalar. For example, the sum over intermediate states includes a quark-antiquark pair with many gluon exchanges, e.g.,



This is a strongly-interacting system—it is possible that one of the contributing intermediate states is a massless spin-0 state (due to the strong binding of the quark/antiquark pair).

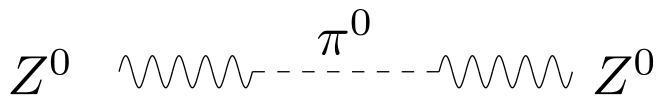
We know that the Z and $W^\pm$ couple to neutral and charged weak currents

$$\mathcal{L}_{\rm int} = -g_Z j^Z_\mu Z^\mu - g_W (j^W_\mu W^{+\mu} + {\rm h.c.})\,,$$

which are known to create neutral and charged pions from the vacuum, e.g.,

$$\langle 0 | j^Z_\mu(0) | \pi^0 \rangle = i f_\pi p_\mu\,.$$

Here, $f_\pi = 93$ MeV is the amplitude for creating a pion from the vacuum. In the absence of quark masses, the pions are massless bound states of $q\bar{q}$ [they are Goldstone bosons of chiral symmetry which is spontaneously broken by the strong interactions]. Thus, the diagram:



yields the leading contribution as $p^2 \to 0$ [shown in red] to the $p_\mu p_\nu$ of $\Pi_{\mu\nu}$,

$$i\Pi_{\mu\nu}(p) = i g_Z^2 f_\pi^2 \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) .$$

Remarkably, the latter is enough to fix the corresponding $g_{\mu\nu}$ part of $\Pi_{\mu\nu}$ [thank you, Lorentz invariance and gauge invariance!]. It immediately follows that

$$\Pi(p^2) = -\frac{g_Z^2 f_\pi^2}{p^2} ,$$

and therefore $m_Z = g_Z f_\pi$. Similarly $m_W = g_W f_\pi$.

Gauge boson mass generation and the Goldstone boson

We have demonstrated a mass generation mechanism for gauge bosons that is both Lorentz-invariant and gauge-invariant! This is the essence of the *Higgs mechanism*. The $p^2 = 0$ pole of $\Pi(p^2)$ corresponds to a propagating massless scalar state called the Goldstone boson. We showed that the W and Z are massive in the Standard Model (without Higgs bosons!!). Moreover, the ratio

$$\frac{m_W}{m_Z} = \frac{g_W}{g_Z} \equiv \cos\theta_W \simeq 0.88$$

is remarkably close to the measured ratio. Unfortunately, since $g_Z \simeq 0.37$ we find $m_Z = g_Z f_\pi = 35$ MeV, which is too small by a factor of 2600.

There must be another source for the gauge boson masses, i.e. new fundamental dynamics that generates the Goldstone bosons that are the main sources of mass for the $W^\pm$ and Z.

How do Goldstone bosons arise?

Suppose a Lagrangian exhibits a continuous global symmetry. If the vacuum state of the theory breaks the global symmetry, then the spectrum contains a massless scalar state—the Goldstone boson. This is a rigorous result of quantum field theory.

Goldstone's theorem can be exhibited in a model of elementary scalar dynamics. Suppose I have a multiplet of real scalar fields ϕ_i with Lagrangian

$$\mathscr{L} = \tfrac{1}{2}\partial_\mu\phi_i\partial^\mu\phi^i - V(\phi_i)\,,$$

which is invariant under $\phi_i \longrightarrow \phi_i + \delta\phi_i$, where

$$\delta\phi_i = -i\theta^a T^a_{ij}\phi_j\,.$$

The generators iT^a are real antisymmetric matrices and the θ^a are real parameters. By assumption, $\delta\mathscr{L} = 0$ which yields

$$\delta V = \frac{\partial V}{\partial\phi_i}\delta\phi_i = \frac{\partial V}{\partial\phi_i}T^a_{ij}\phi_j = 0\,.$$

The global symmetry is spontaneously broken if the vacuum state does not respect the symmetry. That is, the potential minimum occurs at $\phi_i = v_i$ where $\exp(-i\theta^a T^a)v \neq v$ [or equivalently, $T^a v \neq 0$]. Define new fields $\widetilde{\phi}_i \equiv \phi_i - v_i$, in which case

$$\mathscr{L} = \tfrac{1}{2}\partial_\mu \widetilde{\phi}_i \partial^\mu \widetilde{\phi}^i - \tfrac{1}{2} M^2_{ij} \widetilde{\phi}_i \widetilde{\phi}_j + \text{interactions}\,,$$

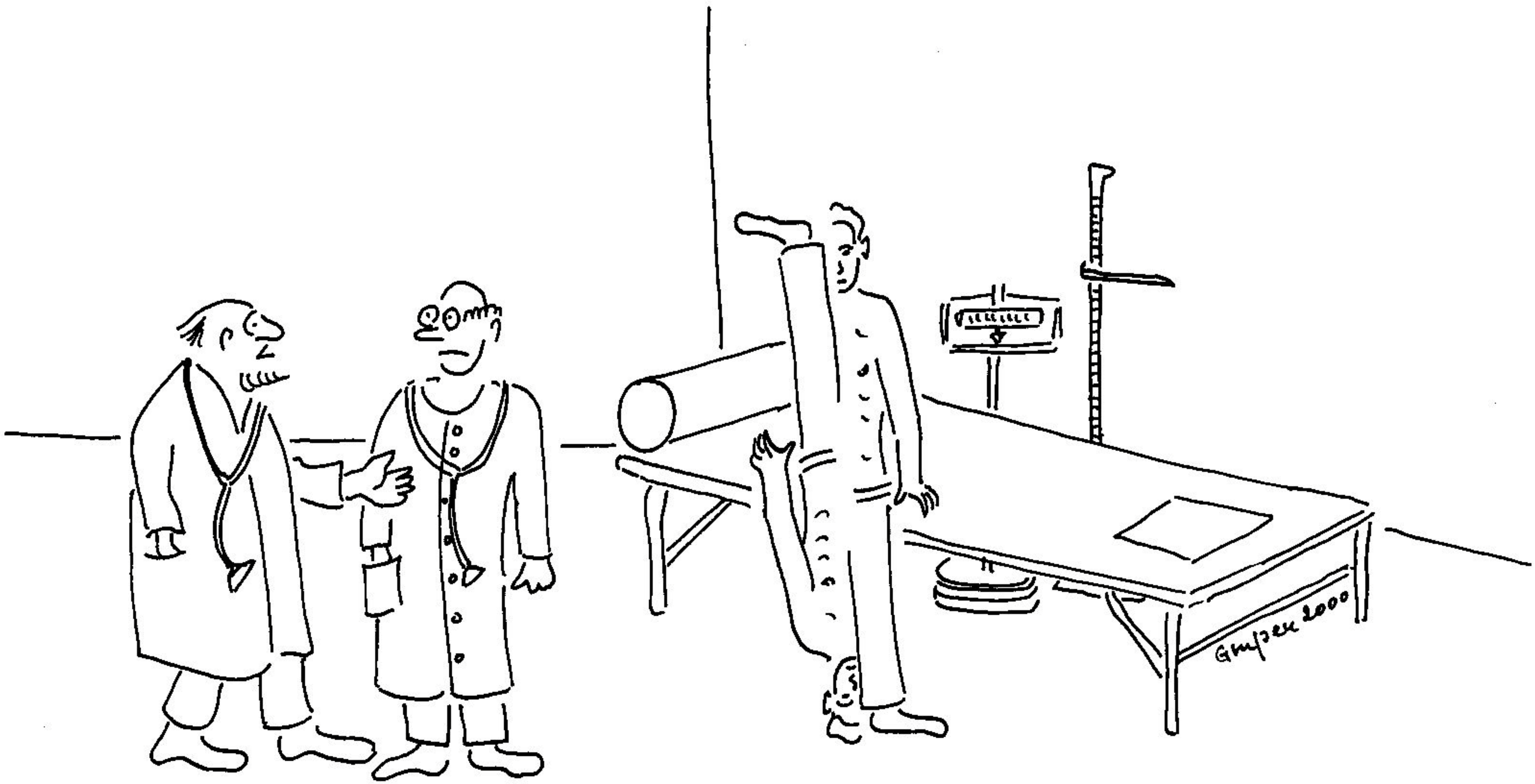
where M^2 is a non-negative symmetric matrix,

$$M^2_{ij} \equiv \left.\frac{\partial V}{\partial \phi_i \partial \phi_j}\right|_{\phi_i = v_i} .$$

Recall the condition for the global symmetry, $(\partial V/\partial\phi_i) T^a_{ij}\phi_j = 0$. Differentiating this equation with respect to ϕ_j and setting $\phi_i = v_i$ and $(\partial V/\partial \phi_i)_{\phi_i = v_i} = 0$ then yields

$$M^2_{ki} T^a_{ij} v_j = 0\,.$$

The T^a (which may be linear combinations of the original symmetry generators) are re-organized to identify the maximal number of unbroken linearly independent generators (i.e. $T^a v = 0$), which determine the residual unbroken symmetry. As for the remaining broken generators (i.e. $T^a v \neq 0$), we see that $(T^a v)_i$ is an eigenvector of M^2 with zero eigenvalue. In particular, there is one Goldstone boson, $G^a \sim i\phi_i T^a_{ij} v_j$ for each broken generator.



"A severe case of symmetry breaking!"

The Higgs mechanism can be exhibited in our simple model of elementary scalar dynamics by promoting the global symmetry to a local symmetry. This is accomplished by introducing a gauge field A^a_μ corresponding to each symmetry generator T^a. The Lagrangian is now

$$\mathscr{L} = \mathscr{L}_{\rm YM} + \tfrac{1}{2}(D_\mu\phi)^T(D^\mu\phi) - V(\phi)\,,$$

where $\mathscr{L}_{\rm YM}$ is the Yang-Mills Lagrangian and D is the covariant derivative

$$D_\mu \equiv \partial_\mu + igT^aA^a_\mu\,.$$

Assuming that the scalar potential is minimized at $\phi_i = v_i$ as before, we again define shifted fields, $\widetilde{\phi}_i \equiv \phi_i - v_i$. Then,

$$(D_\mu\phi)^T(D^\mu\phi) = M^2_{ab}A^a_\mu A^{\mu b} + \cdots\ ,$$

with $M^2_{ab} = g^2 v^T T^a T^b v$. For each unbroken generator, the corresponding vector boson remains massless (due to the residual unbroken symmetry). The remaining vector bosons acquire mass. One can show that the corresponding Goldstone bosons are no longer physical states of the theory. Instead, they are "absorbed" by the corresponding gauge bosons and are realized as the longitudinal spin component of the massive gauge bosons.

Massless and heavy
spin 1 particles

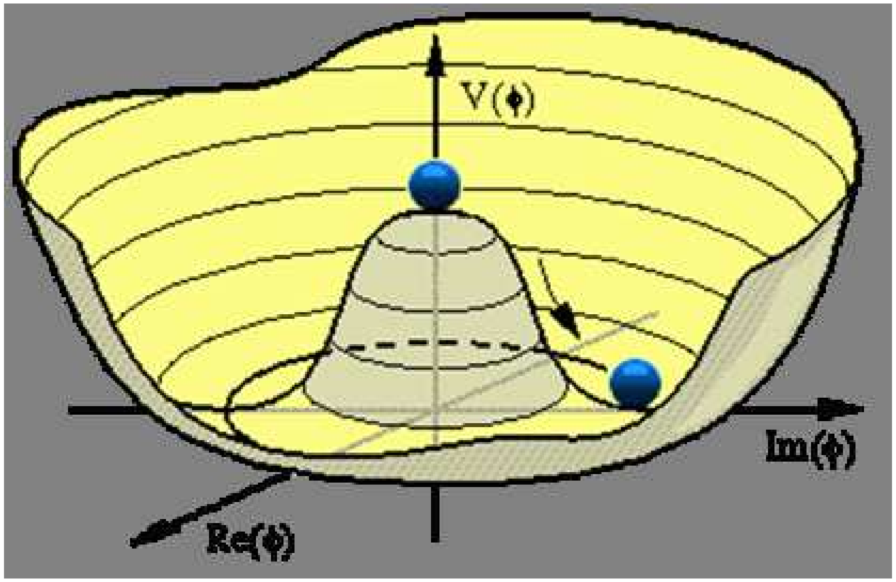
Heavy spin 1 particles can spin in 3 directions:

Massless particles must have their spin-axis either parallel or anti-parallel to their direction of motion:

They can only spin in 2 directions.

Possible choices for electroweak-symmetry-breaking (EWSB) dynamics

- weakly-interacting self-coupled elementary (Higgs) scalar dynamics



- strong-interaction dynamics involving new fermions and gauge fields [technicolor, dynamical EWSB, little Higgs models, composite Higgs bosons, Higgsless models, extra-dimensional EWSB, . . .]

Both mechanisms generate new phenomena with significant experimental consequences.

Fate of the pion

Let us designate by ω^a the triplet of Goldstone bosons that are generated by the additional electroweak symmetry-breaking dynamics. For example, if ω^a is a consequence of elementary scalar dynamics, then the total (axial vector) current that creates π^a and ω^a from the vacuum is given by $j^a_\mu = j^a_{\mu,\rm QCD} + v\partial_\mu\omega_a$, where $v = 246$ GeV and

$$\langle 0|j^a_\mu(0)|\pi^b\rangle = if_\pi p_\mu \delta^{ab}\,, \qquad \langle 0|j^a_\mu(0)|\omega^b\rangle = ivp_\mu\delta^{ab}\,.$$

In this case, the "true" Goldstone bosons of electroweak symmetry breaking are:

$$|G^a\rangle = \frac{1}{\sqrt{f_\pi^2+v^2}}\left[f_\pi|\pi^a\rangle + v|\omega^a\rangle\right]\,,$$

which are absorbed by the $W^\pm$ and Z as a result of the Higgs mechanism, and the physical pions are the states orthogonal to the $|G^a\rangle$,

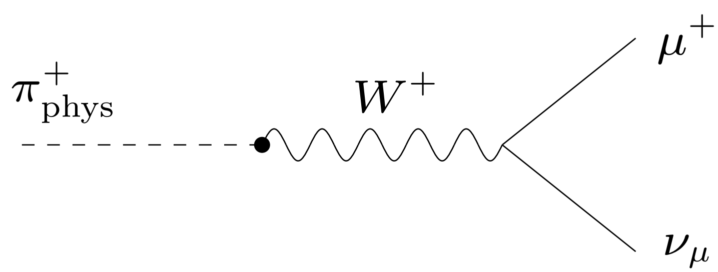
$$|\pi^a\rangle_{\rm phys} = \frac{1}{\sqrt{f_\pi^2+v^2}}\left[v|\pi^a\rangle - f_\pi|\omega^a\rangle\right]\,.$$

One can check that

$$\langle 0|j^a_\mu|G^b\rangle = i(f_\pi^2 + v^2)^{1/2} p_\mu \delta^{ab} \,,$$

$$\langle 0|j^a_\mu|\pi^b\rangle_{\rm phys} = 0 \,.$$

So far so good. But, if you look at old textbooks on the weak interactions, they will insist that the (physical) charged pion decays via



But, the π–W vertex above is proportional to $\langle 0|j^-_\mu|\pi^+\rangle_{\rm phys} = 0$. So how does the charged pion decay?

I learned about this paradox from Marvin Weinstein many years ago. The answer will be given at the beginning of Lecture 2.

Significance of the TeV Scale—Part 1

Let $\Lambda_{\rm EW}$ be energy scale of EWSB dynamics. For example:

- Elementary Higgs scalar ($\Lambda_{\rm EW} = m_h$).
- Strong EWSB dynamics ($e.g.$, $\Lambda_{\rm EW}^{-1}$ is the characteristic scale of bound states arising from new strong dynamics).

Consider $W_L^+ W_L^- \to W_L^+ W_L^-$ ($L =$ longitudinal or equivalently, zero helicity) for $m_W^2 \ll s \ll \Lambda_{\rm EW}^2$. The corresponding amplitude, to leading order in g^2, but to all orders in the couplings that control the EWSB dynamics, is equal to the amplitude for $G^+ G^- \to G^+ G^-$ (where $G^\pm$ are the charged Goldstone bosons). The latter is universal, independent of the EWSB dynamics. This is a rigorous low-energy theorem.

Applying unitarity constraints to this amplitude yields a critical energy $\sqrt{s_c}$, above which unitarity is violated. This unitarity violation must be repaired by EWSB dynamics and implies that $\Lambda_{\rm EW} \lesssim \mathcal{O}\left(\sqrt{s_c}\,\right)$.

Unitarity of scattering amplitudes

Unitarity is equivalent to the conservation of probability in quantum mechanics. A violation of unitarity is tantamount to a violation of the principles of quantum mechanics—this is too sacred a principle to give up!

Consider the helicity amplitude $\mathcal{M}(\lambda_3\lambda_4\,;\,\lambda_1\lambda_2)$ for a $2\longrightarrow 2$ scattering process with initial [final] helicities λ_1, λ_2 [λ_3, λ_4]. The Jacob-Wick partial wave expansion is:

$$\mathcal{M}(\lambda_3\lambda_4\,;\,\lambda_1\lambda_2)=\frac{8\pi\sqrt{s}}{(p_ip_f)^{1/2}}e^{i(\lambda_i-\lambda_f)\phi}\sum_{J=J_0}^{\infty}(2J+1)\mathcal{M}^J_\lambda(s)d^J_{\lambda_i\lambda_f}(\theta)\,,$$

where p_i [p_f] is the incoming [outgoing] center-of-mass momentum, $\sqrt{s}$ is the center-of-mass energy, $\lambda\equiv\{\lambda_3\lambda_4\,;\,\lambda_1\lambda_2\}$ and

$$J_0\equiv\max\{\lambda_i\,,\,\lambda_f\}\,,\quad\text{where}\quad\lambda_i\equiv\lambda_1-\lambda_2\,,\quad\text{and}\quad\lambda_f\equiv\lambda_3-\lambda_4\,.$$

Orthogonality of the d-functions allows one to project out a given partial wave amplitude. For example, for $W_L^+W_L^-\longrightarrow W_L^+W_L^-$ (L stands for *longitudinal* and corresponds to $\lambda=0$),

$$\mathcal{M}^{J=0}=\frac{1}{16\pi s}\int_{-s}^{0}dt\,\mathcal{M}(L,L\,;\,L,L)\,,$$

where $t=-\frac{1}{2}s(1-\cos\theta)$ in the limit where $m_W^2\ll s$.

The $J=0$ partial wave for $W_L^+W_L^- \to W_L^+W_L^-$ in the limit of $m_W^2 \ll s \ll \Lambda_{\rm EW}^2$ is equal to the corresponding amplitude for $G^+G^- \to G^+G^-$:

$$\mathcal{M}^{J=0} = \frac{G_F s}{16\pi\sqrt{2}}\,.$$

Partial wave unitarity implies that:

$$|\mathcal{M}^J|^2 \leq |{\rm Im}\;\mathcal{M}^J| \leq 1\,,$$

which gives

$$({\rm Re}\;\mathcal{M}^J)^2 \leq |{\rm Im}\;\mathcal{M}^J|\left(1-|{\rm Im}\;\mathcal{M}^J|\right) \leq \tfrac{1}{4}\,.$$

Setting $|{\rm Re}\;\mathcal{M}^{J=0}| \leq \frac{1}{2}$ yields $\sqrt{s_c}$. The most restrictive bound arises from the isospin zero channel $\sqrt{\frac{1}{6}}(2W_L^+W_L^- + Z_LZ_L)$:

$$s_c = \frac{4\pi\sqrt{2}}{G_F} = (1.2\ {\rm TeV})^2\,.$$

Since unitarity cannot be violated, we conclude that $\Lambda_{\rm EW} \lesssim \sqrt{s_c}$. That is,

The dynamics of electroweak symmetry breaking must be exposed at or below the 1 TeV energy scale.

EWSB Dynamics of the Standard Model (SM)

- Add a new sector of "matter" consisting of a complex SU(2) doublet, hypercharge-one self-interacting scalar fields, $\Phi \equiv (\Phi^+ \;\; \Phi^0)$ with four real degrees of freedom. The scalar potential is:

$$V(\Phi) = \tfrac{1}{2}\lambda(\Phi^\dagger\Phi - \tfrac{1}{2}v^2)^2\,,$$

  so that in the ground state, the neutral scalar field takes on a constant non-zero value $\langle\Phi^0\rangle = v/\sqrt{2}$, where $v = 246$ GeV. It is convenient to write:

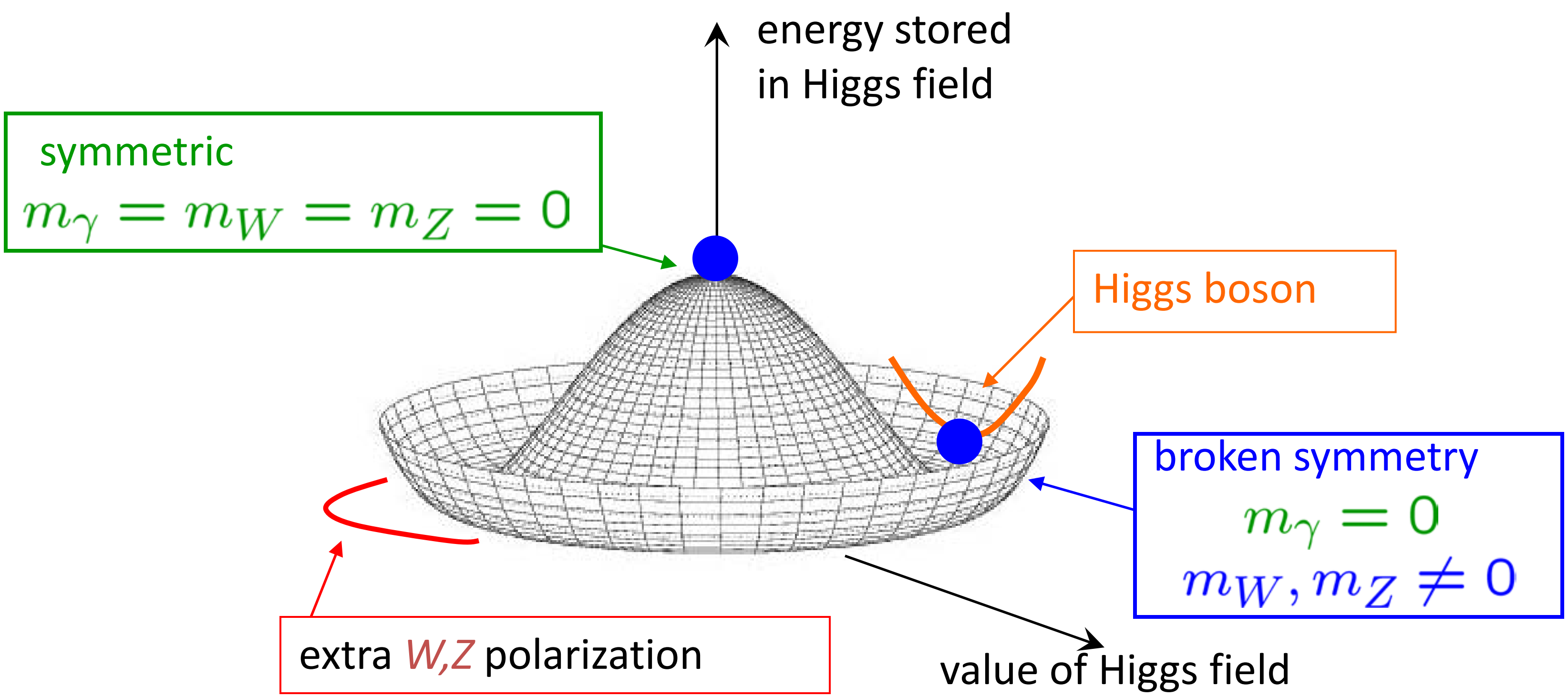
$$\Phi = \begin{pmatrix} \omega^+ \\ \frac{1}{\sqrt{2}}\left(v + h^0 + i\omega^3\right) \end{pmatrix}\,,$$

  where $\omega^\pm \equiv (\omega^1 \mp i\omega^2)/\sqrt{2}$.

- The non-zero scalar vacuum expectation value breaks the electroweak symmetry, thereby generating three Goldstone bosons, ω^a $(a = 1, 2, 3)$.

Breaking the Electroweak Symmetry

Higgs imagined a field filling all of space, with a "weak charge".
Energy forces it to be **nonzero** at bottom of the "Mexican hat".



- The couplings of the gauge bosons to the SU(2)$_L\times$U(1)$_Y$ currents are

$$\mathscr{L}_{\rm int} = \tfrac{1}{2} g W^{\mu a} T^a_{\mu L} + \tfrac{1}{2} g' B^\mu Y_\mu\,.$$

  Decomposing $T_L = \frac{1}{2}(j_V - j_A)$ into vector and axial vector currents and noting that the electric current, $j_Q = T^3 + \frac{1}{2}Y$ is purely vector,

$$\mathscr{L}_{\rm int} = -\tfrac{1}{2} g W^{\mu a} j^a_{A\mu} + \tfrac{1}{2} g' B^\mu j^3_{A\mu} + \text{vector current couplings}\,.$$

  As previously noted, $\langle 0|j^a_{A\mu}|\omega^b\rangle = i v p_\mu \delta^{ab}$. The δ^{ab} factor is a consequence of the global *custodial* SU(2)$_L\times$SU(2)$_R$ symmetry of the scalar potential. Computing the vector boson masses as before yields a 4×4 squared-mass matrix,

$$\frac{v^2}{4}\begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^{\,2} \end{pmatrix}.$$

Diagonalizing this matrix, we identify $Z^\mu = (gW^{\mu 3} - g' B^\mu)/(g^2 + g'^{\,2})^{1/2}$. That is, $g_W = \frac{1}{2}g$ and $g_Z = \frac{1}{2}(g^2 + g'^{\,2})^{1/2}$, which yields

$$m_W^2 = \tfrac{1}{4}g^2 v^2\,, \qquad\qquad m_Z^2 = \tfrac{1}{4}(g^2 + g'^{\,2})v^2 = \frac{m_W^2}{\cos^2\theta_W}\,,$$

and it follows that (at tree-level), the rho-parameter is

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1\,.$$

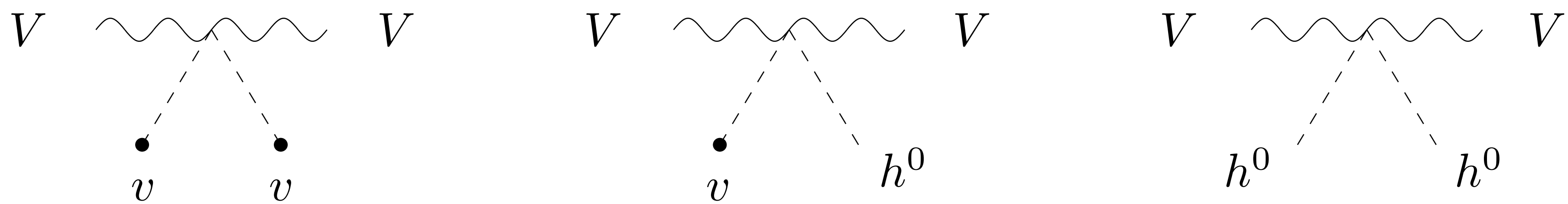
- One scalar degree of freedom is left over—the Higgs boson, with self-interactions

$$V(h) = \tfrac{1}{2}\lambda\left[\left(\frac{h+v}{\sqrt{2}}\right)^2 - \frac{v^2}{2}\right]^2 = \tfrac{1}{8}\lambda\left[h^4 + 4h^3 v + 4h^2 v^2\right]\,.$$

It is a neutral CP-even scalar boson, whose interactions are precisely predicted, but whose mass $m_h^2 = \lambda v^2$ depends on the unknown strength of the scalar self-coupling—the only unknown parameter of the model.

Mass generation and Higgs couplings in the SM

Gauge bosons ($V = W^\pm$ or Z) acquire mass via interaction with the Higgs vacuum condensate.



Thus,

$$g_{hVV} = 2m_V^2/v \,, \qquad \text{and} \qquad g_{hhVV} = 2m_V^2/v^2 \,,$$

i.e., the Higgs couplings to vector bosons are proportional to the corresponding boson squared-mass.

Likewise, by replacing V with the Higgs field h^0 in the above diagrams, the Higgs self-couplings are also proportional to the square of the Higgs mass:

$$g_{hhh} = 3\lambda v = \frac{3m_h^2}{v} \,, \qquad \text{and} \qquad g_{hhhh} = 3\lambda = \frac{3m_h^2}{v^2} \,.$$

Fermions in the Standard Model

Given a four-component fermion f, we can project out the right and left-handed parts:

$$f_R \equiv P_R f\,, \qquad f_L \equiv P_L f\,, \qquad \text{where} \quad P_{R,L} = \tfrac{1}{2}(1 \pm \gamma_5)\,.$$

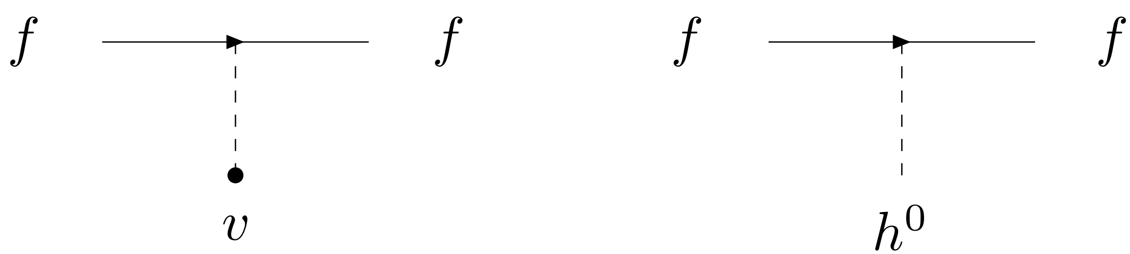
Under the electroweak gauge group, the right and left-handed components of each fermion has different SU(2)×U(1)$_{\rm Y}$ quantum numbers:

fermions	SU(2)	U(1)$_{\rm Y}$
$(\nu\,,\,e^-)_L$	2	-1
e^-_R	1	-2
$(u\,,\,d)_L$	2	$1/3$
u_R	1	$4/3$
d_R	1	$-2/3$

where the electric charge is related to the U(1)$_{\rm Y}$ hypercharge by $Q = T_3 + \frac{1}{2}Y$.

Before electroweak symmetry breaking, Standard Model fermions are massless, since the fermion mass term $\mathcal{L}_m = -m(\bar{f}_R f_L + \bar{f}_L f_R)$ is not gauge invariant.

The generation of masses for quarks and leptons is especially elegant in the SM. The fermions couple to the Higgs field through the gauge invariant Yukawa couplings (see below). The quarks and charged leptons acquire mass when Φ^0 acquires a vacuum expectation value:



Thus, $g_{hf\bar{f}} = m_f/v$, *i.e.*, Higgs couplings to fermions are proportional to the corresponding fermion mass.

It is remarkable that the neutral Higgs boson coupling to fermions is flavor-diagonal. This is a consequence of the Higgs-fermion Yukawa couplings:

$$\mathcal{L}_{\rm Yukawa} = -h_u^{ij}(\bar{u}_R^i u_L^j \Phi^0 - \bar{u}_R^i d_L^j \Phi^+) - h_d^{ij}(\bar{d}_R^i d_L^j \Phi^{0\,*} + \bar{d}_R^i u_L^j \Phi^-) + {\rm h.c.}\,,$$

where i, j are generation labels and h_u and h_d are arbitrary complex 3×3 matrices. Writing $\Phi^0 = (v + h^0)/\sqrt{2}$, we identify the quark mass matrices,

$$M_u^{ij} \equiv h_u^{ij} \frac{v}{\sqrt{2}}\,, \qquad\qquad M_d^{ij} \equiv h_d^{ij} \frac{v}{\sqrt{2}}\,.$$

One is free to redefine the quark fields:

$$u_L \to V_L^U u_L\,, \qquad u_R \to V_R^U u_R\,, \qquad d_L \to V_L^D d_L\,, \qquad d_R \to V_R^D d_R\,,$$

where V_L^U, V_R^U, V_L^D, and V_R^D are unitary matrices chosen such that

$$V_R^{U\,\dagger} M_u V_L^U = \mathrm{diag}(m_u\,,\, m_c\,,\, m_t)\,, \qquad V_R^{D\,\dagger} M_d V_L^D = \mathrm{diag}(m_d\,,\, m_s\,,\, m_b)\,,$$

such that the m_i are the positive quark masses (this is the $singular\ value\ decomposition$ of linear algebra).

Having diagonalized the quark mass matrices, the neutral Higgs Yukawa couplings are automatically flavor-diagonal.* Hence the SM possesses no flavor-changing neutral currents (FCNCs) mediated by neutral Higgs boson (or gauge boson) exchange at tree-level.

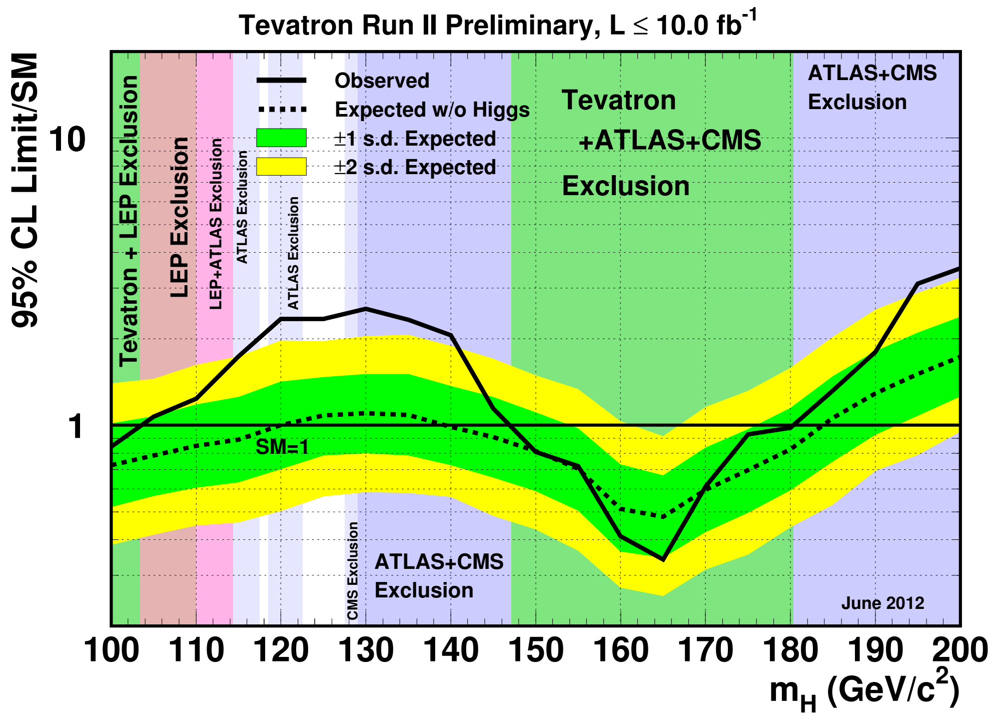
*Independently of the Higgs sector, the quark couplings to Z and γ are automatically flavor diagonal. Flavor dependence only enters the quark couplings to the $W^\pm$ via the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $K \equiv V_L^{U\,\dagger} V_L^D$.

Expectations for the SM Higgs mass

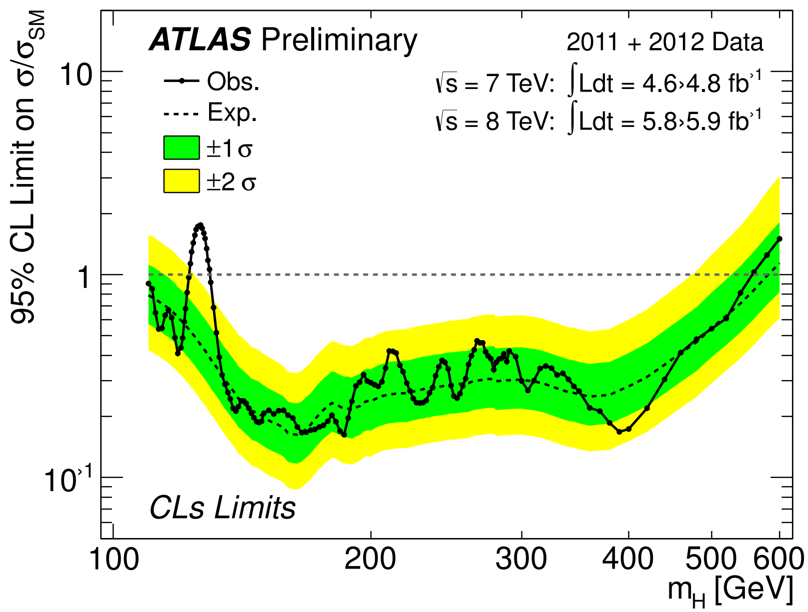
1. Higgs mass bounds from searches at LEP, the Tevatron and LHC.

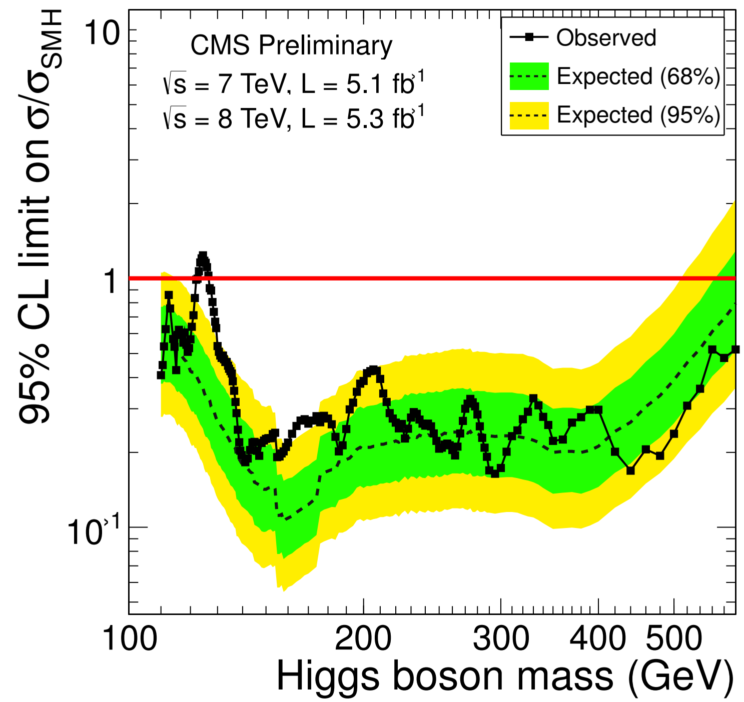
From 1989–2000, experiments at LEP searched for $e^+e^- \to Z \to h^0 Z$ (where one of the Z-bosons is on-shell and one is off-shell). A bound was obtained on the SM Higgs mass, $m_h > 114.4$ GeVat 95% CL.

Tevatron data extended the Higgs mass exclusion region to $147 \text{ GeV} < m_h < 180 \text{ GeV}$ at 95% CL.



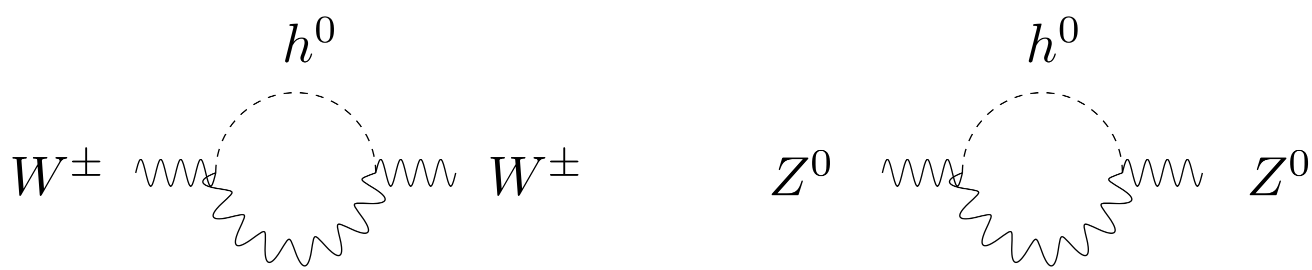
ATLAS and CMS extend the 95% CL exclusion regions further. For example, the CMS excluded mass regions are 110 GeV$< m_h <$122.5 GeV and 127 GeV $< m_h \lesssim 600$ GeV.





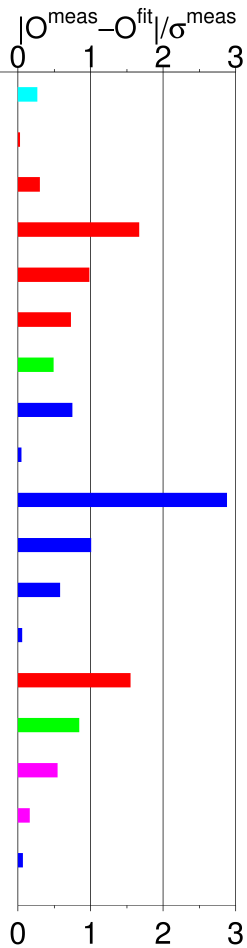
2. Consequences of precision electroweak data.

Very precise tests of the Standard Model are possible given the large sample of electroweak data from LEP, SLC and the Tevatron. Although the Higgs boson mass (m_h) is unknown, electroweak observables are sensitive to m_h through quantum corrections. For example, the W and Z masses are shifted slightly due to:



The m_h dependence of the above radiative corrections is logarithmic. Nevertheless, a global fit of many electroweak observables can determine the preferred value of m_h (assuming that the Standard Model is the correct description of the data).

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^0 [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26



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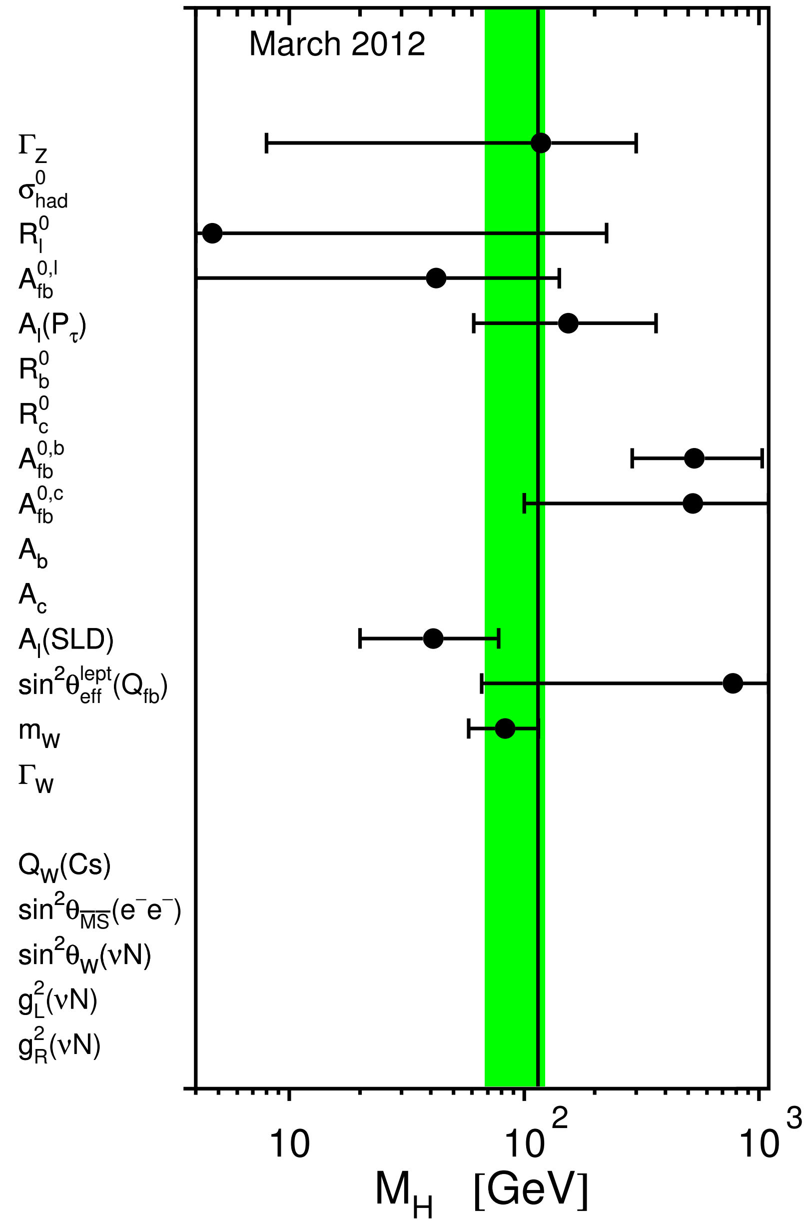
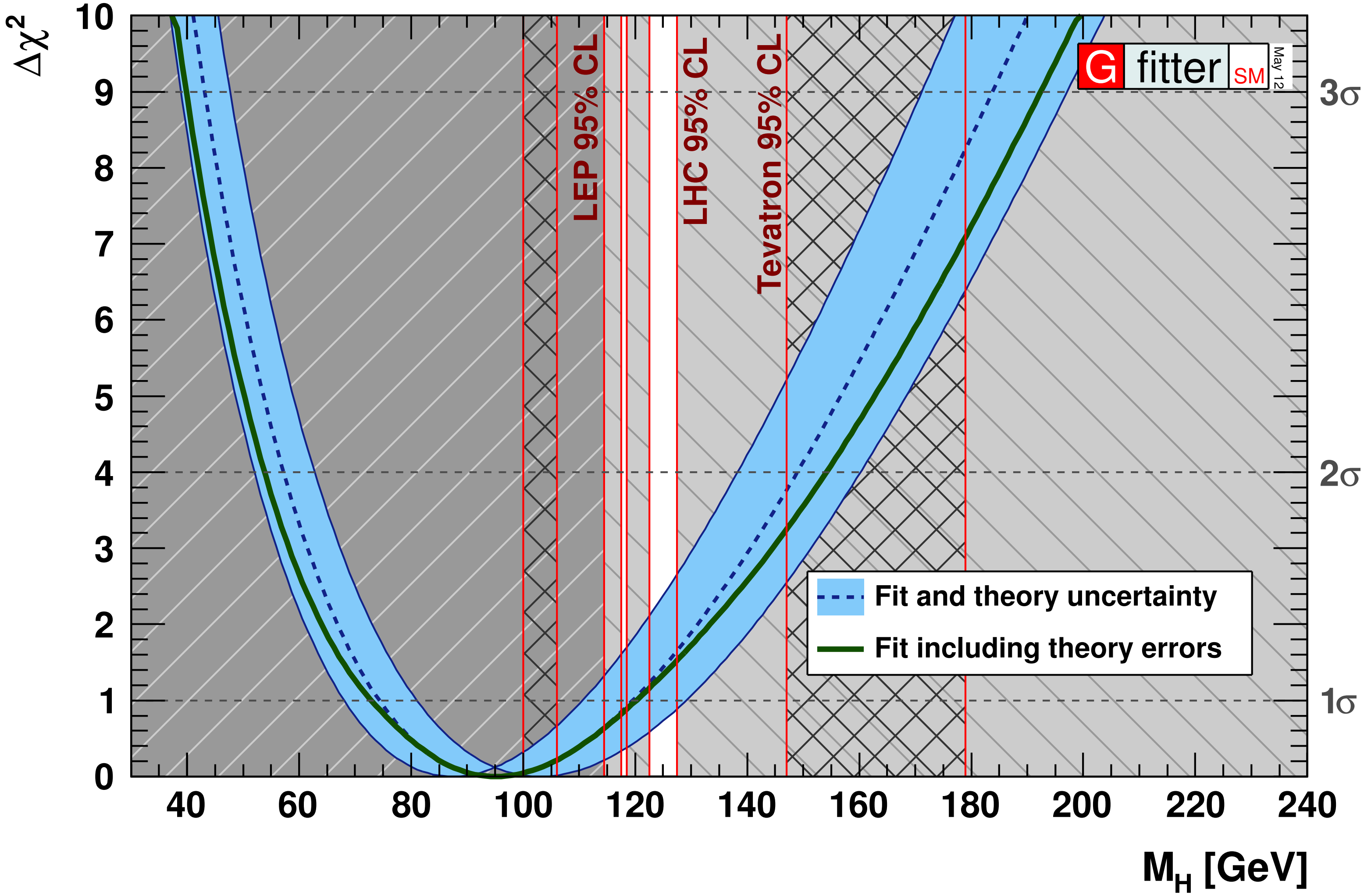
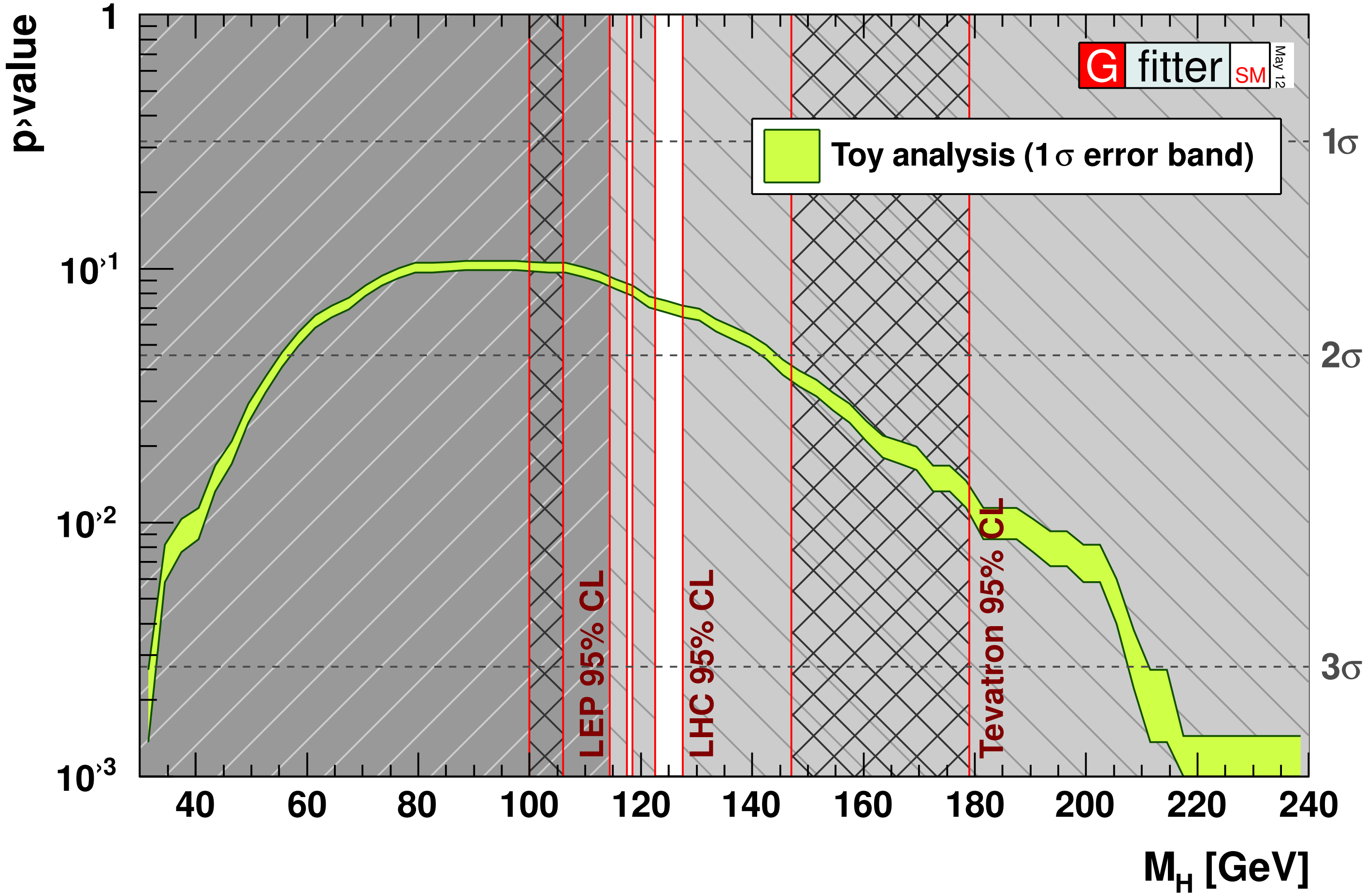


from the LEP, Tevatron and SLD Electroweak Working Groups



The blue band, which does *not* employ the direct Higgs search limits, corresponds to a upper bound of $m_h < 153$ GeV at 95% CL. A similar result of the LEP Electroweak Working group quotes $m_h < 152$ GeV at 95% CL.



The GFITTER collaboration evaluates the p-value of the global SM fit using a toy MC simulation with 10000 experiments, These are generated using as true values for the SM parameters the outcomes of the global fit. For each toy simulation, the central values of all of the observables used in the fit are generated according to Gaussian distributions around their expected SM values (given the parameter settings) with standard deviations equal to the full experimental errors taking into account all correlations. Fair agreement is observed between the empirical toy MC distribution and the χ^2 function expected for Gaussian observables.

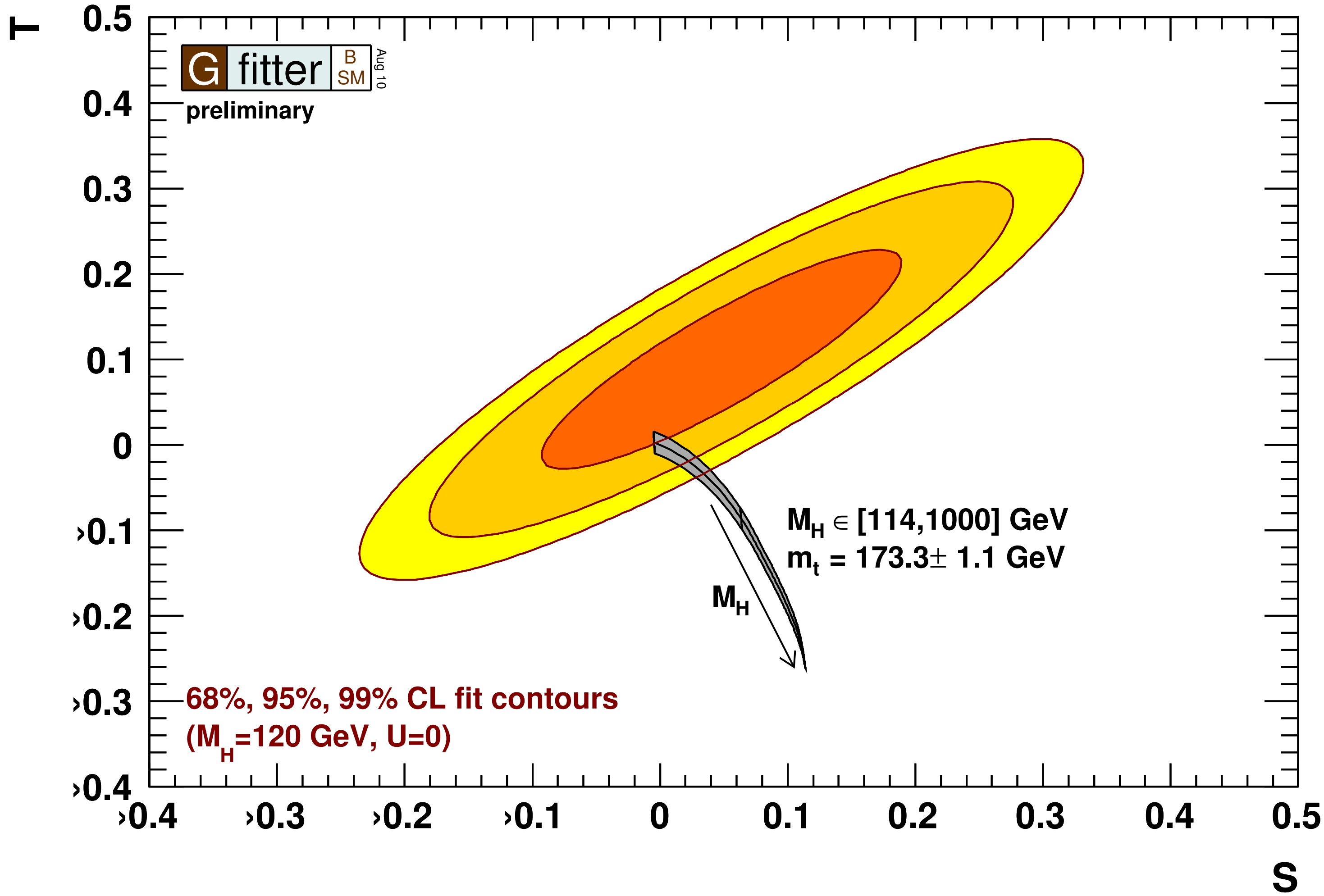
Can a Light Higgs Boson be avoided?

If new physics beyond the Standard Model (SM) exists, it almost certainly couples to W and Z bosons. Then, there will be additional shifts in the W and Z mass due to the appearance of new particles in loops. In many cases, these effects can be parameterized in terms of two quantities, S and T [Peskin and Takeuchi]:

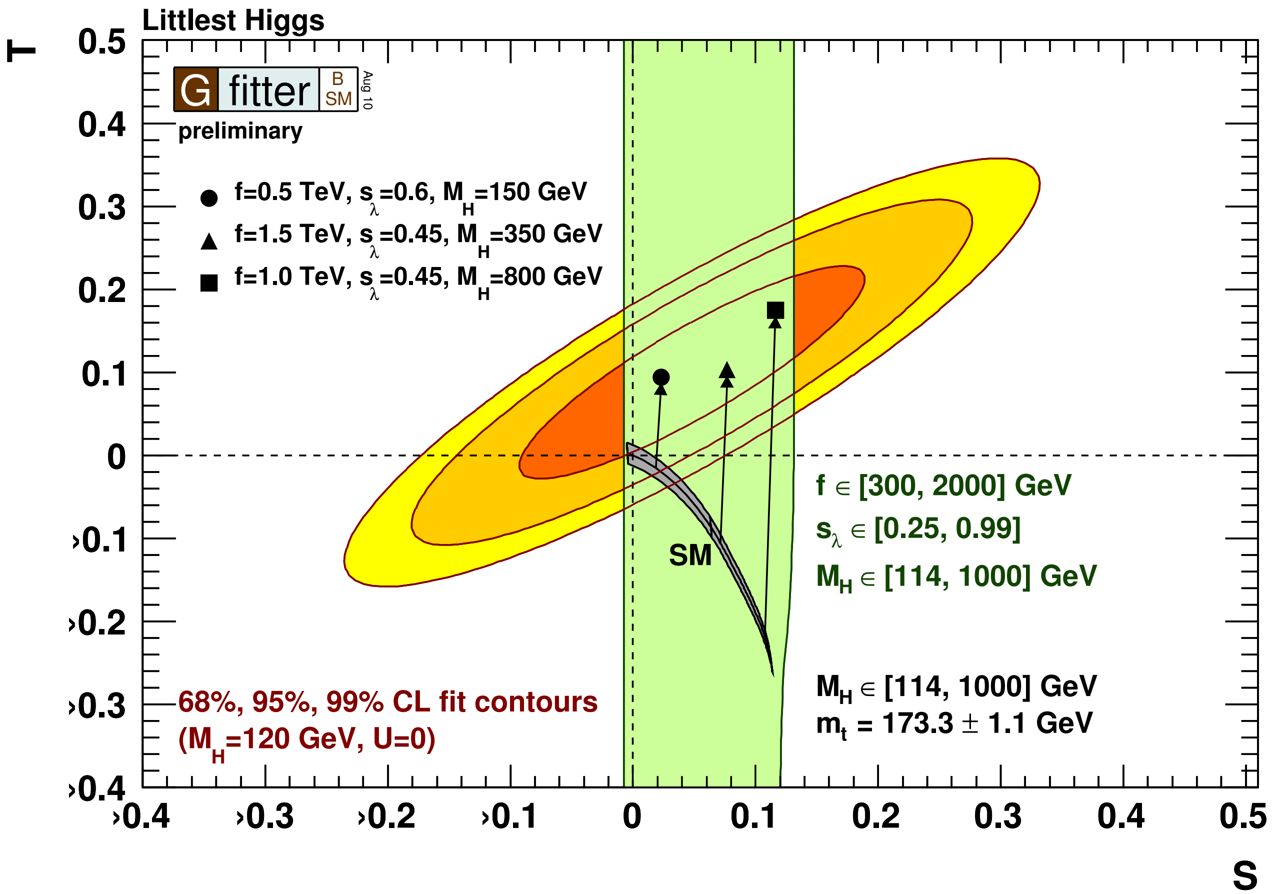
$$\overline{\alpha}\,T \equiv \frac{\Pi^{\rm new}_{WW}(0)}{m_W^2} - \frac{\Pi^{\rm new}_{ZZ}(0)}{m_Z^2}\,,$$

$$\frac{\overline{\alpha}}{4\overline{s}^2_Z\overline{c}^2_Z}\,S \equiv \frac{\Pi^{\rm new}_{ZZ}(m_Z^2)-\Pi^{\rm new}_{ZZ}(0)}{m_Z^2} - \left(\frac{\overline{c}^2_Z-\overline{s}^2_Z}{\overline{c}_Z\overline{s}_Z}\right)\frac{\Pi^{\rm new}_{Z\gamma}(m_Z^2)}{m_Z^2} - \frac{\Pi^{\rm new}_{\gamma\gamma}(m_Z^2)}{m_Z^2}\,,$$

where $s \equiv \sin\theta_W$, $c \equiv \cos\theta_W$, and barred quantities are defined in the $\overline{\rm MS}$ scheme evaluated at m_Z. The $\Pi^{\rm new}_{V_aV_b}$ are the new physics contributions to the one-loop V_a—V_b vacuum polarization functions.



In order to avoid the conclusion of a light Higgs boson, new physics beyond the SM must be accompanied by a variety of new phenomena at an energy scale between 100 GeV and 1 TeV.



This new physics will be detected at future colliders

- either through direct observation of new physics beyond the SM
- or by improved precision measurements that can detect small deviations from SM predictions.

Although the precision electroweak data is suggestive of a weakly-coupled Higgs sector, one cannot definitively rule out another source of EWSB dynamics (although the measured S and T impose strong constraints on alternative approaches).

In alternative models of EWSB, there may be a scalar state with the properties of the Higgs boson that is significantly heavier. Unitarity of $W_L^+ W_L^-$ scattering (which is violated in the SM in the absence of a Higgs boson) can be restored either by new physics beyond the Standard Model (e.g., the techni-rho of technicolor or Kaluza-Klein states of extra-dimensional models) or by the heavier Higgs boson itself. Suppose we assume the latter. How heavy can this Higgs boson be?

Can the Higgs Boson mass be large?

A Higgs boson with a mass greater than 200 GeV requires additional new physics beyond the Standard Model. A SM-like Higgs boson with mass above 600 GeV is not yet excluded by LHC data. But, how heavy can this Higgs boson be?

Let us return to the unitarity argument. Consider the scattering process $W_L^+(p_1)W_L^-(p_2) \to W_L^+(p_3)W_L^-(p_4)$ at center-of-mass energies $\sqrt{s} \gg m_W$. Each contribution to the tree-level amplitude is proportional to

$$[\varepsilon_L(p_1)\cdot\varepsilon_L(p_2)]\,[\varepsilon_L(p_3)\cdot\varepsilon_L(p_4)] \sim \frac{s^2}{m_W^4}\,,$$

after using the fact that the helicity-zero polarization vector at high energies behaves as $\varepsilon_L^\mu(p) \sim p^\mu/m_W$. Due to the magic of gauge invariance and the presence of Higgs-exchange contributions, the bad high-energy behavior is removed, and one finds for $s,\, m_h^2 \gg m_W^2$:

$$\mathcal{M} = -\sqrt{2}G_F m_H^2\left(\frac{s}{s-m_h^2}+\frac{t}{t-m_h^2}\right)\,.$$

Projecting out the $J = 0$ partial wave and taking $s \gg m_h^2$,

$$\mathcal{M}^{J=0} = -\frac{G_F m_h^2}{4\pi\sqrt{2}}\,.$$

Imposing $|\mathrm{Re}\ \mathcal{M}^J| \leq \frac{1}{2}$ yields an upper bound on m_h. The most stringent bound is obtained by all considering other possible final states such as $Z_L Z_L$, $Z_L h^0$ and $h^0 h^0$. The end result is:

$$\boxed{m_h^2 \leq \frac{4\pi\sqrt{2}}{3G_F} \simeq (700\ \mathrm{GeV})^2\,.}$$

However, in contrast to our previous analysis of the unitarity bound, the above computation relies on the validity of a tree-level computation. That is, we are implicitly assuming that perturbation theory is valid. If $m_h \gtrsim 700$ GeV, then the Higgs-self coupling parameter, $\lambda = 2m_h^2/v^2$ is becoming large and our perturbative analysis is becoming suspect.

Nevertheless, lattice studies suggest that an upper Higgs mass bound below 1 TeV remains valid even in the strong Higgs self-coupling regime.